Ch-1: Electric Charges and Fields

Property of charges:

- Quantization of charge, where n=integer multiple and e =-1.6×10^{-19} C

$$q=\pm ne$$

- Additivity of charges,

$$Q_{net} = \sum_{i=1}^{n} q_i$$

- Conservation of charges

$$Q_{net}\ (isolated) = Constant.$$

Coulomb's Force

- The below formula can be used when force on multiple individual charge is being calculated.

$$\vec{F_j} = \sum \vec{F}_{ij} = \frac{q_j}{4\pi\varepsilon}\sum_{i=1}^{n}\frac{q_i(\overrightarrow{r_{ij}})}{|\overrightarrow{r_{ij}}|^3}$$

- Coulomb's force between charge q_1 and q_2 separated by distance r and placed in vacuum, can also be expressed in vector form as-

$$\vec{F} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1q_2}{r^2}\grave{r}$$

- In magnitude form, Coulomb force can be written as -

$$F = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1q_2}{r^2} \text{ i. e. } F\propto \frac{1}{r^2} \text{ and } F \propto q_1q_2$$

- Coulomb's force in vector form shows that it obeys Newton's third law of motion as-

$$\overrightarrow{F_{12}} = -\overrightarrow{F_{21}}$$

And we read this as ''force on charge q_2 due to charge q_1 = negative of force on charge q_1 due to q_2''.

$$\vec{F_j} = \frac{q_j}{4\pi\varepsilon}\sum_{i=1}^{n}\frac{q_i(\vec{r}-\vec{r_i})}{|\vec{r}-\vec{r_i}|^3}$$

The above formula can be used when force on any one charge is being calculated.

We read as force on charge j having position r due to charges q_1, q_2, q_3, ----having position r_1, r_2, r_3---- placed in a medium. This is also called ''Principle of Superposition'' for force determination due to several charges.

Electric Field Intensity

- Electric field intensity E can be defined as $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$

Where q0 is the test charge and is so small and not to disturb the strength of field produced by Source or point charge

- Electric field intensity at a point whose position is r due to other charges whose positions are r_1, r_2, r_3, -------can be expressed as- (Principle of Superposition for Electric field intensity)

$$\overrightarrow{E_J} = \frac{1}{4\pi\varepsilon} \sum_{i=1}^{n} \frac{q_i(\vec{r} - \vec{r_i})}{|\vec{r} - \vec{r_i}|^3}$$

- Electric field intensity E at a point placed at distance r due to point or source charge q kept in free space is given by in vector form as-

$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{q}{r^2}\, \grave{r}$$

- In magnitude form, Electric field intensity at any point in the region of electric field is given by-

$$E = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{q}{r^2} \text{ i.e. } E \propto \frac{1}{r^2} \text{ and } E \propto q$$

Dipole

- To find the net electric field intensity at Axial Point (A) at distance x from dipole centre
  $\overrightarrow{E_A} = \frac{1}{4\pi\varepsilon} \frac{4aqx\acute{\imath}}{[x^2 - a^2]^2}$
- Dipole moment (p) and its direction from –q to +q separated by 2a and placed in X-direction is given by –

$$\vec{p} = 2aq\grave{\imath}$$

- The net electric field intensity at Axial point If x>>a is given by-

$$\overrightarrow{E_A} = \frac{1}{4\pi\varepsilon} \frac{2\vec{p}}{x^3}$$

- In unit vector net field intensity in the direction X-axis at axial position is-

$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{2p\grave{\imath}}{x^3}$$

$In\ m$agnitude form,

$$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2p}{x^3} \text{ This shows that } E \propto \frac{1}{x^3}$$

- To find the electric field intensity when dipole makes an angle θ with the horizontal x-axis

$$\overrightarrow{E_A} = \frac{1}{4\pi\varepsilon}\frac{4aqx(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})}{[x^2 - a^2]^2}$$

- To find the net field intensity at the equatorial point (B) of the dipole at distance x from the centre of dipole

$$\overrightarrow{E_B} = \frac{1}{4\pi\varepsilon}\frac{2aq(-\hat{\imath})}{[x^2 + a^2]^{\frac{3}{2}}}$$

- To find the field intensity at the equatorial point (B) of the dipole at distance x from the centre of dipole and when x>>a

$$\overrightarrow{E_B} = \frac{1}{4\pi\varepsilon}\frac{-\vec{p}}{x^3}$$

- In magnitude form,

$$\overrightarrow{|E_B|} = \frac{1}{4\pi\varepsilon}\frac{\overrightarrow{|p|}}{x^3}$$

- The ratio of net field intensity at axial point to the equatorial point is given by-

$$\frac{E_{axial}}{E_{equatorial}} = 2:1$$

- Torque in a dipole

$$\vec{\tau} = \vec{p}x\vec{E}$$

  Where $\vec{p} = p_x\hat{\imath} + p_y\hat{\jmath} = p\cos\theta\,\hat{\imath} + p\sin\theta\,\hat{\jmath}\ and\ \vec{E} = E\hat{\imath}$

  Therefore, $\vec{\tau} = pE\sin\theta\left(-\hat{k}\right)$ that is, the direction of torque is both dipole moment and electric field and pointing downward (going into the plane of paper) in clockwise acting from p to E at an angle θ between p and E.

- In magnitude form, Net torque

$$\tau = pE\sin\theta$$

- Potential energy in rotating a dipole at an angle θ is given by

$$U = -\vec{p}.\vec{E}$$

- In magnitude form, Potential in the dipole is

$$U = -pE\cos\theta$$

  The dipole is in stable equilibrium when $\theta=0^0$, U is minimum and equals to – pE. The dipole is in unstable equilibrium, when $\theta=180^0$, U is maximum and equals to +pE.

  In both stable and unstable equilibrium, $\tau_{net} = 0\ and\ F_{net} = 0$

Long straight charged conductor (infinite)

- Electric field intensity at distance r on x axis from the long charged conductor (infinite) having linear charge density λ is given by

$$E=\frac{\lambda}{2\pi\varepsilon_0 r}$$

i.e. $E \propto \frac{1}{r}$ and the graph between E vs. r is rectangular hyperbola.

Field due to a uniform ring

- Electric field intensity due to a ring having radius R and linear charge density λ, at a distance x from the centre of the ring on axial position is given by-

$$E_x = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{qx}{\left((x^2+R^2)^{\frac{3}{2}}\right)}$$

And we see that $E_x \propto \frac{1}{x^2}$

- When x=0, field is zero at the centre of the ring.
- $E_x = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$ For x>>R when point is much farther from the ring, its field is same as that of a point charge.

It will be maximum when $\frac{dE_x}{dx}$=0 and $(E_x)_{max} = \frac{2}{\sqrt[3]{3}}\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{R^2}$

Electric Flux

- Total electric flux through a surface

$$d\varphi_E = \vec{E}.\overrightarrow{dS} \text{ or, } \varphi_E = \int \vec{E}.\overrightarrow{dS}$$

For uniform field, $\varphi_E = \vec{E}.\vec{S}$

Here, $\overrightarrow{dS}$ is a small area element vector perpendicular (normal) to a plane surface and pointing outward.

- In magnitude form,

$\varphi_E = ES\ \cos\theta$ where θ is the angle between Electric field $\vec{E}$ and $\overrightarrow{dS}$.

If any square or rectangular slab placed in co-ordinate axes , the field is uniform, the angle θ value is given by according to position of plane of the slab as mentioned below-

- If the plane is parallel to the electric field, θ=90^0
- If the plane is perpendicular to the field, θ=0^0
- If the plane makes an angle say 30^0, 60^0 etc, θ=90^0 – 30^0 or 60^0

Gauss's Law

$$\varphi_E = \oint \vec{E}.\overrightarrow{dS} = \frac{q_{in}}{\varepsilon_0}$$

Electric field intensity due to infinite plane sheet (charge on one side of sheet) is given by-

$$E = \frac{\sigma}{2\varepsilon_0}$$

Electric field intensity due to a charged conductor –charged on both sides (near the conductor)

$$E = \frac{\sigma}{\varepsilon_0}$$

$$E_{in} = 0$$

Electric field intensity in between and outside two long parallel conductors having surface charge densities ±σ is as-

(i) Outside parallel conductor, E=0

(ii) Between parallel conductor, $E = \frac{\sigma}{\varepsilon_0}$

Electric field intensity in between and outside two long parallel conductors having surface charge densities +σ on each conductor is as-

(i) Outside parallel conductor, $E = \frac{\sigma}{\varepsilon_0}$

(ii) Between parallel conductor, E=0

Electric field intensity due to thin spherical shell or hollow shell having charge +q on its surface (surface charge density σ) and radius R.

(i) Inside the shell (r<R) E=0 as $q_{in} = 0$

(ii) Outside the shell (r>R), $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

(iii) On the surface of shell, r=R, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

- Electric Potential V =constant (inside the shell)
- Electric potential V = $\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ (on the surface)
- Electric Potential V = $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ (outside the sphere)

Electric field intensity due to solid spherical shell having charge +q distributed inside sphere (volume charge density ρ) and radius R

(i) Inside the shell (r<R) $E = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}$

(ii) Outside the shell (r>R), $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

(iii) On the surface of shell, r=R, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

For solid sphere,

- Electric potential V =$= \frac{1}{4\pi\varepsilon_0}\frac{q}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right]$ inside the solid sphere
- At the centre, r=0 and $V_c = \frac{3}{2}V_s$

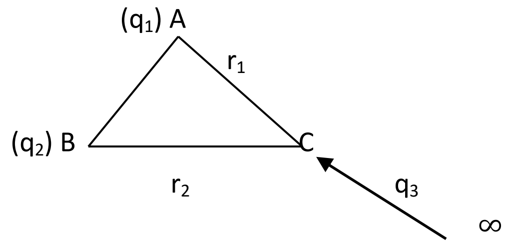
(To find the electric potential, apply the formula dV=- Edr)

Ch-2: Electrostatic Potential and Capacitance

1. Relation between work done and potential energy:
   $W_{i\to f} = U_f - U_i$: ΔU, Against the electric force or by external force (external agent)
   $W_{i\to f} = U_i - U_f$: $-\Delta U$, By the electric force
   [since, Work done = - $\Delta U = -(U_f - U_i)$]
2. Potential energy of two point charge system (if charges q1 and q2 are separated by distance r)
   $q_1 \bullet \longleftarrow r \longrightarrow \bullet q_2$
   $$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$
3. Potential energy of system of charges (due to multiple charges) say, charges kept at vertices of triangle or square or rectangle or at any geometrical shape.
   $$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}^{n}\frac{q_j q_i}{|\overrightarrow{r_{ji}}|}$$
   For example, in a triangle, if charges q_1, q_2 and q_3 are placed at vertex A, B and C respectively of a triangle ABC. The distance is r_{32} between charges q_3 and q_2, distance r_{31} between charges q_3 and q_1, distance r_{21} between charges q_2 and q_1, then
   $$U= \frac{1}{4\pi\epsilon_0}\left[\frac{q_3 q_2}{r_{32}} + \frac{q_3 q_1}{r_{31}} + \frac{q_2 q_1}{r_{21}}\right]$$
4. If potential energy (U) of system of charges is negative, work done will be positive and if potential energy is positive, work done will be negative. [By conservative force]
   That is, if work done decreases (example, when charge is brought from any point or from infinity (initial point) to a point (final point) by electric force), potential energy increases and vice-versa.
5. Relation between electric potential and potential energy
   $$V = \frac{U}{q_0}$$
   A. If a test charge q_0 is brought from a point to another point against the electric force or by external force or by electric force.
      $$V_i - V_f = \frac{U_i - U_f}{q_0} = W_{i\to f} \text{ By Electric force}$$
      $$V_f - V_i = \frac{U_f - U_i}{q_0} = W_{i\to f} \text{ By external agent or force}$$
   B. Similarly, if initial point is at ∞ and final point is say f so work done to bring a charge (say q_0) from infinity to a point (say f) by external force or against electric force is given by-
      $$V_f - V_\infty = \frac{U_f - U_\infty}{q_0} = W_{\infty\to f}$$
   [Here V_∞ and U_∞ are zero as seen that $U \propto \frac{1}{r}$ and $\propto \frac{1}{r}$]
6. Work done to bring a charge say q_3 if two point charges q_1 (at point A) and q_2(at point B) kept in free space. A third point charge q_3 is moved from infinity to a point (say C , then formula of work done shall be applied as-
   $W_{\infty\to C} = q_3(V_C - V_\infty) = q_3 V_C$, where $V_\infty = 0$

Given r_1 is the distance from point C to charge q_1 at point A, r_2 is the distance of point C to charge q_2 at point B as shown in the below figure of triangle ABC. Where V_C is

$V_C = V_{CA} + V_{CB} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$,



7. Relation between electric potential and electric field intensity

$$dV = -\vec{E}.\overrightarrow{dr}$$

Or,

$$\int_{V_B}^{V_A} dV = -\int_{r_B}^{r_A} \vec{E}.\overrightarrow{dr}$$

8. To find electric potential difference say V_{AB} or $V_A - V_B$ in a uniform electric field

$$V_A - V_B = \vec{E}.(\overrightarrow{r_B} - \overrightarrow{r_A})$$

So,

$$V_B - V_A = \vec{E}.(\vec{r}_A - \vec{r}_B)$$

9. Electric potential at a point (say point P) due to a point or source charge q placed at distance r is given by

q• ⟵ r ⟶ P

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r}\right)$$

10. Electric potential due to a point or source charge kept at position vector r_0 at a point whose position vector r is given by

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{|\vec{r} - \overrightarrow{r_0}|}\right)$$

11. Electric potential at a point whose position vector is r due to system of charges q1, q2, q3..... Kept at positions r1, r2, r3 ....respectively is given by

$$V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{|\vec{r} - \vec{r_i}|}$$

12. Electric potential due to a system of charges at point is given by

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q_{net}}{r}\right)$$

13. Electric potential V is also written as

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r}$$

14. Electric potential in a dipole at any point p, at distance r from the centre of dipole, making angle θ from the dipole axis is given by-

$$V_p = \frac{1}{4\pi\epsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

A. when θ=0^0, (axial position of dipole)

$$V_p = \frac{1}{4\pi\epsilon_0}\left(\frac{p}{r^2}\right)$$

B. When θ=90^0, (at the equatorial point), V=0

15. Electric potential in case of hollow spherical conductor,

    A. Inside the hollow spherical conductor (r<R) where R is the radius of conductor and q is the charge distributed on the surface of conductor

    $$V_{inside} = V_{surface} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{R}\right)$$

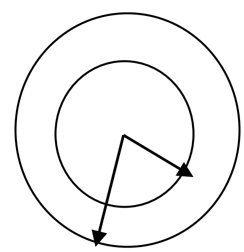
    B. Outside at any point at distance r from the centre of the spherical shell

    $$V_r = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r}\right)$$

16. Electric potential in case of two metallic hollow concentric shells:
    If the inner shell A has radius R_1 and charge distributed is q_1 and has outer shell B has radius R_2 and charge distributed is q_2, then potential V_A and V_B are given as

$$V_A = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{R_1} + \frac{q_2}{R_2}\right)$$

$$V_B = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{R_2} + \frac{q_2}{R_2}\right)$$



Thus, potential difference is given as

$$V_A - V_B = \frac{1}{4\pi\epsilon_0} q\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Capacitance:

17. Capacitance C is given if potential difference is V and q the charge on a conductor, as
    Q = CV
18. Capacitance C of a metallic spherical conductor of radius R and charge q on its surface is given by

    $$C = 4\pi\epsilon_0 R$$

19. Parallel Plate Capacitor:

    A. Capacitance C of a parallel plate capacitor having area A with charge q and –q on each plate placed in medium K (K is a dielectric constant) and separation between the plates d is given by-

    $$C = \frac{\epsilon_0 KA}{d}$$

    B. Instead of two plates if there are n similar plates of equal separation from each other and alternate plates are connected together, the capacitance C is given by-

$$C = (n-1)\frac{\epsilon_0 A}{d}$$

C. The capacitance C of a parallel plate capacitance of separation d and area A if a dielectric slab K of thickness t with same area is placed in between it, is given by

$$C = \frac{\epsilon_0 A}{d-t+\frac{t}{K}} \text{ --------------- (1)}$$

Different cases-

i. If more than one dielectric slabs are placed between the parallel plates capacitor, the capacitance C is given as-

$$C = \frac{\epsilon_0 A}{(d - t_1 - t_2 - \cdots..) + (\frac{t_1}{K_1} + \frac{t_2}{K_2} + \cdots.)}$$

ii. If the space between the plates is completely filled by the slab of dielectric constant K, then t=d and hence from equation (1) will be as –

$$C = \frac{\epsilon_0 K A}{d}$$

iii. If the space between the plates is completely filled by the conducting slab instead of di-electric slab of dielectric constant K, then d=t and K =∞ for conductor and hence C =∞

iv. If the di-electric slab K is replaced by the conducting slab of same thickness, then K=∞ and Capacitance C is given by –

$$C = \frac{\epsilon_0 A}{d-t}$$

v. Suppose in two parallel plates a and b, If the plate b is earthed and charge q is given to plate a, then whole charge is transferred to earth as the capacity of earth is very large and capacitance of capacitor becomes infinite.

20. Energy stored in capacitor is given by

$$U = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{1}{2}\frac{q^2}{C}$$

21. Energy density: The potential energy of a charged conductor or a capacitor is stored in the electric field (E). The energy per unit volume is called the energy density (u). Energy density in a dielectric medium(dielectric constant K) is given by

$$u = \frac{1}{2}(\epsilon_0 K E^2)$$

The total stored energy in an electrostatic field (E field between the plates of capacitor is uniform) given by

$$U = u(total\ volume)$$

22. Electric field intensity on the –q plate due to positive charge in case of two parallel pales is given by

$$E = \frac{q}{2A\epsilon_0}$$

23. Force of attraction with which both parallel plates (charge q and –q and area A with separation d) attract each other is given by

$$F = \frac{q^2}{2A\epsilon_0}$$

24. Work done by the external force if one of parallel plates is moving from fixed plate to a distance d1 from the earlier arrangement when distance was d, is given by

$$W = F(d_1 - d) \; or \; W_i \rightarrow f = U_f - U_i$$

Here, $F = \frac{q^2}{2A\epsilon_0}$

$$U_f = \frac{1}{2}\left(\frac{q^2}{C_f}\right) \; and \; U_i = \frac{1}{2}\left(\frac{q^2}{C_i}\right)$$

Here,

$$C_f = \frac{\epsilon_0 A}{d_1} \; and \; C_i = \frac{\epsilon_0 A}{d}$$

25. Electric field intensity (E) in the dielectric (dielectric constant K) inserted between parallel plates is given by

$$E = \frac{\sigma}{\epsilon} = E_0 - E_i$$

In free space between the plates, External electric field (E0) is given by

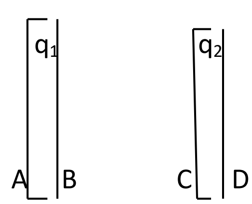
$$E_0 = \frac{\sigma}{\epsilon_0}$$

Where σ is the surface charge density and E_i is the induced electric field in the dielectric, so,

$$E = \frac{E_0}{K} or, E_0 - E_i = \frac{E_0}{K} \; or, K = \frac{E_0}{E_0 - E_i}$$

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right) \; so, q_i = q\left(1 - \frac{1}{K}\right)$$

26. If two parallel plates a and b given charge as q1 and q2 then to find charge at the point outer surface of a (point A), inner surface of a (point B), inner surface of b (point C) and outer surface of b (point D):



Charge on point A and D is given as, $q_A = q_D = \frac{q_1+q_2}{2}$

$$q_1 = Charge \; at \; point \; A + Charge \; at \; point \; B$$

Similarly q_2 = Charge at point C + Charge at point D

So, charge at point B, $q_B = \frac{q_1-q_2}{2}$

Similarly, Charge at point C , $q_C = -\left(\frac{q_1-q_2}{2}\right)$

27. Charge redistribution: When two conductors of charge q_1 and q_2 on its surfaces and having capacitances as C_1 and C_2 with radii as R_1 and R_2 are joined by a conducting wire, the charges on conductors after joining the wire are in the ratio of their capacities as given-

$$q_1' = C_1 V' \; and \; q_2' = C_2 V' \; where \; V' = Common \; potential = \frac{q_1+q_2}{C_1+C_2}$$

Since, $C \propto R$ $So, q_1' = (q_1 + q_2)\left(\frac{R_1}{R_1+R_2}\right)$

28. Loss of energy during redistribution of charge $\Delta U = U_i - U_f = \frac{C_1C_2}{2(C_1+C_2)}((V_1 - V_2)^2)$

    Where, $U_i = \frac{1}{2}\left(\frac{q_1^2}{C_1}\right) + \frac{1}{2}\left(\frac{q_2^2}{C_2}\right)$ and $U_f = \frac{1}{2}\left[\frac{(q_1+q_2)^2}{C_1+C_2}\right]$

29. Capacitors in series and parallel

    If C_1, C_2 capacitors are connected in -

    (i) Series (equal charge on and unequal voltage across each capacitor)

    $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \ldots ..$$

    (ii) Parallel (unequal charge on and equal voltage across each capacitor)

    $$C = C_1 + C_2 + \cdots \ldots \ldots$$

30. Law of Capacitors-

a) Law of conservation of charge

    (i) In case of a battery, both terminals of battery supply an equal amount of charge

    (ii) Net charge on plates of capacitors which are not connected to either battery or earth is zero.

b) Sum of potential rise and potential drop = 0 or, Potential rise = Potential drop in a loop

    Potential rise is taken as positive (from –ve to +ve) and potential drop is taken as –ve (from +ve to -ve)

31. About di-electric slabs placed in two parallel plates:

    (i) When more than two di-electric slabs with different di-electric constant divide the separation between the plates d as d/2 or d/3 or to any fraction of d, while area A is fixed, those two or more di-electric slabs are in Series and accordingly equivalent capacitance can be calculated.

    (ii) When slabs divide the area while separation between the plates is fixed, those slabs are in parallel and accordingly equivalent capacitance can be calculated.

    (iii) When slabs divide in such a way that any one divides the area and other two slabs divide the separation or vice-versa, the dielectric slabs will be in series and parallel combination both and accordingly equivalent capacitance can be calculated.

    Those arrangements are shown here in Fig. -1, Fig-2 and Fig-3 respectively of what mentioned above in point (i), (ii) and (iii) respectively.

    The formula for capacitance is – $C = \frac{\epsilon_0 KA}{d}$

    Here d and A are area and separation between the plates and can be changed as per question.

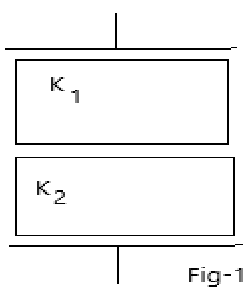


Fig-1

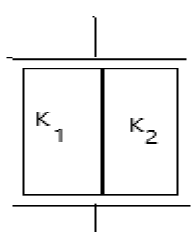


Fig-2

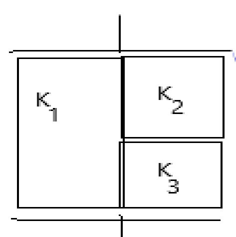


Fig-3

32. Relation between electrical susceptibility and polarisation
Polarisation is defined as dipole moment per unit volume and its magnitude is referred to as polarisation density. Electrical susceptibility is given by

$$\chi = \frac{\vec{p}}{\epsilon_0 \vec{E}}$$

33. Relation between dielectric constant (K) and electrical susceptibility (χ)

$$K = 1 + \chi$$

34. Polarisation density (P) is given by

$$P = \frac{q_i t}{At} = \frac{dipole\ moment\ in\ the\ slab}{volume} = \frac{q_i}{A} = \sigma_i$$

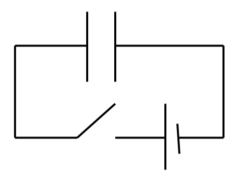
Where A is the area of dielectric slab, t the thickness of slab, q_i the induced charge in the dielectric slab.

35. Effect of dielectric on various parameters

A. Effect of dielectric when battery is kept disconnected from the capacitors

Let q_0, V_0, C_0, E_0 and U_0 be the charge, potential difference, capacitance, electric field Intensity and potential energy stored respectively before dielectric slab is inserted.
Then,

$$V_0 = \frac{q_0}{C_0}\ and\ V_0 = E_0 d\ and\ U_0 = \frac{1}{2} C_0 V_0^2$$



a. When supply is disconnected as shown in Fig. 2. and dielectric slab is inserted:

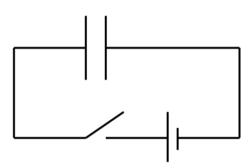
(i) Charge:

$$q = q_0$$

The charge on capacitor plate remains q0 as battery has been disconnected before the insertion of dielectric slab.

(ii) Electric field:

$$E = \frac{E_0}{K}$$



(iii) Potential difference:

$$V = Ed = \frac{E_0}{K} d = \frac{V_0}{K}$$

(iv) Capacitance: $C = \frac{q_0}{V} = \frac{q_0}{\frac{V_0}{K}} = C_0 K$

(v) Potential energy stored: $U = \frac{1}{2} CV^2 = \frac{U_0}{K}$

B. Effect of dielectric when battery remains connected across the capacitor

(i) Potential difference: $V = V_0$

(ii) Electric field intensity $E = E_0$

(iii) Capacitance: $C = C_0 K$

(iv) Charge: $q = CV = C_0 K V_0 = K q_0$

(v) Potential energy stored: $U = \frac{1}{2} CV^2 = K U_0$

Chapter-3: Current Electricity Formulae

1. Relation among charge (q) to flow through a conductor of uniform cross-sectional area, current (I) and time t is: $I = \frac{q}{t} = \frac{ne}{t}$

   For positive q, current is in forward direction and for negative q, it is in backward direction.

2. Relation between very small charge dq to flow for small time dt through a non uniform cross sectional area is given by $I = \frac{dq}{dt}$ or $q = \int_{t_1}^{t_2} Idt$

3. **Current density (J)** is the current (I) distributed uniformly over the cross section area (A) of the conductor and is given by $J = \frac{I}{A}$ or I = JA, also for non-uniform area of conductor, $I = \int JdA$; also, $I = \vec{J} \cdot \vec{A}$; It is a vector quantity.

4. **By Ohm's law**, $I \propto V \; or \; V \propto I \Rightarrow V = RI$; Or, $R = \frac{I}{A}$ = constant, provided that temperature, pressure remains constant.

   The value of R depends upon length, shape and nature of the material.

5. **Resistance (R) of a conductor**

   It depends upon the following factors as-

   (i) Resistance(R) is directly proportional to length (l) of conductor

   (ii) It is inversely proportional to the area (A) of the cross section of the conductor if the temperature remains constant, that is, $R \propto l, R \propto \frac{1}{A} \; So, R = \rho \frac{l}{A}$

   (iii) It depends upon temperature, (increases with temperature)

   (iv) It depends on type of materials, different for different materials

   Where ρ is the resistivity or specific resistance of the conductor and ρ depends upon nature of the material and temperature of the conductor.

6. **Effect of temperature on resistance**:

   Resistance of a metallic conductor at temperature t ^{0}C is given by $R_t = R_0(1 + \alpha t + \beta t^2)$

   Where α, β are the temperature coefficients of resistance

   If the temperature is not sufficiently large, as in practical cases, equation (1) can be written as- $R_t = R_0(1 + \alpha t) \; or \; \alpha = \frac{R_t - R_0}{R_0 t} = \frac{Change \; in \; resistance}{original \; resistance \; x \; rise \; in \; temperature}$

7. **Temperature dependence of resistivity**

   Resistivity of a metal conductor is given by $\rho_T = \rho_0[1 + \alpha(T - T_0)]$

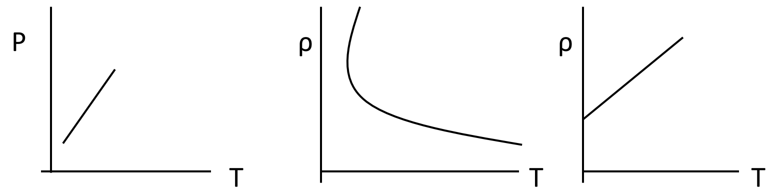
   Where, ρ is the resistivity at temperature T (in Kelvin), and resistivity at temperature T_0 is ρ_0.

   Or, $\alpha = \frac{\rho - \rho_0}{\rho_0(T - T_0)} = \frac{d\rho}{\rho_0}\left(\frac{1}{dT}\right)$

So, temperature coefficient of electrical resistivity is defined as fractional change in electrical resistivity per unit change in temperature.

For metals, α is positive, so, resistivity of metal increases with increase in temperature.

For semiconductor, resistivity decreases with decrease in temperature and for alloys, the resistivity is very large and weak dependence on temperature.



Metal Semiconductor Alloys

8. **Conductance (G) and conductivity (σ):**
   Conductance (G) is given by reciprocal of resistance and will be written as: $G = \frac{1}{R}$
   And conductivity (σ) is given by reciprocal of resistivity and will be written as: $\sigma = \frac{1}{\rho}$
9. **Drift velocity and relaxation time**
   The average drift velocity v_d can be written by $v_d = \frac{-eE\tau}{m}$

Here, negative sign shows that vd is opposite the direction of E. So, average drift velocity is given by

$$|v_d| = \frac{eE}{m}\tau$$

Average relaxation time is defined as mean free path of electron per drift speed of electron.

10. **Relation between average drift velocity, relaxation time and current is given by**

$$I = \frac{q}{t} = \frac{Ne}{\tau} = \frac{Ne}{\frac{l}{v_d}} = \frac{Nev_d}{l} = \frac{nev_d(Volume)}{l} = \frac{nev_d(Area\ x\ l)}{l} = neAv_d$$

Here, n number of electrons per unit volume or electron density or number density and N = number of electrons in a metal conductor of length l and cross section A, so volume =Al
Deduction of ohm's law from the expression of drift velocity

$$I = neAv_d = neA\frac{eE\tau}{m} = \frac{ne^2A\tau V}{ml} \Rightarrow V = \frac{ml}{ne^2A\tau}I \Rightarrow V = RI, \text{ where, E=V/l and,}$$

$$R = \frac{ml}{ne^2A\tau} = constant$$

11. **Relation between resistivity (ρ), number density (n) and relaxation time (τ)**

$$R = \rho\frac{l}{A} = \left(\frac{m}{ne^2\tau}\right)\frac{l}{A} \Rightarrow \rho = \frac{m}{ne^2\tau}, \text{ Therefore, } \rho = \frac{m}{ne^2\tau}$$

From equation (1), $\rho \propto \frac{1}{\tau}\ for\ metals\ and\ \rho \propto \frac{1}{n}\ for\ semiconductor$, further

a) **For conductor**, as temperature increases, relaxation time decreases and resistivity of metal increases so, conductivity decreases. (Relaxation time depends upon temperature of conductor)
b) **For semiconductor**, as temperature increases, number of free electrons increases as resistivity decreases. (Number of free electrons depend upon nature of material)

12. **Mobility** is defined as magnitude of drift velocity per unit electric field applied and given by

$$\mu = \frac{\frac{qE\tau}{m}}{E} = \frac{q\tau}{m}$$

a) Mobility due to charge carrier electrons in a conductor is given by $\mu_e = e\frac{\tau_e}{m_e}$
b) Mobility due to charge carrier holes in a conductor is given by $\mu_h = e\frac{\tau_h}{m_h}$
c) The total current in a conducting material is the sum of current due to the positive current carriers and negative current carriers.

13. Kirchhoff's Law:

Kirchhoff's First law is based on conservation of charge and Kirchhoff's Second law is the law of conservation of energy. These two laws are used in solving the circuit parameters as to find current in the branch, potential difference across two points of resistors etc.

a) **Kirchhoff's First law** is also called Kirchhoff's Current Law (KCL): It is applied at a junction. According to this law:
The algebraic sum of current at any junction is equal to zero. $\sum_{junction} I = 0$

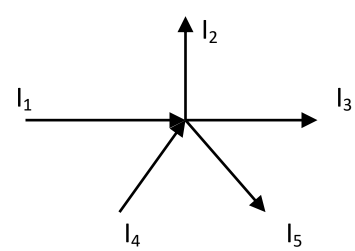
That is, net current entering the junction = net current leaving the junction

Here, I_1 and I_4 entering

the junction O

and I_2, I_3 and I_5 leaving

the junction O,

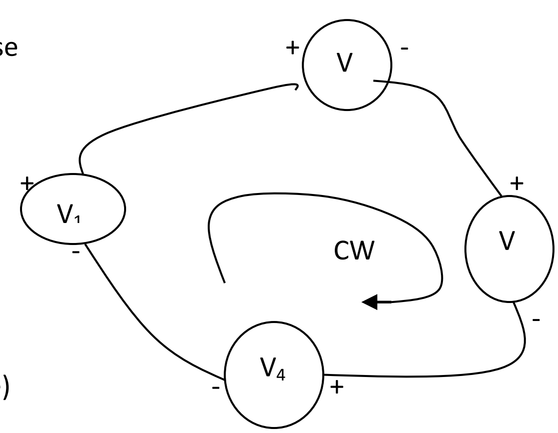


SO, $I_1 + I_4 = I_2 + I_3 + I_5$

b) **Kirchhoff's Voltage Law** (KVL): It is applied to a closed loop circuit.

The net voltage in a closed loop is equal to zero. $\sum_{closed\ loop} V = 0 \ or \ \sum E_{net} = \sum IR$

That is, Voltage drop = Voltage rise



Here, $V_1 - V_2 - V_3 - V_4 = 0$

V_1 is voltage rise (from –ve to +ve)

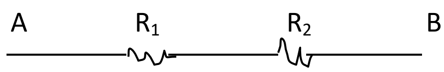
V_2, V_3 and V_4 are voltages drop (from +ve to –ve)

Take voltage rise as + and voltage drop as -.

14. Arrangements of resistances

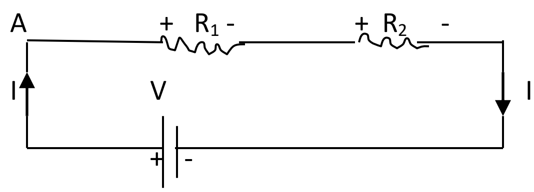
a) Resistances in series

Suppose two resistances R1 and R2 are connected in series as shown below, to find the equivalent resistance, the following method is being adopted.



a) Connect the voltage source (say a battery or mains supply) across terminal A and B
b) The current flows in wire AB is same while potential difference across R1 and R2 is different
c) Find net or equivalent resistance across AB

The arrangement is shown below:

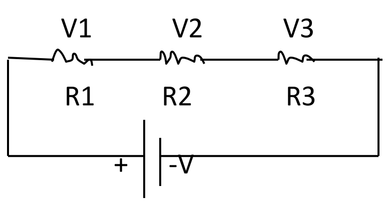


$$V = I(R_1 + R_2) \Rightarrow IR_{eqvt} = I(R_1 + R_2) \Rightarrow R_{eqvt} = R_1 + R_2$$

Voltage Distribution: V_1 across R_1 and V_2 across R_2 can be written as:

$$V_1 = IR_1 = \frac{V}{R_{eqvt}}(R_1) = V\left(\frac{R_1}{R_1 + R_2}\right)$$

$$V_2 = IR_2 = \frac{V}{R_{eqvt}}(R_2) = V\left(\frac{R_2}{R_1 + R_2}\right)$$



Given, R1=R, R2=2R, R3=3R

Find, V1, V2 and V3

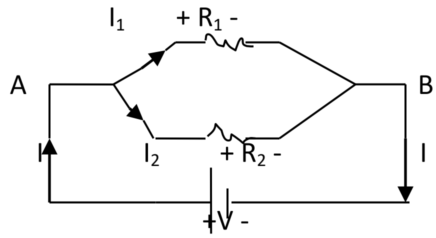
$V = IR \Rightarrow V \propto R, keeping\ current\ same$

So, $V_1 : V_2 : V_3 = R : 2R : 3R\ hence, V_1 = V\left(\frac{1}{6}\right), V_2 = V\left(\frac{2}{6}\right), V_3 = V\left(\frac{3}{6}\right)$

d) Two resistances are connected in parallel

Equivalent Resistance and current distribution

In this arrangement, voltage is same across each resistor and current flows in each branch is different as shown below.



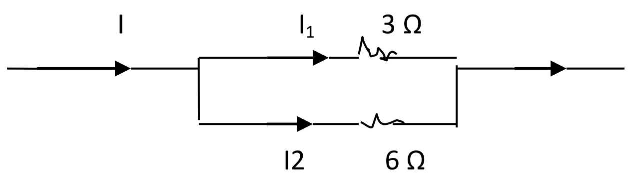
$$I = I_1 + I_2 \Rightarrow \frac{V}{R_{eqvt}} = \frac{V}{R_1} + \frac{V}{R_2} \Rightarrow \frac{1}{R_{eqvt}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eqvt} = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \frac{V}{R_1} = \frac{I R_{eqvt}}{R_1} = I \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_1} = I \frac{R_2}{R_1 + R_2}$$

$$I_2 = \frac{V}{R_2} = I \frac{R_{eqvt}}{R_2} = \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_2} = I \frac{R_1}{R_1 + R_2}$$

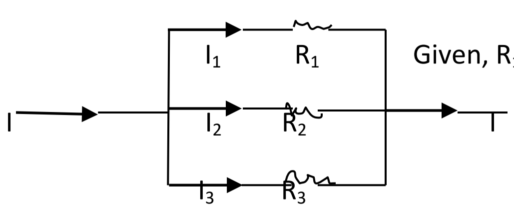
For example, the current of I= 9A can be divided in two resistances 3 Ω and 6 Ω in parallel as:



$$I_1 = 9\left(\frac{6}{6+3}\right) = 6\ A\ and\ I_2 = 9\left(\frac{3}{6+3}\right) = 3\ A$$

Suppose n resistances of same ohmic resistance are connected in series and same n resistances are connected in parallel, then R $_{eqvt}$ in both cases are given by: $R_{eqvt} = nR, \frac{1}{R_{eqvt}} = \frac{n}{R} \Rightarrow R_{eqvt} = \frac{R}{n}$

Suppose three resistances R, 2R and 3R are connected in parallel, then current in three branches:



Given, R_1=R, R_2=2R and R_3=3R, Find I_1, I_2 and I_3

$$I = \frac{V}{R} \Rightarrow I \propto \frac{!}{R}\ keeping\ V\ as\ constant$$

$$I_1 : I_2 : I_3 = \frac{1}{R} : \frac{1}{2R} : \frac{1}{3R} = 6 : 3 : 2$$

Then, $I_1 = I\left(\frac{6}{11}\right), I_2 = I\left(\frac{3}{11}\right), I_3 = I\left(\frac{2}{11}\right)$

15. EMF (E) of cell and internal resistance (r)

1) EMF (E) of cell is defined as work done by the cell in moving unit positive charge through the whole circuit (including the cell) is called electromotive force (emf) of the cell.
   $E = \frac{W}{q}$
2) Internal resistance (r) of cell: The potential difference across a real source in a circuit is not equal to the emf of a cell. The reason is that charge moving through the Electrolyte of the cell encounters resistance and this resistance is called internal resistance (r) of the cell.

Internal resistance of the cell depends upon the following factors:

a. It depends upon the nature, concentration and temperature of the electrolyte and increases with increase in concentration
b. It is directly proportional to the separation two plates of the cell
c. It is inversely proportional to the area of plate dipped into the electrolyte
d. It decreases with increase in temperature.
   While taking the loop, the equations can be written as:

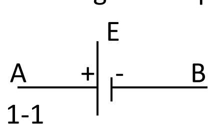


Fig. 1-1

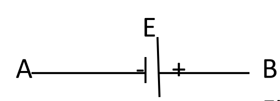


Fig. 1-2

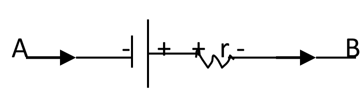
Path Path

In Fig. 1-1,

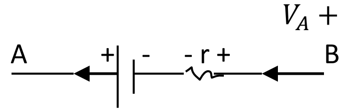
$$V_A - E - V_B = 0 \Rightarrow \Delta V = V_B - V_A = -E$$

In Fig. 1-2

$$-V_A + E + V_B = 0$$



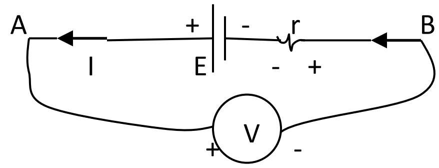
$$V_A + E - Ir - V_B = 0$$



$$V_B - Ir + E - V_A = 0$$

When current leaves the terminal, the voltage is taken as positive and when current enters the terminal, the voltage is taken as negative. Voltage rise can be taken from negative to positive in cell and voltage drop can be taken from positive to negative across resistance.
Voltage rise as from negative to positive is taken plus sign before it and voltage drop as from positive to negative to minus sign before it.

16. **Cell EMF with internal resistance r and PD across two points A and B can be written as**:



The potential at point B for the current I leaving the terminal B is taken as +ve while potential at point A for the current I entering the terminal as taken as –ve in the closed loop as shown above and by KVL,

Condition 1: Current supplied by batteries when battery is charged (combination of cells)

$$V_B - Ir + E - V_A = 0 \Rightarrow V_A - V_B = E - Ir \Rightarrow V = E - Ir$$

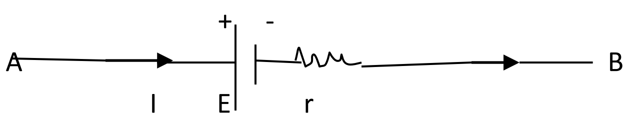
If no current flows through the cell, V=E; If the cell is short circuited, V=0

In case of charged cell, V<E

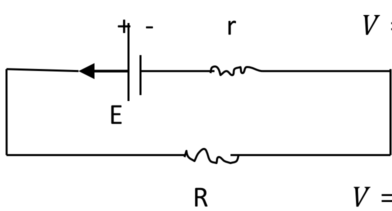
Condition 2: When battery is discharged (it draws the current)

$$V = E + Ir$$

In case of discharged cell, V>E



When cell is connected with external resistance R,



$$V = IR = R\frac{E}{r+R}$$

Net EMF=E, Net resistance=r+R

$$V = E_{net} - IR_{net}$$

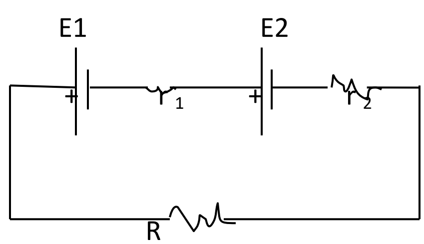
17. Power supplied by a battery = EI

    Power consumed by a battery = -EI

18. Grouping of cells

a. **Series Grouping**:

    1) Case 1: Two cells connected in series



If external resistance R is not connected, then

$$V = V_1 + V_2 = (E_1 - Ir_1) + (E_2 - Ir_2) \Rightarrow E_{eqvt} - Ir_{eqvt} = (E_1 + E_2) - I(r_1 + r_2)$$

Comparing both sides, $E_{eqvt} = E_1 + E_2 \; and \; r_{eqvt} = r_1 + r_2$

If external resistance R is connected in fig. 1, using KVL in a closed loop,

$$+E_1 - IR - Ir_2 + E_2 - Ir_2 = 0 \Rightarrow I = \frac{E_1 + E_2}{r_1 + r_2 + R}$$

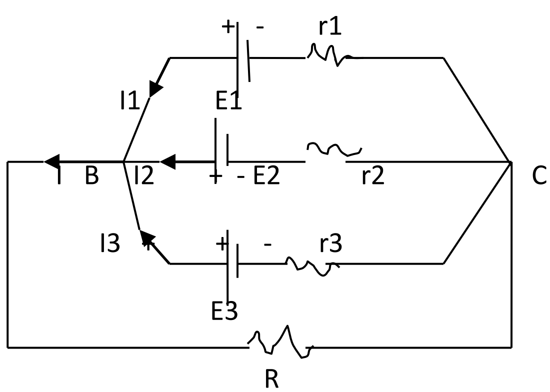
S0, it can be said that two cells connected in series, $Net\ EMF = E_1 + E_2 Net\ Resistance = Total\ internal\ resistance + Total\ External\ Resistance$

$$I = \frac{Net\ EMF}{Net\ Resistance}$$

2) Case 2: N cells of same emf and same internal resistance are connected in series, then,$I = \frac{nE}{nr+R}$
3) Case 3: If polarity of m cells is reversed, then equivalent emf= (n-2m)E while total resistance is still nr+R. $I = \frac{(n-2m)E}{nr+R}$

b. Parallel Grouping

Case 1: When E and r of each cell is different and positive terminals of all cells connected at one junction and negative terminals connected to another junction.



$I = I_1 + I_2 + I_3$

$I = \frac{E-V}{r}$

$\frac{E_{eqvt}-V}{r_{eqvt}} = \frac{E_1-V}{r_1} + \frac{E_2-V}{r_2} + \frac{E_3-V}{r_3} = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$ or,

$\frac{E_{eqvt}}{r_{eqvt}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} \; and \; \frac{1}{r_{eqvt}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$ and,

$I = \frac{E_{eqvt}}{R_{eqvt}} = \frac{\sum\frac{E}{r}}{1+R\sum\frac{1}{r}}$ where,

$$\boxed{E_{eqvt} = \frac{\sum\frac{E}{r}}{\sum\frac{1}{r}} \; and \; R_{eqvt} = R + \frac{1}{\sum\frac{1}{r}}}$$

If n cells of same emf and same internal resistance are connected in parallel, then

$\sum\frac{E}{r}=\frac{nE}{r}\ and\ \sum\frac{1}{r}=\frac{n}{r}$ Therefore, $I=\frac{\frac{nE}{r}}{1+R\frac{n}{r}}=\frac{E}{R+\frac{r}{n}}$

Note: In parallel grouping, E_{eqvt} = E if each cell equals to E and each resistance equals to r.

Case II: When positive terminals of few cells are connected to negative terminals of others.

Suppose there are three cells E1, E2 and E3 with internal resistance r1, r2 and r3 are connected in parallel. E2 is of opposite polarity and E1 and E3 are of same polarity, then

$$I=\frac{\frac{E_1}{r_1}-\frac{E_2}{r_2}+\frac{E_3}{r_3}}{1+R\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)}$$

c. **Mixed Grouping**:
There are n identical cells in a row and number of rows are m. EMF of each cell is E and internal resistance is r. Net EMF=nE and Total internal resistance = $\frac{nr}{m}$
Total external resistance=R, current through the external resistance is given by

$$I=\frac{nE}{R+\frac{nr}{m}}$$

Current I will be maximum when √(mR) =√(nr) or, $R=\frac{nr}{m}$
Hence, power transferred to the load is maximum when load resistance =total internal resistance (Maximum Power Transfer Theorem)

19. **Electrical Power (P) and Heat generated across a resistor**:

$$\int dW=\int V\,dq\Rightarrow W=VI\int dt\Rightarrow W=VIt=Pt\Rightarrow P=VI$$

a. Electrical power (P) dissipated or generated across a resistor is given by $P=VI=I^2R=\frac{V^2}{R}$

b. Heat generated across a resistor is given by $H=VIt=I^2Rt=\frac{V^2}{R}t$

20. Principle of bulb:
Resistance of a bulb is given by
R= $\frac{V^2}{P}\Rightarrow R\propto\frac{1}{P}\Rightarrow P\propto\frac{1}{R}\mathit{keeping\ the\ voltage\ fixed\ across\ the\ bulb}$

a. In series (suppose two bulbs are connected in series having the same rated voltage but rated power of both bulbs are P_1 and P_2 respectively), total power is given by (as $\frac{1}{P}\propto R$)

$$\frac{1}{P}=\frac{1}{P_1}+\frac{1}{P_2}\Rightarrow P=\frac{P_1P_2}{P_1+P_2}$$

b. In parallel (suppose two bulbs are connected in parallel having the same rated voltage but rated power of both bulbs are P1 and P2 respectively), total power is given by (as $P\propto\frac{1}{R}$)

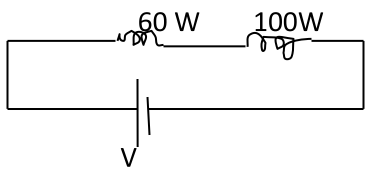
$$P=P_1+P_2$$

c. In $R = \frac{V^2}{P}$ V and P are rated values (written) of that bulb.

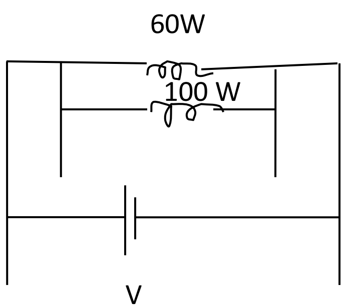
d. In parallel, a bulb having more rated power, glows more brightly ($H \propto \frac{V^2}{R}$) as R becomes less while in series, a bulb having less rated power, glows more brightly ($H \propto I^2R$).

e. In series, since I is same, hence, $P \propto R \ or \ \frac{P_1}{P_2} = \frac{R_1}{R_2}$

f. In parallel, since V is same, hence, $P \propto \frac{1}{R} \ or \ \frac{P_1}{P_2} = \frac{R_2}{R_1}$



60 W bulb will glow more in this series combination and total power is 75/2 W.

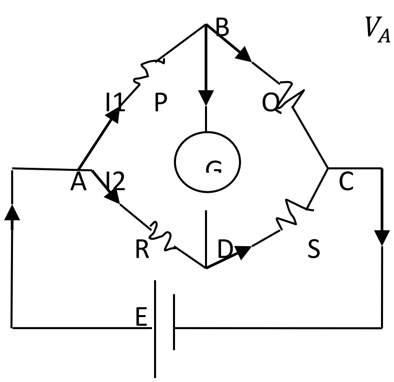


Here, 100 W bulb will glow more brightly.

Total power =160 W

21. **Wheatstone bridge**:

By this bridge, the unknown resistance can be found out in terms of known resistance.
Principle: The bridge is said to be balanced when deflection in the galvanometer is zero. That is I_g=0 and hence, V_B=V_D



$$V_A - V_B = V_A - V_D$$

$$I_1P = I_2R \Rightarrow \frac{I_1}{I_2} = \frac{R}{P}$$

Similarly,

$$V_B - V_C = V_D - V_C$$

$$I_1Q = I_2S \Rightarrow \frac{I_1}{I_2} = \frac{S}{Q}$$

Hence,

$$\frac{P}{Q} = \frac{R}{S}$$

One of the four resistances is unknown say, P and another one is variable, say Q. The value of Q is so adjusted so that deflection through galvanometer is zero and in this case, unknown resistance is

$$P = \left(\frac{R}{S}\right) Q$$

Following two points are important regarding a Wheatstone bridge,

a) In Wheatstone bridge, cell and galvanometer arms are interchangeable. In both cases, condition of balanced bridge is same.
b) If bridge is not balanced, current will flow from D to B if, $PS > QR$

Difference between EMF (electromotive force) and Potential Difference (Terminal Voltage)

Ser. No.	Electromotive Force (EMF) (E)	Potential Difference (V)
1.	It is work done by a source in taking a unit charge once round the complete circuit.	It is the amount of work done in taking a unit charge from one point to another.
2.	It is equal to the maximum potential difference between the two terminals of a source when it is in an open circuit.	Potential difference may exist between any two points of a closed circuit.
3.	It exists even when circuit is not closed.	It exists only when circuit is closed.
4.	It has non-electrostatic origin	It originates from the electrostatic field
5.	It is a cause. When emf is applied in a circuit, potential difference is caused.	It is an effect.
6.	It is larger than PD	It is smaller than emf
7.	It is independent of external resistance in the circuit.	It depends upon it

Chapter 4: Moving Charges and Magnetism-Important Formulae

1. Force on conductor 2 due to conductor 1 is given by $\overrightarrow{dF_{12}} = \left(\frac{\mu}{4\pi}\right)\frac{(I_2\overrightarrow{dl_2})x(I_1\overrightarrow{dl_1}\ x\ \overrightarrow{r_{12}})}{|\vec{r}_{12}|^3}$
2. Magnetic force on a moving charge $\vec{F}_m = q\vec{v}x\vec{B}\ \ or\ \ F_m = qvB\sin\theta$
3. Path of charged particle in uniform magnetic field
   (i) If θ = 0^0 or 180^0, path is straight line. (ii) If θ = 90^0, Path is circle.
   (ii) For any other angle, path is helix.
4. List of formulae in circular path
   i. Centripetal force = magnetic force $m\frac{v^2}{r} = qvB$
   ii. Radius of the circle (r), momentum (p), kinetic energy(K): $r = \frac{\sqrt{2Km}}{Bq} = \frac{\sqrt{2qVm}}{Bq}$
   iii. Time period, angular velocity and frequency $T = \frac{2\pi m}{Bq}, \omega = \frac{Bq}{m}, f = \frac{Bq}{2\pi m}$
5. List of formulae in helical path
   i. Radius of helical path $r = \frac{mv\sin\theta}{Bq}$
   ii. Time period and frequency of helical path $T = \frac{2\pi m}{Bq}\ , f = \frac{Bq}{2\pi m}$
   iii. Pitch of the helical path $p = (v\cos\theta)T = \frac{2\pi mv\cos\theta}{Bq}$
6. Magnetic force on a current carrying conductor $\vec{F}_m = I(\vec{l}x\vec{B})\ or\ F_m = IlB\sin\theta$

   Here θ is the angle between $\vec{l}\ and\ \vec{B}$ or between direction of current and magnetic field.

7. Force between a parallel wires (conducting current) $dF = \frac{\mu_0}{2\pi}\left(\frac{I_1I_2dl}{d}\right) \Rightarrow \frac{dF}{dl} = \frac{\mu_0}{2\pi}\left(\frac{I_1I_2}{d}\right)$
   [If the current in both wires flows in the same direction, force is attractive and if flows in the opposite direction, force is repulsive.]
8. Lorentz Force: The total force on a charge Q moving in the presence of both electric and magnetic field is given by $\vec{F} = \vec{F}_e + \vec{F}_m = Q(\vec{E} + \vec{v}X\vec{B})$
9. Velocity selector: Path of a charged particle in uniform electric and magnetic field will remain unchanged if, $\vec{F}_{net} = 0\ or\ q(q\vec{E} + q\vec{v}x\vec{B}) = 0\ or\ \vec{E} = -\vec{v}x\vec{B}\ or\ \vec{E} = \vec{B}x\vec{v}$
10. Cyclotron: It is a device (machine) to accelerate charged particles (such as alpha particles, protons and deuterons or ions to high energies to investigate nuclear structure

a) The frequency of revolution of the particle is given by $f = \frac{qB}{2\pi m}$
b) The energy gained by ions as $K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{q^2B^2R^2}{m}\right)\ [As\ \frac{mv^2}{R} = qvB \Rightarrow v = \frac{qBR}{m}]$

11. Magnetic dipole: Every current carrying loop is a magnetic dipole.

a) Magnetic dipole moment of magnetic dipole is given by $M = NIA$
b) Direction of $\vec{M}$ is the perpendicular to the plane of loop and given by right hand screw law.
c) Magnetic dipole in uniform magnetic field (i) Net force $\vec{F} = 0$, (ii) Torque $\vec{\tau} = \vec{M}x\vec{B}\ \ or\ \tau = MB\sin\theta$, Potential energy $U = -\vec{M}.\vec{B}\ \ or\ U = -MB\cos\theta$

$$W_{\theta_1\to\theta_2} = U_{\theta_2} - U_{\theta_1} = MB(\cos\theta_2 - \cos\theta_1)$$

d) If θ = 0^0, it is stable equilibrium position of the loop. In this condition,
   F = 0, $\tau = 0\ and\ U = -MB = Minimum$

e) If $\theta = 180^0$, it is unstable equilibrium position of the loop. Under thus condition,

$$F = 0, \tau = 0 \; and \; U = +MB = Maximum$$

12. Biot-Savart Law $d\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{I\vec{dl}x\vec{r}}{r^3}\right)$ $or\ dB = \frac{\mu_0}{4\pi}\left(\frac{Idl\sin\theta}{r^2}\right)$
13. Magnetic field of a moving point charge: The magnetic field vector $\vec{B}$ at point P at position vector $\vec{r}$ from the charge q moving with a velocity $\vec{v}$ is given by

$$\vec{B} = \frac{\mu_0}{4\pi}\left[\frac{q(\vec{v}x\vec{r})}{r^3}\right] \; or \; B = \frac{\mu_0}{4\pi}\left(\frac{qvB\sin\theta}{r^2}\right)$$

Here θ is the angle between $\vec{v}$ and $\vec{r}$ and field is zero when $\theta = 0^0$ or $180^0$and maximum at $\theta = 90^0$, direction of $\vec{B}$ is along $(\vec{v}x\vec{r})$ if q is positive and opposite to $(\vec{v}x\vec{r})$ if q is negative.

14. Applications of Biot-Savart Law:

(i) Magnetic field due at a point due to a straight wire of finite length (wire is held vertically) $B = \frac{\mu_0 I}{4\pi x}(\cos\theta_1 - \cos\theta_2) = \frac{\mu_0 I}{4\pi x}(\sin\alpha + \sin\beta)$

(ii) Magnetic field due at a point due to a straight wire of infinite length $B = \frac{\mu_0 I}{2\pi x}$

(iii) Magnetic field at a point due a straight wire one end finite and other end infinite

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi x}\right)$$

15. **Rules for finding direction of magnetic field in a straight conductor:** Right hand thumb rule-If we held the straight conductor in the grip of right hand in such a way that extended thumb points in the direction of current, then the direction of curl of fingers will give the direction of magnetic field.
16. **Direction of Magnetic field in circular coil**:Right hand thumb rule (in case of axial position) or Cork Screw Rule (in case of centre of a circle)-The curl finger will point out to the direction of current and the thumb will give the direction of magnetic field.

a) Magnetic field on the axis (either x, y or z-axis) of a circular loop (say, if coil placed in yz plane so, $B_y = 0$, $B_z = 0$ and B_x is given as$B_x = \frac{\mu_0 I R^2}{2(R^2+x^2)^{\frac{3}{2}}}$

b) Magnetic field at the centre of ring (or circular loop) $B_o = \frac{\mu_0 I}{2R}$

c) Magnetic field when x>>R for N circular loops on axial position

$$B_x = \frac{\mu_0 N I R^2}{2x^3} = \frac{\mu_0 NI}{4\pi}\left(\frac{2\pi R^2}{x^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{2NIA}{x^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{2M}{x^3}\right)$$

d) Magnetic field due to an arc of a circle at the centre $B = \left(\frac{\theta}{2\pi}\right)\frac{\mu_0 I}{2R}$

17. Magnetic field on the axis of a solenoid $B = \frac{1}{2}\mu_0 nI(\cos\theta_1 - \cos\theta_2)$

a) Magnetic field at the centre (on the axis) of a long solenoid $B(centre) = \mu_0 nI$

b) Magnetic field at ends of a long solenoid $B(end) = \frac{1}{2}\mu_0 n$

18. Magnetic field due to toroid: $B = \frac{\mu_0 NI}{2\pi r} = \mu_0 nI$ where $n = \frac{N}{2\pi r} = \frac{n}{l}$
19. Ampere's Circuital law: It states that line integral of $\vec{B}$ around a closed path is equal to μ_0 times current enclosed by that path, provided that electric field inside the loop remains constant. That is, $\oint \vec{B}.\vec{dl} = \mu_0 I_{net}$

20. Speed of light $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

21. Magnetic field created by long current carrying wire at r>R is $\frac{\mu_0 I}{2\pi r}$ and r<R is $\frac{\mu_0 I r}{2\pi R^2}$

Chapter-5: Magnetism and Matter-Formulae

1. Uniform magnetic field:-

a) At a given place, magnetic field of the earth can be considered uniform.

b) Magnetic field acting in plane of paper is represented by equidistant parallel lines.

$\vec{B}$

a) Magnetic field acting perpendicular to the plane of paper and directed outwards is represented by dot. ⊙ (North pole) ⊙ ⊙ ⊙ ⊙

b) Magnetic field acting perpendicular to the plane of paper and directed inwards is represented by cross. (south pole) ⊗ ⊗ ⊗ ⊗

2. **Bar magnet as a dipole**:

a) A small current loop carrying a current I, produces a magnetic field at an axial point is-

$$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{2\vec{M}}{x^3}\right)$$

b) $\vec{M} = I\vec{A}$ is the magnetic dipole moment of the current loop.

c) A current loop placed in a magnetic field $\vec{B}$ experiences a torque $\vec{\tau} = \vec{M} x \vec{B}$

d) There are two types of magnetic charges, positive magnetic charge and negative magnetic charge.

e) A magnetic charge m placed in a magnetic field $\vec{B}$ experiences a force $\vec{F} = m\vec{B}$

f) The force on a positive magnetic charge is along the field and force on a negative magnetic charge is opposite to the field.

g) A magnetic charge m produces a magnetic field $B = \frac{\mu_0}{4\pi}\left(\frac{m}{r^2}\right)$

h) Coulomb's law of magnetic force

$$F = \frac{\mu_0}{4\pi}\left(\frac{m_1 m_2}{r^2}\right)$$

i) Magnetic dipole and magnetic dipole moment: An arrangement of equal and opposite magnetic poles separated by a small distance is called a magnetic dipole.

-m 2 l +m $\vec{M} = m(\overrightarrow{2l})$

Where m is the pole strength and 2l is the magnetic length. It is a vector quantity and represented from south to north pole.

3. Magnetic field of a bar magnet at an axial point (end on position) (x-direction)

$$B_{axial} = B_1 + B_2 = \frac{\mu_0}{4\pi}\left(\frac{4xml}{(x^2 - l^2)^2}\right)\ if\ x \gg l, then\ B_{axial} = \frac{\mu_0}{4\pi}\left(\frac{2M}{x^3}\right)$$

In vector form, $\vec{B}_{axial} = \frac{\mu_0}{4\pi}\left(\frac{2\vec{M}}{x^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{4ml}{x^3}\right)\hat{\imath}$

4. Magnetic field of a bar magnet at an equatorial point (broad on position) (y-direction)

$$B_{equitorial} = \frac{\mu_0}{4\pi}\left(\frac{M}{y^3}\right) \ or \ \vec{B}_{equitorial} = \frac{\mu_0}{4\pi}\left(\frac{\vec{M}}{y^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{2ml}{y^3}\right)(-\hat{\imath})$$

5. **Some Important Definitions**:

a) The number of magnetic lines of induction inside a magnetic substance crossing unit area normal to their direction is called the magnitude of magnetic induction, or magnetic flux density inside the substance.'' It is denoted by $\vec{B}$. Its unit is tesla (T) or weber/meter^2 (Wb/m^2). The CGS unit is gauss (G). 1 Wb/m^2 = 1 T =10^4 G

b) Intensity of Magnetisation$(\vec{I})$: It is defined as magnetic moment per unit volume of the magnetised substance. This basically represents the extent to which the substance is magnetised. Thus, $I = \frac{M}{V}$, The S I unit is Amp/m.

c) Magnetic Intensity or magnetic field strength$(\vec{H})$: When a substance is placed in an external magnetic field, the actual magnetic field inside the substance is the sum of external magnetic field and the field due to its magnetisation (or intensity of magnetisation). $\vec{B} = \vec{B}_0 + \vec{B}_M \Rightarrow \vec{B} = \mu_0\vec{H} + \mu_0\vec{I}$

d) The capacity of magnetising field to magnetise the substance is expressed by means of a vector$(\vec{H})$, called the magnetic intensity of the field.

e) The SI unit of magnetic intensity is same as that of I that is A/m. The CGS unit is Oersted.

f) Magnetic permeability(μ): It is defined as the ratio of magnetic induction $(\vec{B})$ inside the magnetised substance to the magnetic intensity $(\vec{H})$ of the magnetising field. $\mu = \frac{B}{H}$

g) It is measure of the extent to which material can be penetrated or permeated by a magnetic field. It is basically a measure of conduction of magnetic lines of force through it. The SI unit of magnetic permeability is Wb/A-m.

h) Relative magnetic permeability(μ_r): Permeability of various substances can be compared with one another in terms of relative permeability. It is defined as ratio of permeability of the medium to the permeability of the free space. $\mu_r = \frac{\mu}{\mu_0}$

i) For vacuum, $\mu_r = 1$, It is also defined as ratio of magnetic field B in the substance when placed in magnetic field B_0. Thus, $\mu_r = \frac{B}{B_0}$

j) Magnetic susceptibility (χ_m): It is a measure of how easily a substance is magnetised in a magnetic field. It is defined as the ratio of intensity of magnetisation to the magnetic intensity of the magnetising field. $\chi_m = \frac{I}{H}$

k) Relation between relative permeability and magnetic susceptibility: $\mu_r = 1 + \chi_m$

6. **Hysteresis**: The phenomenon of lagging of magnetic induction (B) behind the magnetising field (H) is called Hysteresis. It is a Greek word, meaning delayed.
7. Hysteresis Loop: The closed curve ABCDEFA which represents a cycle of magnetisation of a ferromagnetic sample is called its hysteresis loop.
8. Significance of the area of hysteresis loop: The product B-H has the dimension of energy per unit volume. $BH = B\left(\frac{B}{\mu}\right) = \frac{B^2}{\mu_0\mu_r} = B^2v^2 = \frac{F^2}{q^2v^2}v^2 = \frac{Energy}{Volume}$

9. Area within the B-H loop represents energy dissipated per unit volume in materials when it is carried through a cycle of magnetisation.
10. Curie's Law: According to Curie's law, magnetic susceptibility of a paramagnetic substance is inversely proportional to the absolute temperature T. $\chi_m \propto \frac{1}{T}$ or $\chi_m = \frac{C}{T}$
11. Curie-Weiss law: The susceptibility of ferromagnetic material decreases with temperature according to $\chi_m = \frac{C}{T-T_C}$

12. **Comparison of magnetic properties of soft iron and steel**:
a) Soft iron has greater permeability (μ) i. e. (B) than steel
b) Soft iron has greater susceptibility ($\chi_m = \mu_r - 1$)
c) Retentivity of soft iron is greater than that for steel
d) Coercivity of soft iron is less than that of steel
e) Soft iron has less hysteresis loss than steel.
13. **Neutral Point**: It is the point where the magnetic field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field. The resultant magnetic field at the neutral point is zero. If a compass needle is placed at such a point, it can stay in any position.
a) Suppose a small bar magnet is placed such that the north pole of the magnet is towards the magnetic south pole of the earth, then neutral points are obtained both sides on the axis of the magnet. If the distance of each neutral point from the middle point of the magnet be r and the magnitude of magnetic moment be M, then $B_H = \frac{\mu_0}{4\pi}\left(\frac{2M}{r^3}\right)$
b) When North Pole of the bar magnet is towards magnetic north pole of the earth, the neutral points are obtained on perpendicular bisectors of the magnet. $B_H = \frac{\mu_0}{4\pi}\left(\frac{M}{r^3}\right)$
14. **Elements of earth's magnetic field**:
a) Magnetic declination (α): The angle between geographical meridian and the magnetic meridian at a place is called the magnetic declination at that place.
b) Angle of dip or magnetic inclination (δ): The angle made by the earth's total magnetic field B with the horizontal direction in the magnetic meridian is called angle of dip.
c) Horizontal component of earth's magnetic field: It is the component of earth's total magnetic $\vec{B}$ in the horizontal direction in the magnetic meridian. If δ be the angle of dip at any place, then $B_H = B\cos\delta$
15. **Tangent Galvanometer**: Tangent law: This law states that if a freely suspended small magnet is acted upon by two uniform mutually perpendicular magnetic fields B_1 and B_2 simultaneously, then the magnets comes to rest in such a position that the tangent of the angle θ that the magnet makes with B_1 is equal to the ratio B_2/B_1 of the two fields. That is, $\tan\theta = \frac{B_2}{B_1}$
16. **Time period** of oscillation of freely suspended magnet is $T = 2\pi\sqrt{\frac{I}{MB}}$
17. **Difference between magnetic dipole and electric dipole**
a) **Magnetic Dipole**:
    i. Current coil loop, solenoid etc. Are just like a magnetic dipole whose dipole moment M = NIA

ii. Direction of Magnetic dipole moment $(\vec{M})$: It is from south pole (S) to north pole (N).

iii. The behaviour of magnetic dipole (may be a bar magnet) is similar to electric dipole. The only difference is that the electric dipole moment $\vec{p}$ is replaced by magnetic dipole moment $\vec{M}$, charge q is replaced by pole strength m, $\vec{E}$ by $\vec{B}$ and the constant $\frac{1}{4\pi\epsilon_0}$ is replaced by $\frac{\mu_0}{4\pi}$

18. Comparative study of properties of para, dia and ferromagnetic materials

Parameters	Paramagnetic	Diamagnetic	Ferromagnetic
Effects of magnets	Feebly attracted by magnets	Feebly repelled by magnets	Strongly attracted by magnets
In external magnetic field	Acquires feeble magnetism in the direction of magnetizing field	Acquires feeble magnetism in the opposite direction of magnetizing field	Acquires strong magnetism in the direction of magnetic field
In non uniform magnetic field	Tends to move slowly from weaker to stronger	Tends to move slowly stronger to weaker	Tends to move quickly from weaker to stronger
In uniform magnetic field	Freely suspended rod aligns itself parallel to field	Freely suspended rod aligns perpendicular to field	Freely suspended rods aligns parallel to field
μ_r	>1	< 1 or $0 \le \chi_m < 0$	>> 1 or $\mu_{r>}$>1000
X_m	Slightly positive	Slightly negative, or $-1 \le \chi_m < 0$	Positive and very large, or $\chi_m > 1000$
μ	$\mu > \mu_0$	$\mu < \mu_0$	$\mu >> \mu_0$
Effect of temperature	Inversely as temp.	Independent of temp.	$\chi_m \propto \frac{1}{T-T_C}$, $T > T_C$
Removal of magnetising field	As soon as magnetising field is removed, magnetism is lost	Magnetisation lasts as long as magnetising field is applied	Magnetisation is retained even after magnetising field is removed
Variation of I with H	I changes linearly with H and attains saturation	I changes linearly with I	I changes non linearly with H and ultimately attains saturation

Hysteresis effect	Shows no hysteresis	Shows no hysteresis	Shows hysteresis
Physical state of material	Solid, liquid or gas	Solid, liquid or gas	Normally solids only

19. **Moving coil galvanometer**:

a) The moving coil is a device used to measure an electric current.

b) Principle: Action of moving coil galvanometer is based upon the principle that when a current carrying coil is placed in a magnetic field, it experiences a torque whose magnitude depends on the magnitude of current. $Current\ I \propto Deflection\ (\varphi)$

c) In the equilibrium, torque experienced in a magnetic field = Spring torque, that is $\tau = MB\sin\theta = (NIA)B\sin\theta = k\varphi$ where k is the torsional or spring constant.

d) $Galvanometer\ constant = \frac{k}{NAB}$

20. $Current\ sensitivity = \frac{\varphi}{I} = \frac{NAB}{k} Voltage\ sensitivity = \frac{\varphi}{V} = \frac{\varphi}{IR} = \frac{NBA}{kR}$ of galvanometer

21. Conversion of galvanometer into voltmeter, $V = I(R_g + R_s)$, $R_V = R_g + R_s \gg R_g$

22. Conversion of galvanometer into ammeter, $I_g = I\frac{R_{sh}}{R_{sh}+R_g}$, $R_A = \frac{R_g R_{sh}}{R_g+R_{sh}} < R_{sh}$

23. **Magnetic dipole moment of a revolving electron in terms of Bohr's magneton**

a) Magnetic dipole moment $M = \frac{eL}{2m}$ where $L = mvr = \frac{nh}{2\pi}$ $or\ M = \frac{e}{2m}\left(\frac{nh}{2\pi}\right) = nM_{min}$

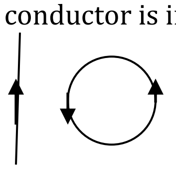
b) Bohr magneton = $M_{min} = \frac{eh}{4\pi m}$

Chapter-6: Electromagnetic Induction –Formulae

1. Magnetic flux $\varphi = \int \vec{B}.\overrightarrow{dS}$ or $\varphi = BS\cos\theta$ when $\vec{B}$ is uniform over a plane surface with total area S.
2. The following points regarding magnetic flux:
   a) The magnetic flux is a scalar quantity like electric flux
   b) The SI unit of magnetic flux is T-m^2 or weber (Wb).
   c) 1 Wb = 1 T-m^2 = 1 N-m/A
   d) In the special case in which $\vec{B}$ is uniform over a plane surface with total area S,
   $$\varphi = BS\cos\theta$$
   e) If $\vec{B}$ is perpendicular to the surface, θ = 0^0 and $\varphi = BS$
3. Gauss's law of magnetism $\oint \vec{B}.\overrightarrow{dS} = 0$ as there is no monopole in magnetism.
4. Faraday's law of electromagnetic induction $e = -\frac{d\varphi}{dt}$
5. Neumann's law could be stated as '' when the magnetic flux linked with a coil (or circuit) is changed in any manner, then an emf is set up in the circuit such that the produced emf is proportional to the rate of change of the flux linkage with the circuit.''
   The induced emf around the closed path is given by the line integral of the induced electric field E around the closed path. $e = \oint \vec{E}.\overrightarrow{dl} \Rightarrow -\frac{d\varphi}{dt} = \oint \vec{E}.\overrightarrow{dl}$ or $-\frac{d}{dt}\oint \vec{B}.\overrightarrow{dS}$
   Hence, we can say that the electric fields associated with changing magnetic fields are non-conservative while the fields associated with stationary charges are conservative.
6. Thus the Faraday's law can be stated as, 'The line integral of electric field intensity around any closed path is equal to the time rate in decrease of magnetic flux induction through the surface enclosed by that curve''.
7. If a circuit in coil contains N loops or turns, in the same area and if φ is the flux through one loop, an emf is induced in every loop, thus the total induced emf in the coil is given by $e = -N\frac{d\varphi}{dt}$
8. Induced emf is produced only when there is a change in a magnetic flux passing through a loop. The flux passing through the loop is given by $\varphi = BS\cos\theta$
9. Flux can be changed only when
   a) The magnitude of $\vec{B}$ can change with time, that is, $B = B(t)$
   b) The current producing in magnetic field can change with time. That is, I = i(t)
   c) The area enclosed by the loop can change with time, that is, S = S (t).This can be done by pulling a loop inside (or outside) a magnetic field.
   d) The angle θ between $\vec{B}$ and normal to the loop can change with time. This can be done by rotating a loop in a magnetic field.
10. When the magnetic flux passing through a loop is changed, an induced emf and hence, an induced current is produced in the circuit. If R is the resistance of the circuit, then induced current is given by $i = \frac{e}{R} = \frac{1}{R}\left(\frac{-d\varphi}{dt}\right)$
11. Current starts flowing in the circuit means flow of charge takes place. Charge flows in the circuit is given by $dq = idt = -\frac{d\varphi}{R}$

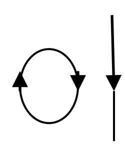
12. For a time interval Δt, $e = -\frac{\Delta\varphi}{\Delta t}, i = \frac{1}{R}\left(-\frac{\Delta\varphi}{\Delta t}\right)$ and $\Delta q = \frac{1}{R}(\Delta\varphi)$
    We can see that e and i are inversely proportional to Δt while Δq is independent of Δt.
13. Faraday's experiment
    a) **Experiment 1**: Induced emf with a stationary coil and moving magnet
        i. When a north pole of the magnet is brought closer to the coil, its face towards the magnet develops north polarity (direction of current in coil is anticlockwise).
        ii. Similarly, when north pole the magnet is moved away from the coil, its face towards magnet develops south polarity (direction of current in the coil is clockwise)
        iii. When a south pole of the magnet is brought closer to the coil, its face towards the magnet develops south polarity and
        iv. When south pole the magnet is moved away from the coil, its face towards magnet develops north polarity
    b) **Experiment 2**: Induced emf with a stationary magnet and moving coil:
        When the relative motion between the coil and the magnet is fast, the deflection in the galvanometer is large and when the relative motion is slow, the galvanometer deflection is small.
14. Lenz's law: ''It states that the direction of induced current in a circuit is such that it opposes the cause or the change which produces it.''
15. **To know the direction of induced current in a loop, apply**
    RIN: R stands for right, I stands for increasing and N for north pole (anticlockwise). It means if a loop is placed on the right side of a straight current carrying conductor and the current I in the conductor is increasing, then induced current in the loop is anticlockwise.

I increasing



(Current in the loop is anticlockwise)

I decreasing



(Current in the loop is clockwise)

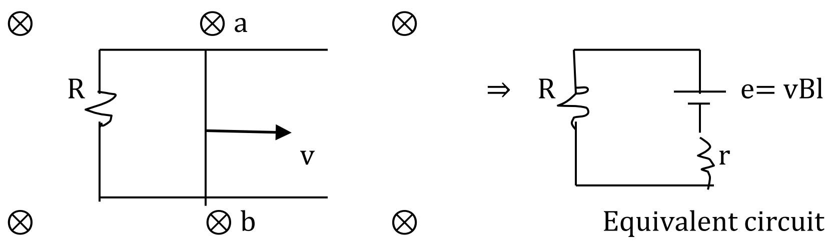
16. ⊗ IN: Suppose the magnetic field in the loop is perpendicular to paper inwards (⊗) and this field is increasing, then induced current in the loop is anticlockwise.
    ⊙ IN: If field increasing, induced current in the loop will be clockwise
    While in magnetic field, ⊗ (cross) means South Pole so magnetic field will be in clockwise and ⊙ (dot) means North Pole so magnetic field will be in anticlockwise direction as in case of solenoid.
17. Fleming Right hand Rule: ''If we stretch thumb and two fingers of our right hand in mutually perpendicular direction and if the forefinger points in the direction of magnetic field, thumb in the direction of motion of the conductor; then the central fingers points in the direction of current induced in the conductor.''
18. Motional emf
    a) On a straight conducting wire, $e = vBl$ = motional emf = ΔV
    b) On a rotating conducting wire about one end, $e = \frac{B\omega l^2}{2}$ where v = ωl

19. Current induced in the loop: $i = \frac{e}{R} = \frac{vBl}{R}$

20. Force experienced in the movable conductor of length l carrying current I, is given by

$$F = Ilb \sin 90^0 = IlB = \frac{vBl}{R}(lB) = \frac{B^2 l^2 v}{R}$$

This force (due to induced current) acts in the outward direction (By Fleming Left hand Rule) opposite to the velocity of the arm in accordance with Lenz's law. Hence to move the arm with a constant velocity v, it should be pulled with a constant force F.



Equivalent circuit

(From magnetic circuit to electric circuit)

The moving rod ab is replaced by a battery of emf vBl with the positive terminal at a and the negative terminal at b. The resistance r of the rod ab may be treated as the internal resistance of the battery. Hence, the current in the circuit is, $i = \frac{e}{R+r} = \frac{vBl}{R+r}$

21. Power delivered by the external force: $P_{del} = Fv = \frac{B^2 l^2 v}{R} v = \frac{B^2 l^2 v^2}{R}$
22. Power dissipated as Joule loss: $P_{dissipated} = i^2 R = \frac{B^2 l^2 v^2}{R^2} R = \frac{B^2 l^2 v^2}{R}$

Note 1: $e = -\frac{d\varphi}{dt}$ is best applied to the problem in which there is a changing flux through a closed loop while e= Blv is applied to problem in which conductor moves through a magnetic field (fixed)

Note 2: If conductor is moving in a magnetic field but circuit is not closed, then only P.D. will be asked between two points of the conductor. If the circuit is closed, then current will be asked in the circuit.

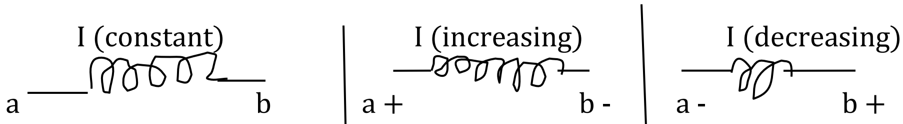
23. Self Inductance: Definition

a) Total flux linked with the coil is directly proportional to current in the coil. That is, $N\varphi \propto i \text{ or } N\varphi = Li \Rightarrow L = \frac{N\varphi}{i}$
b) it is total flux linkage per unit current. The SI unit of self inductance is Henry (H).

c) Hence, coefficient of self induction (or self Inductance) $L = \frac{N\varphi}{i} = \left|\frac{-e}{\frac{di}{dt}}\right|$

24. An inductor is a circuit element which opposes the change in current through it. It may be a circular coil, solenoid etc.

25. Potential difference across an inductor:



$\frac{di}{dt} = 0$, e =0, V_{ab} =0 | $\frac{di}{dt} > 0$, $e_{ab} < 0$, V_{ab}>0 | $\frac{di}{dt} < 0$, e_{ab} >0, V_{ab} <0

26. Assume that inductor has negligible resistance, so that PD $V_{ab} = V_a - V_b$. Here, V_{ab} = -e = $L\frac{di}{dt}$
27. Difference between Resistor and inductor: A resistor opposes the current i while inductor opposes the change in current (di/dt).
28. Kirchhoff's Second law with a inductor connected in series with resistor R and both are connected with a battery supplying current i, $E - iR - L\frac{di}{dt} = 0$
29. **Methods of finding self inductance of a circuit**:
a) Assume that current i flowing through the circuit
b) Determine the magnetic field B produced by the current
c) Obtain the magnetic flux
d) With the flux known, self inductance can be found from $L = \frac{N\varphi}{i}$
30. Self inductance of the solenoid $L = \frac{\mu_0 N^2 A}{l}$
31. This shows that L depends upon as: $L \propto N^2, L \propto A\ and\ L \propto \frac{1}{l}$
32. Energy (potential energy) stored in an inductor in magnetic field, $U = \frac{1}{2}Li^2$
33. Energy density in magnetic field, $u = \frac{U}{V} = \frac{1}{2}(\frac{B^2}{\mu_0})$
34. This expression is similar to energy density in electric field as $u = \frac{1}{2}\epsilon_0 E^2$.
35. A change in current in one circuit can induce a current in a circuit, is called mutual induction.
36. Mutual Inductance (M): $M = \frac{N_2\varphi_2}{I_1}\ or\ e_2 = -M\frac{di_1}{dt}$
37. Coefficient of mutual inductance of two closely wound circular coils, $M \propto N_1 N_2$
38. **The following steps worth noting for calculating the mutual inductance of two circuits**:
a) Assume any one of the circuits as primary (first) and the other as secondary (second).
b) Suppose a current i_1 flows through the primary circuit
c) Determine the magnetic field $\vec{B}$ produced by current i_1
d) Obtain the magnetic flux ϕ_2 (in the secondary circuit) as $\varphi_2 = B_1 A_2 = \frac{\mu_0 N_1 i_1}{l} A_2$ (Incase of two solenoids placed co-axially)
e) With the flux known, mutual inductance can be found from, $M = \frac{N_2\varphi_2}{i_1}$
39. Coefficient of coupling, $M = K\sqrt{L_1 L_2}$

Where, K = Coefficient of coupling and is a number depending on the geometry of the coils and their relative closeness, having value between 0 and 1 (K≤ 1).

40. Combination of inductances (when spacing between two coils is more)

a) Inductance in series, $L_{eqvt} = L_1 + L_2$

b) Inductances in parallel, $\frac{1}{L_{eqvt}} = \frac{1}{L_1} + \frac{1}{L_2}$ $or\ L_{eqvt} = \frac{L_1 L_2}{L_1 + L_2}$

41. Combination of inductances (when two coils are closely spaced) : Series combination only

a) Let the series connection be such that the current flows in the same sense in the two coils.

$$L_{eq} = L_1 + L_2 + 2M$$

b) Let the series combination be such that flows in opposite senses in the two coils

$$L_{eq} = L_1 + L_2 - 2M$$

42. Current growth in an L-R circuit, $i = i_0\left(1 - e^{-\frac{t}{\tau}}\right)$

43. Current decay in L-R circuit, $i = i_0 e^{-\frac{t}{\tau}}$

44. $\tau = Time\ constant = \frac{L}{R}$ and its unit is second.

45. Oscillations in L-C circuit: In L-C circuit: charge, current and rate of change of current oscillate simple harmonically. These oscillations are similar to oscillations of spring-block system. $\omega = \sqrt{\frac{1}{LC}}$

46. Comparison of an oscillation of a mass spring system and an LC circuit:

Mass Spring system	L-C circuit
Displacement x	Charge q
Velocity v	Current i
Acceleration a	Rate of change of current di/dt
$\frac{d^2x}{dt^2} = -\omega^2 x$ where $\omega = \sqrt{\frac{K}{m}}$	$\frac{d^2q}{dt^2} = -\omega^2 q$ where $\omega = \frac{1}{\sqrt{LC}}$
$x = A\sin(\omega t \pm \varphi)\ or\ x = A\cos(\omega t \pm \varphi)$	$q = q_0\sin(\omega t \pm \varphi)\ or\ q = q_0\cos(\omega t \pm \varphi)$
$\frac{1}{K}$	Capacitance C
Mass m	Inductance L
$\lvert v_{max}\rvert = A\omega$	$i_{max} = q_0\omega$
Kinetic energy $\frac{1}{2}mv^2$	Magnetic energy $\frac{1}{2}Li^2$
Potential energy $\frac{1}{2}kx^2$	Potential energy $\frac{1}{2}\frac{q^2}{C}$

47. Difference between capacitance and inductance keeping in view in the following circuit:

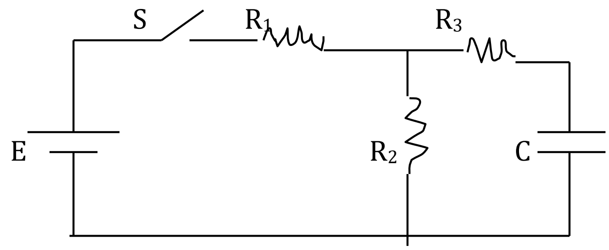


Figure-1

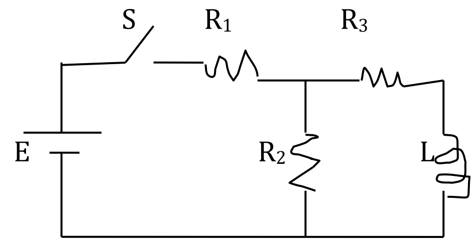


Figure-2

In figure-1, resistance offers by capacitor immediately after switch is closed, is zero (that is capacitor is shorted).

In figure-2, resistance offers by inductor immediately after switch is closed, is infinite (that is inductor is open).

In figure-1, resistance offers by capacitor after long time of closing the switch (in steady state condition), is infinite that is, capacitor is open circuited.

In figure-2, resistance offers by inductor after long time of closing the switch (in steady state condition), is zero that is, inductor is short circuited.

In figure-2, A long time after switch is reopened, current through all resistor will be zero.

Chapter-7: Alternating Current Formulae

1. $V = V_m \sin(\omega t \pm \varphi)$ or $V = V_m \cos(\omega t \pm \varphi)$
   Similarly, $i = I_m \sin(\omega t \pm \varphi)$ or $i = I_m \cos(\omega t \pm \varphi)$
   Here V and i are instantaneous value of voltage and current respectively. The phase angle ϕ is between voltage and current.
2. $T = \frac{2\pi}{\omega}$ or $\omega = 2\pi f$
3. Average value of current over positive half cycle if $i = I_m \sin \omega t$
$$I_{av} = < i > = \frac{\int_0^{\frac{T}{2}} I_m \sin(\omega t)\, dt}{\int_0^{\frac{T}{2}} dt} = \frac{2I_m}{\pi} = 0.637 I_m$$
4. Average value of voltage over positive half cycle if $V = V_m \sin \omega t$
$$V_{av} = < V > = \frac{\int_0^{\frac{T}{2}} \mathrm{V_m} \sin(\omega t)\, dt}{\int_0^{\frac{T}{2}} dt} = \frac{2V_m}{\pi} = 0.637 V_m$$
5. RMS value: It is the square root of the mean of the squares of the instantaneous current or voltage.
6. Root mean square (rms) value of current over one full cycle,
$$I_{rms} = \sqrt{< i^2 >} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$
7. Root mean square (rms) value of voltage over one full cycle
$$V_{rms} = \sqrt{< V^2 >} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$
8. The average value of sin ωt, cos ωt, sin 2ωt and cos 2ωt over one complete cycle is zero.
9. The average value of $sin^2 \omega t$ and $cos^2 \omega t$ over one cycle is 1/2.
10. Unless specified in the equation, take rms values of voltage or current.
11. Resistance of the circuit = R
12. Inductive reactance is $X_L = \omega L = 2\pi f L$
13. Capacitive reactance is $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$
14. Impedance of the circuit is Z.
15. The unit of resistance, inductive reactance, capacitive reactance and impedance is Ohm.
16. The reciprocal of reactance is susceptance and the reciprocal of impedance is admittance (Y). Its unit is Ω^{-1}.
17. $Power\ factor = \frac{Actual\ Power}{Apparent\ Power} = \cos \varphi = \frac{R}{Z}$
18. In purely resistive circuit, take $V = V_m \sin \omega t$ then $i = I_m \sin \omega t$
    The voltage and current are in same phase. The phase difference (ϕ) is zero and the power factor (cos ϕ) is 1.
19. In purely inductive circuit, take $V = V_m \sin \omega t$ then $i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$
    The instantaneous current lags the instantaneous voltage by 90^0 (or voltage leads the current by 90^0). The phase angle is + 90^0 and hence power factor = cos 90^0 = 0

20. In purely capacitive circuit, take $V = V_m \sin\omega t$ then $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$
    The instantaneous current leads the instantaneous voltage by 90^0 (or voltage lags the current by 90^0). The phase angle is - 90^0 and hence power factor = cos 90^0 = 0
21. In R-L circuit, take $V = V_m \sin\omega t$ then $i = I_m \sin(\omega t - \varphi)$. The current lags the voltage by phase difference ϕ. The power factor $PF = \cos\varphi = \frac{R}{Z}$ here $z = \sqrt{R^2 + \omega^2 L^2}$
22. In R-C circuit, take $V = V_m \sin\omega t$ then$i = I_m \sin(\omega t + \varphi)$. The current leads the voltage by phase difference ϕ. The power factor $PF = \cos\varphi = \frac{R}{Z}$ here $z = \sqrt{R^2 + {X_C}^2}$
23. The tangent of phase difference in R-L circuit is $\tan\varphi = \frac{X_L}{R} = \frac{\omega L}{R}$
24. The tangent of phase difference in R-C circuit is $\tan\varphi = \frac{X_C}{R} = \frac{1}{\omega CR}$
25. In phasor representation in R-L circuit (inductive circuit), $\bar{Z} = R + jX_L\ or$
    $Z = \sqrt{R^2 + {X_L}^2}$ or $\bar{V} = V_R + jV_L\ \ or\ V = \sqrt{{V_R}^2 + {V_L}^2}$
26. In phasor representation in R-C circuit (capacitive), $\bar{Z} = R - jX_C\ or$
    $Z = \sqrt{R^2 + {X_C}^2}$ or $\bar{V} = V_R - jV_C\ \ or\ V = \sqrt{{V_R}^2 + {V_C}^2}$
27. Here V is the resultant voltage, V_R is rms value of voltage across resistance, V_L is the rms value of voltage across inductance and V_C is the rms value of voltage across capacitance.
28. In inductive circuit, V_L leads the V_R by 90^0 by 90^0 phase difference.
29. In capacitive circuit, V_C lags the V_R by 90^0 phase difference.
30. If $\bar{Z} = Z\angle\phi$ it means current lags the voltage by phase difference ϕ and further means that the circuit is inductive which means that power factor is lagging.
31. If $\bar{Z} = Z\angle - \phi$ it means current leads the voltage by phase difference ϕ and further means that the circuit is capacitive which means that power factor is leading..
32. In L-C-R series circuit, ϕ = phase difference between instantaneous current and instantaneous voltage = $cos^{-1}\frac{R}{Z} = tan^{-1}\frac{X_L - X_C}{R}$
33. If $X_L > X_C$ then current lags the voltage and if $X_C > X_L$ then current leads the voltage.
34. Average power consumed in one cycle $P_{av} = \langle p \rangle = \frac{\int_0^T p dt}{\int_0^T dt} = V_{rms} I_{rms} \cos\phi = I^2 R$
35. $I_{max} = \frac{V_{max}}{Z}$ and $I_{rms} = \frac{V_{rms}}{Z}$
36. RMS values of
    a) $V_R = I_{rms} R, V_L = I_{rms} X_L$ and $V_C = I_{rms} X_C$
    b) $V_{applied} = I_{rms} Z = \sqrt{{V_R}^2 + (V_L - V_C)^2}$
    c) $\omega = \omega_r = \frac{1}{\sqrt{LC}}$=Resonant frequency
37. At $\omega = \omega_r$
    a) $X_L = X_C$
    b) Z = minimum value = R
    c) I_{rms} = maximum value = $\frac{V_{rms}}{Z} = \frac{V_{rms}}{R}$
    d) $I_m = maximum\ value = \frac{V_m}{Z} = \frac{V_m}{R}$

e) Power factor $= \cos\phi = 1$

38. In one complete cycle, power is consumed only by resistance. No power is consumed by a capacitor or an inductor.
39. Regarding graph between resistance, reactance and impedance vs. angular frequency
    a) $X_L = \omega L \ or \ X_L \propto \omega \ \ and \ X_C = \frac{1}{\omega C} \ or \ X_C \propto \frac{1}{\omega}$
    b) R does not depend upon ω and it is constant. At $\omega = \omega_r : X_L = X_C \ and \ Z = Z_{min} = R$
40. In view of Euler's identity, $e^{j\theta} = \cos\theta + j\sin\theta$
41. There are three equivalent notations for a phasors.
    a) Polar form: $\vec{V} = V\angle\theta$
    b) Rectangular form: $\vec{V} = V(\cos\theta + j\sin\theta)$
    c) Exponential form: $\vec{V} = Ve^{j\theta}$
42. Impedance (Z): If impedance is given as $Z = \frac{V\angle\varphi}{I\angle\theta}$
    a) If impedance angle is positive (ϕ > θ), current lags the voltage and circuit is R-L circuit.
    b) If impedance angle is negative (ϕ < θ), current leads the voltage and circuit is R-C circuit.
    c) If impedance angle is 0^0, purely resistive circuit
    d) If impedance angle is π/2, current lags the voltage by π/2, the circuit is purely inductive.
    e) If impedance angle is – π/2, current leads the voltage by π/2, the circuit is purely capacitive.
43. **Leads/Lags mean**:
    a) If voltage maximum comes first than that of current at given time, voltage leads current. The circuit is R-L circuit.
    b) If current maximum comes first than that of voltage, current leads the voltage. The circuit is purely capacitive.
    c) If voltage and current maximum comes at same time, voltage is in phase with current and circuit is purely resistive.
44. Quality factor Q is defined as $Q = \frac{Resonant\ frequency}{Bandwidth} = \frac{\omega_r}{2\Delta\omega} = \frac{\omega_r}{\omega_2 - \omega_1}$
45. Quality factor at resonance is $Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r L}{R}$
46. Resonant frequency is, $\omega_r = \frac{1}{\sqrt{LC}}$
47. Quality factor can be in another form as, $Q = \frac{1}{R}\left(\sqrt{\frac{L}{C}}\right)$
48. Q is large if R is small or L is large, this means resonance is sharp or series resonance circuit is more selective.
49. Wattless Current: The current is ac circuit is said to be wattless if the average power consumed in the circuit is zero.
50. Choke coil: It is simply an inductor with large inductance which is used to reduce current in ac circuit without much loss of energy.
51. **L-C oscillations: Qualitative approach**

a) When switch is OFF: Initially the charge on capacitor is maximum (switch S is open) and electrical energy stored in the capacitor is $U_E = \frac{1}{2}\frac{{q_0}^2}{C}$

b) The energy in the inductor is zero.

c) When switch S is closed: The magnetic energy stored in the inductor for a maximum current is, $U_B = \frac{1}{2} L{I_0}^2$

d) Energy stored by a capacitor is, $U_E = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{{q_m}^2}{C} cos^2 \omega t$,

e) Energy stored by inductor is, $U_B = \frac{1}{2} Li^2 = \frac{1}{2} L{q_m}^2 \omega^2 sin^2 \omega t = \frac{{q_m}^2}{2C} sin^2 \omega t$, Since, at resonance,

f) $\omega^2 = \frac{1}{LC}$, so total energy $E = \frac{{q_m}^2}{C}$

52. **Transformer**:

a) It is a device (electrostatic) which is used to decrease (step down) or increase (step up) the voltage in ac circuit through mutual induction. A transformer consists of two coils wound over the same core.

$$\frac{\varphi_p}{N_p} = \frac{\varphi_s}{N_s} \Rightarrow \frac{1}{N_p}\frac{d\varphi_p}{dt} = \frac{1}{N_s}\frac{d\varphi_s}{dt} \Rightarrow \frac{e_p}{N_p} = \frac{e_s}{N_s} \Rightarrow \frac{e_s}{e_p} = \frac{N_s}{N_p}$$

b) In an ideal transformer, there is no loss of power. That is, $ei = constant, \frac{e_s}{e_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s}$

53. Regarding a transformer, **followings are few important points**:

a) In step up transformer, $N_s > N_p$. It increases voltage and reduces current
b) In step down transformer, $N_p > N_s$. It reduces voltage and increase current
c) It works only on AC.
d) A transformer cannot increase or decrease voltage or current simultaneously as ei = constt.
e) Some power is always lost due to eddy current or hysteresis etc.

$Output\ Power\ of\ transformer = Input\ Power\ x\ Efficiency\ x\ PF = VI \cos\varphi\,\eta$

Ch-8: Electromagnetic wave

1. Displacement Current (i_d) : It is current inside a capacitor plates when it is getting charged. In the expression, ϵ_0 is permittivity of free space, φ is the flux due to electric field.

$$i_d = \epsilon_0 \frac{d\varphi}{dt}$$

2. Modified Ampere-circuital law: In this expression, i_c is the conduction current in the wire to which a capacitor is connected.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0(i_c + i_d)$$

3. Maxwell's four set of equations:

Gauss law of electrostatics-

This law tells that electric flux through a closed surface (or surface integral of electric field around a closed surface) is 1/Ɛ0 times charge enclosed by that surface.

$$\varphi = \oint \vec{E} \cdot \vec{dS} = \frac{q}{\epsilon_0}$$

Charge on an insulated conductor resides only on its outer surface.

Gauss law of magnetism-

This law tells that magnetic flux through any closed surface is zero.

$\varphi = \oint \vec{B} \cdot \vec{dS}$ =0

It explains that isolated magnetic poles do not exist.

Faraday's law of electromagnetic induction-

This law tells that a changing magnetic field induces an electric field. The induced emf set up in a closed circuit is equal to the rate of change of magnetic flux (φ)linked with that circuit.

$$\boldsymbol{e = -\frac{d\varphi}{dt} = \frac{d}{dt}\oint \vec{B} \cdot \vec{dS} = \oint \vec{E} \cdot \vec{dl}}$$

Modified Ampere's law-

This law states that line integral of magnetic field around any closed circuit is equal to µ0 times the total current (the sum of conduction and displacement current) threading the closed circuit.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0(i_c + i_d) = \mu_0\left(i_c + \epsilon_0 \frac{d\varphi}{dt}\right)$$

4. Speed of electromagnetic wave in vacuum is:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 m/s$$

5. Maxwell's prediction of electromagnetic wave:
   - A time varying magnetic field is a source of changing electric field
   - A time varying electric field is a source of changing magnetic field.
6. The expression of electromagnetic wave is given as:

$$E = E_0 \sin(kx - \omega t) \ and \ B = B_{-} \sin(kx - \omega t)$$

In the above expression, the negative sign between $kx\ and\ \omega t$ shows that waves are travelling in positive x −direction. If it were positive, it means the wave is travelling in negative x-direction.

The $x-, y-, z-$ direction shows by which one (x or y or z) is multiplied with k. Here k is wave number and ω is angular frequency.

7. Wave number k is $k = 2\pi/\lambda$, where λis the wavelength.
8. Angular frequency $\omega = 2\pi f = \frac{2\pi}{T}$,where f and T are frequency and time period respectively of the wave.
9. Speed of light in vacuum is $c = E_0/B_0$,where E_0 (in N/C) is maximum value of electric field intensity and B_0 (in Tesla) is maximum value of magnetic field induction.
10. If the wave is travelling in positive $x - direction$,then direction of electric field and magnetic field oscillations can be determined by the following expression as:

$$\vec{E} \times \vec{B} = \vec{v}$$

Suppose, EM wave is propagating with velocity($\vec{v}$) in +x-direction, then, it is in unit vector $\hat{\imath}$,so,$\hat{\jmath} \times \hat{k} = \hat{\imath}$,hence electric field oscillates in y- direction and magnetic field will oscillate in z-direction. Suppose, wave is propagating in + z-direction and electric field is in y-direction, the then direction of magnetic field oscillation will be as:

$$\hat{\jmath} \times (-\hat{\imath}) = \hat{k}$$

So, magnetic field will oscillate in negative x-direction.

Ch-9: Ray Optics Formulae

Plane Mirror

1. Suppose a mirror is rotated by angle θ (say anticlockwise), keeping the incident ray fixed, then the reflected ray rotates by 2θ along the same sense (anticlockwise).
2. Suppose the incident ray makes an angle i, after rotating the mirror by angle θ CCW, the reflected ray angle i - 2θ from the Y-axis.
3. The minimum length of plane mirror to see one's full height (H) in it is H/2.
4. A man is standing exactly mid-way between a wall and a mirror and he wants to see the full height of the wall (behind him) in a plane mirror in front of him. The minimum length of mirror in this case is H/3.
5. Focal length as well as the radius of curvature of a plane mirror is infinity.
6. Power of a plane mirror is zero.
7. When two plane mirrors are kept facing to each other at an angle θ and an object is placed between them, then
    i. Number of images, $n = \left(\frac{360}{\theta} - 1\right)$, if $\frac{360^0}{\theta}$ is even.
    ii. Number of images, $n = \frac{360^0}{\theta}$, if 360/θ is odd.

    Or, number of images formed is $n = \frac{360}{\theta} - 1$ if θ is a sub multiple of 180^0. If θ is not sub multiple of 180, then number of images formed is the integers next higher to it.

    Example-Let θ = 72^0, then 360/72 = 5 (odd), so number of images is 5. If θ = 90^0, the number of images formed is 3.
8. A man approaches a vertical plane at a speed of 2 m/s; at what rate does he approach his image? [Ans. 4m/s] or a plane mirror is approaching you at a speed of 10 m/s, at what speed will your image approach you.

Spherical Mirror Formula

9. Mirror formula (for all types of Mirrors):

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$where\ v$ is distance of image. u is distance of object and f is focal length from the pole of the mirror. For a concave mirror(converging mirror) , f is negative, and for a convex mirror (diverging mirror), f is positive.

10. Magnification formula in mirror: Linear magnification or transverse magnification or lateral magnification of a mirror is given by:

$$m = \frac{h_I}{h_O} = -\frac{v}{u}$$

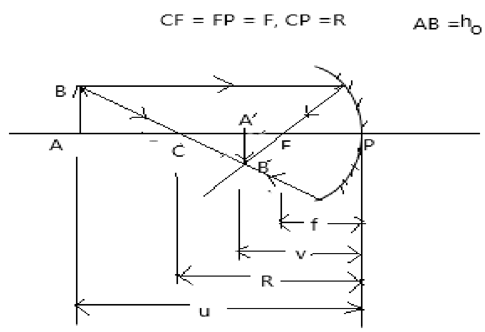
Where, $h_I = height\ of\ image, h_O = height\ of\ object$

Sign Convension in Mirror

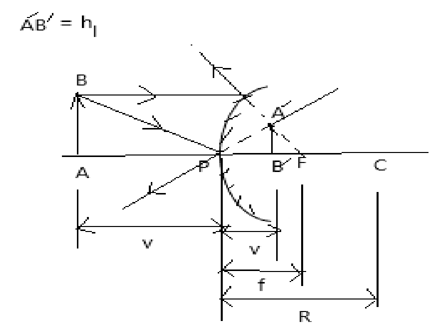
11. All distances are measured from the pole of the mirror.
12. Distance measured along the direction of incident ray from the pole of the mirror are taken positive and distance measured opposite to the direction of incident ray is taken negative.
13. Height above the principal axis is taken positive and height below the principal axis is taken negative.

Other information about the mirror:

14. For real image, magnification (m) is negative and for virtual image, magnification (m) is positive.



Concave Mirror
Fig-1



Convex Mirror
Fig-2

Position of Image for different Position of Object:

For concave Mirror:

Ser. No.	Position of Object	Details of Image
1	At ∞	At F, real, inverted, $\|m\| \ll 1$
2	Between C and ∞	Between F and C, real, inverted, $\|m\| < 1$
3	At C	At C, real, inverted, $\|m\| = 1$
4	Between F and C	Between C and ∞, real, inverted, $\|m\| > 1$
5	At F	At infinity, real, inverted, $\|m\| \gg 1$
6	Between F and P	Behind the mirror, virtual, erect, $\|m\| > 1$

For Convex Mirror:

Ser. No.	Position of Object	Details of Image
1	At ∞	At F, virtual, erect, $\|m\| \ll 1$
2	In front of mirror	Between P and CF, virtual, erect, $\|m\| < 1$

15. Two laws of reflection
    a) ∠i = ∠r
    b) Incident ray, reflected ray and normal lie on the same plane
16. Reflection from plane surface (or from plane mirror
    a) If object is real, its image from a plane mirror is virtual, erect, same size and at same perpendicular distance from the mirror.

b) F = ∞, P = 0

17. Reflection from spherical surface (concave or convex mirror)
    a) $f = \frac{R}{2}$
    b) $P\ (dioptre) = \frac{-1}{f\ (metre)}$
    c) Image speed $= (m^2)\ Object\ speed$
    They travel in the opposite directions.

Spherical Lens

18. Lens formula:
    For all type of Lenses, lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Where, u =distance of object from optical centre of lens, v = distance of image from optical centre, f = focal length of lens from optical centre oa a lens

19. Lens magnification formula:

$$m = \frac{h_I}{h_0} = \frac{v}{u}$$

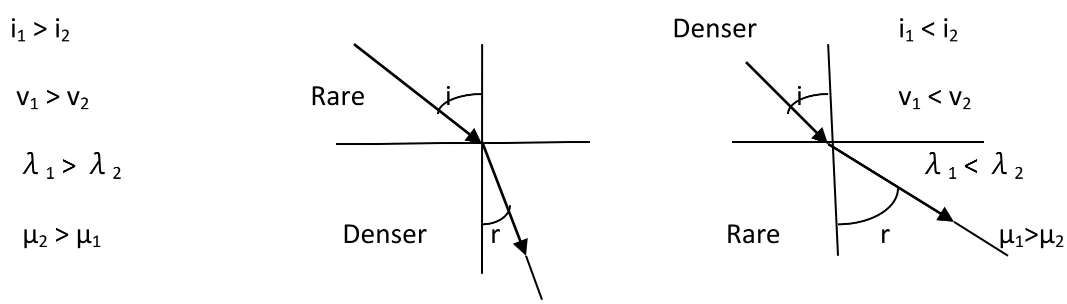
20. Here also in lenses, for all real image, magnification is negative and for virtual image, magnification is positive.
21. Convex lens is converging lens and has positive focal length f (Second Focus F_2),while concave lens is diverging lens, and has focal length negative (First Focus F_1).
22. Sign convention in lens is same as that in mirror.
23. **Snell's Law**

For two particular media, the ratio of sine of the angle of incidence to the sine of angle of refraction is constant. That is,

$$\frac{\sin i}{\sin r} = Constant = 1\mu_2 = \frac{\mu_2}{\mu_1}$$

Equation (1) is known as Snell's law.

The incident ray, the normal and the refracted ray all lie in the same plane.



The constant ratio is called the refractive index (Equation 1 above) for light passing from the first to the second medium (or refractive index of medium 2 with respect to medium 1). It is denoted by $1\mu_2$.

For two media,

$$\mu_1 \sin i = \mu_2 \sin r$$

If medium 1 is vacuum (say air), we refer $1\mu_2$ as μ_2 or simply μ. $((\mu_{air} = 1)$.

If $\mu_2 > \mu_1, i > r$, if a ray of light passes from rare to denser medium, it bends towards normal and vice-versa. Again,

$$_1\mu_2 = \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$$

Here, v_1 is the speed of light in medium 1 and v_2 is the speed of light in medium 2. Correspondingly, λ_1 *and* λ_2 are wave lengths in medium 1 and medium 2 respectively.

If $\mu_2 > \mu_1$ then $v_1 > v_2$ and $\lambda_1 > \lambda_2$, i. e. in a rare medium, speed and wavelength of light is more.

In general, speed of light in any medium is less than its speed in vacuum.

So, Definition of refractive index (μ) is as

$$\mu = \frac{speed\ of\ light\ in\ vacuum}{speed\ of\ light\ in\ medium} = \frac{c}{v}$$

As ray of light moves from medium 1 to medium 2, its wave length changes but its frequency remains constant.

Principle of reversibility: This principle states that if the final path of a ray of light after it has suffered several reflections and refractions, is reversed, it retraces its path exactly.

24. Expression for lateral displacement

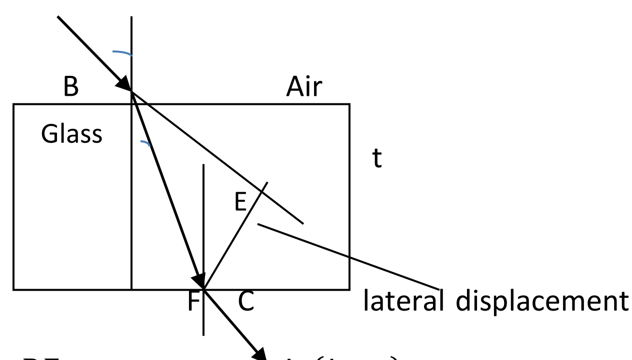
i=incidence angle ,r = refracted angle

t =thickness of glass

Lateral displacement (d):

d=CE=BC sin (i-r)

Hence,



$$d = BC \sin(i - r) = \frac{BF}{\cos r} \sin(i - r) = t\frac{\sin(i - r)}{\cos r}$$

$$d = t \sin i \left[1 - \frac{\cos i}{(\mu^2 - sin^2 i)^{\frac{1}{2}}}\right]$$

25. Relation between actual (real) depth, apparent depth and refractive index

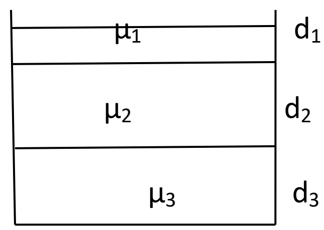
$$Apparent\ depth = \frac{Real\ depth}{refractive\ index}$$

26. Normal shift(t): d=real depth, μ is refractive index

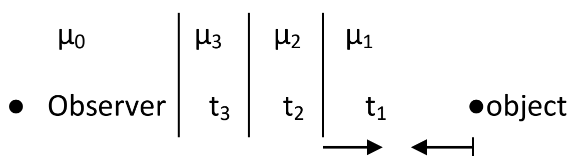
$$d = t\left(1 - \frac{1}{\mu}\right)$$

27. Refraction through composite slabs or through a number of parallel media as seen from μ_0

Three immiscible liquids of refractive indices μ_1, μ_2 and μ_3 ($\mu_3 > \mu_2 > \mu_1$) are filled in a vessel. Their depths are d_1, d_2 and d_3 respectively. The apparent depth (for normal incidence) when seen from top of first liquid will be



$d_{app} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3}$ when top of first liquid is air.



$$Apparent\ depth\ D_{app} = \sum \frac{d}{\mu_{rel}} = \frac{t_1}{\frac{\mu_1}{\mu_0}} + \frac{t_2}{\frac{\mu_2}{\mu_0}} + \frac{t_3}{\frac{\mu_3}{\mu_0}} + - - -$$

$$Apparent\ shift\ s = t_1\left(1 - \frac{1}{\mu_{1rel}}\right) + t_2\left(1 - \frac{1}{\mu_{2rel}}\right) + t_3\left(1 - \frac{1}{\mu_{3rel}}\right) + - - - -$$

Total Internal Reflection:

28. Necessary condition for total internal reflection:

1. Light must travel from an optically denser to rarer medium.
2. The angle of incidence in the denser medium must be greater than the critical angle for the two media.

Refraction of light through curved convex surface

29. When the light passes from rarer to denser medium, object lies in rarer medium, whether the surface is convex or concave and the image is real or virtual, the formula is given as

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

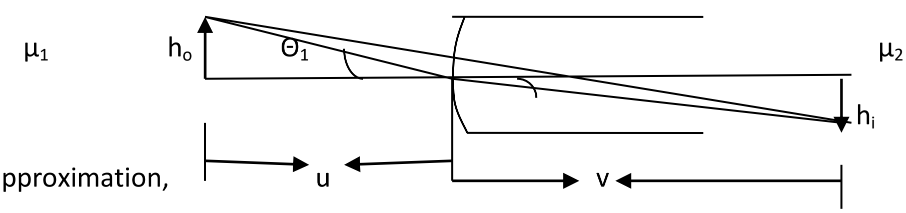
30. When the light passes from dense to rarer medium, object lies in denser medium, whether the surface is convex or concave and the image is real or virtual, the formula is given as

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

(Note: All formulae must be derived with sign convention)

31. Lateral magnification through a curved surface:

We take the curved surface as convex. The figure is shown below-



With the paraxial approximation,

$$\sin\theta_1 = \theta_1 = \tan\theta_1 = \frac{h_o}{-u} \text{ and } \sin\theta_2 = \theta_2 = \tan\theta_2 = \frac{-h_i}{v}$$

Applying Snell's law, $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

So, Magnification, $m = \frac{h_i}{h_o} = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right)$

32. **Lens Maker Formula,** If lens has refractive index μ_2 and the medium outside lens,both sides is μ_1, then focal length of lens is

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = Power\ (P)$$

33. If media is different on two sides a bi-convex lens,then lens maker formula becomes as:

$$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$$

$$For\ u = \infty, v = f, then\ lens\ make\ formula\ becomes\ as\ \ \frac{\mu_3}{f} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$$

34. When lenses placed at distance from each other, then two cases,

$Case - I$: Lenses placed apart (distance d from each other) but object is at infinity, then

$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$Case - II$: Lenses placed apart and object at finite distance, then apply lens formula for each lens separately. The image formed by first lens acts as an object for the second lens and so on so forth. For lens L_1,apply $\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$; find v_1 for u_1 and f_1 given. Then for second lens $L_{2,}$ take u_2 from optical centre of lens L_2 by subtracting or adding v_1. If image from lens L_1 is formed right side from optical centre of L_1 lens ,then subtract it from u_2 as $u_2 = d_1 - v_1$ but if image distance v_1 is left of optical centre of L_1,then $u_2 = d_1 + v_1$

35. **Two thin lenses in contact**: Suppose a lenses of focal length f_1 is placed in contact with another lens of focal length f_2, then combined focal length is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

36. Similarly, if more than two lenses in contact, the equivalent focal length is

$$\frac{1}{f}=\sum_{i=1}^{i=n}\frac{1}{f_i}$$

Here f_1, f_2 etc. are substituted with sign.

37. For n thin lenses in contact, the equivalent power is given by

$$P=P_1+P_2+P_3\pm----$$

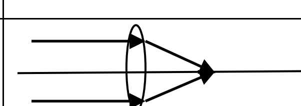
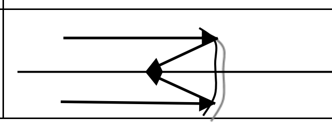
38. Construct the ray diagram to show the nature of the images as for convex lens:

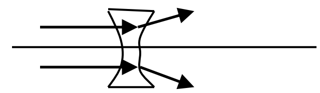
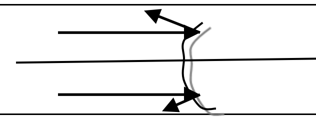
Object Position	Image Position	Size of image.	Nature of image
At infinity	At focus	Very small (Diminished)	Real & inverted
Between 2F and ∞	Between 2F & F	small	Real & inverted
At 2F	At 2F	Equal as object	Real & inverted
Between 2 F & F	Between 2F & ∞	Large	Real & inverted
At F	At infinity	Very large	Real & inverted
Between F and Optical centre	Beyond first focus	Large	Virtual & erect

Construct the ray diagram to show the nature of the images as for concave lens:

Object position	Image Position	Size of image	Nature of image
At any position on principal axis	Between F and Optical centre	Smaller	Virtual and Erect

39. The shorter the focal length of a lens or mirror, the more it converges or diverges, i. e. the bending of light is more for shorter focal length.

Nature of lens/Mirror	Focal length (f)	Power	Converging /diverging	Ray diagram
Convex lens	Positive	$P=\frac{1}{f}$ (Positive)	Converging	
Concave mirror	Negative	$P=-\frac{1}{f}$ (Positive)	Converging	

Concave lens	Negative	$P = \frac{1}{f}$ (Negative)	Diverging	
Convex mirror	Positive	$P = -\frac{1}{f}$ (Negative)	Diverging	

40. **Refraction of light through a prism**: If the minimum deviation is δ_{min},

δ_{min} = 2i – A or $i = \frac{A+\delta_m}{2}$ and A = r_1 + r_2 = 2r or $r = \frac{A}{2}$ So, refractive index

$$\mu_g = \frac{\sin i}{\sin r} = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$$

41. Minimum deviation occurs when refracted ray becomes parallel to the base of the prism.

42. Simple Microscope

(i) When final image is formed at least distance of distinct vision(D = 25 cm):Magnifying power or angular magnification M_D is:

$$M_D = \left(\frac{1}{D} + \frac{1}{f}\right)D = 1 + \frac{D}{f}$$

(ii) When final image is formed at infinity, the magnifying power is:

$$M_\infty = \frac{D}{f}$$

43. Compound Microscope:

(i) When final image is formed at least distance of distinct vision(D = 25 cm):Magnifying power or angular magnification M_D is

$$M_D = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

(ii) When final image is formed at infinity, the magnifying power is:

$$M_\infty = -\frac{v_0}{u_0}\left(\frac{D}{f_e}\right)$$

(iii) Tube length or distance between a convex lens of small aperture and focal length (objective) (f_o) and another convex lens of bigger aperture and large focal length (eye-piece) (f_e) when final image is formed at least distance of distinct vision is,

$$L_D = v_0 + \frac{Df_e}{D + f_e} \quad and \quad L = v_0 + u_e$$

(iv) Tube length or distance between a convex lens of small aperture and focal length (objective) (f_o) and another convex lens of bigger aperture and large focal length (eye-piece) (f_e) when final image is formed at least distance of distinct vision is,

$$L_\infty = v_0 + f_e$$

44. Astronomical Telescope: Refraction type (Objective is large and Eye-piece is small)

(i) Final image formed at least distance of distinct vision,

$$M_D = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

(ii) When final image is formed at infinity,

$$M_\infty = -\frac{f_0}{f_e}$$

(iii) Tube length when final image formed at D:

$$L_D = f_o + |u_e|$$

(iv) Tube length when final image is formed at infinity,

$$L_\infty = f_o + f_e$$

Ch-10:Wave Optics Formula sheet

1. Expression of resultant amplitude and resultant intensity of two waves in interference:

$$A^2 = {A_1}^2 + {A_2}^2 + 2A_1A_2\cos\varphi$$

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\varphi$$

Where,φ is the phase difference between two waves in interference, $A_1\ and\ A_2$ are respective amplitudes and $I_1\ and\ I_2$ are respective intensity.

2. For constructive interference, $\cos\varphi = 1$, (Maxima or bright frimges) or in general, Phase difference $\varphi = 2n\pi$, path difference, $\Delta x = d\sin\theta = n\lambda$; where, n = 0, $\pm1, \pm2, \pm3, -----$
For Central maxima, n = 0, for first bright, n = 1
3. For destructive interference, $\cos\varphi = -1$ (Minima or dark fringes); or in general, phase difference, $\varphi = (2n-1)\pi$ and path difference, $\Delta x = d\sin\theta = (2n-1)\lambda/2$,where n = $\pm1, \pm2, \pm3, -----$
4. ***Comparison of intensities at maxima and minima***:
$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \frac{(A_1+A_2)^2}{(A_1-A_2)^2}$$
5. Resultant intensity when $I_1 = I_2 = I$,
$$I_r = 4Icos^2\left(\frac{\varphi}{2}\right) = I_0cos^2\left(\frac{\varphi}{2}\right)$$
Where, I_0 is the maximum intensity at central maxima, at central maxima, $\varphi = 0^0$
6. Relation between phase difference(φ) and path difference (Δx),
$$\varphi = \frac{2\pi}{\lambda}\times\Delta x$$
7. In YDSE, if $w_1\ and\ w_2$ are slit widths, then,
$$\frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{{A_1}^2}{{A_2}^2}; \qquad (I \propto A^2)$$
8. Position of bright fringes from Central maxima in YDSE:
(i) n^{th} distance of bright fringes from central maxima,
$$(y_n)_{bright} = \frac{nD\lambda}{d}, where\ n = 0, \pm1, \pm2, ----$$
(ii) n^{th} distance of dark fringes from central maxima;
$$(y_n)_{dark} = \frac{(2n-1)D\lambda}{2d}, where\ n = \pm1, \pm2, ----$$

Where, D = separation between slits and screen, d = slits separation, λ is wavelength of light.

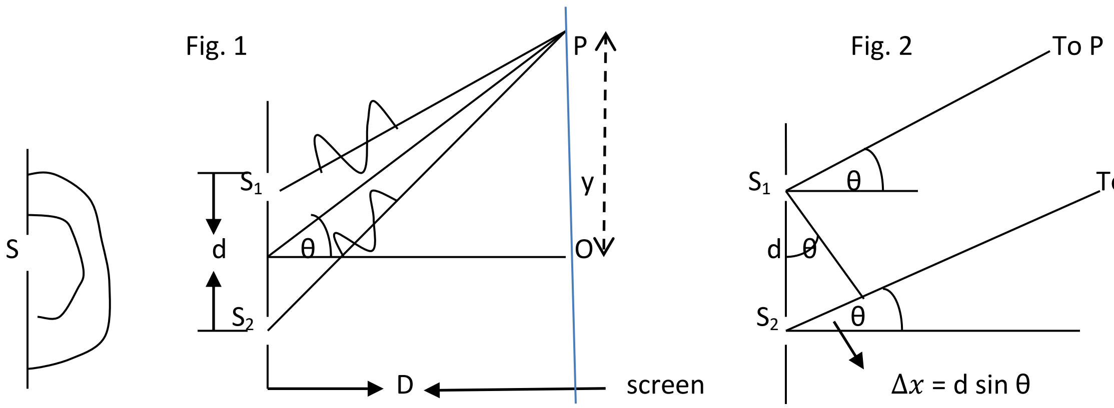
9. **Fringe width (β):It is the difference between two successive bright or two successive dark fringes.**

$$\beta = \frac{D\lambda}{d}$$

10. **Angular fringe width**: $\alpha = \frac{\beta}{d} = \frac{\lambda}{d}$
11. If YDSE apparatus is immersed in liquid of refractive index μ the wavelength of light and hence fringe width decreases μ times.

$$\mu\beta = constant$$

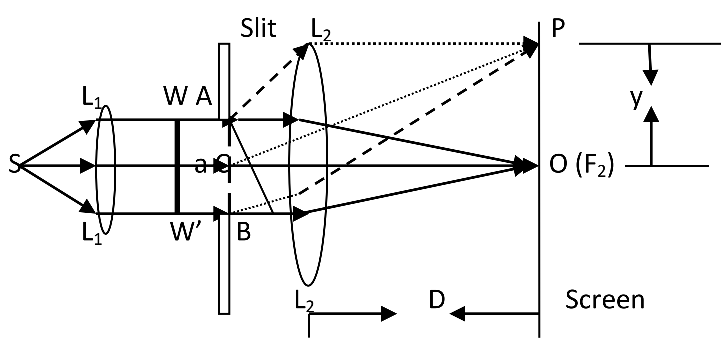
12. Two assumptions: If D >> d, then two waves becomes parallel. If d >> λ, so,angle becomes small and then $\tan\theta \approx \sin\theta = \frac{y}{D} = \frac{n\lambda}{d}$



Diffraction of light:

Diffraction occurs on account of mutual interference of secondary wavelets which are not blocked by obstacle or from portions of wave fronts which are allowed to pass through the aperture.

Diffraction at single slit:



Fraunhofer Diffraction

13. **Position of n^{th} secondary minima**:
    The general condition for destructive interference, $path\ difference = a\ \sin\theta = n\lambda$ or,

$\sin\theta = \frac{n\lambda}{a}, n = \pm1, \pm2, \pm3\_\_\_$

Position of n^{th} minima from the centre of the screen:

$Since, \sin\theta = \theta = \tan\theta = \frac{y}{D} = \frac{n\lambda}{a}$

$$\therefore\ \boldsymbol{y_n = \frac{nD\lambda}{a}} - - - (1)$$

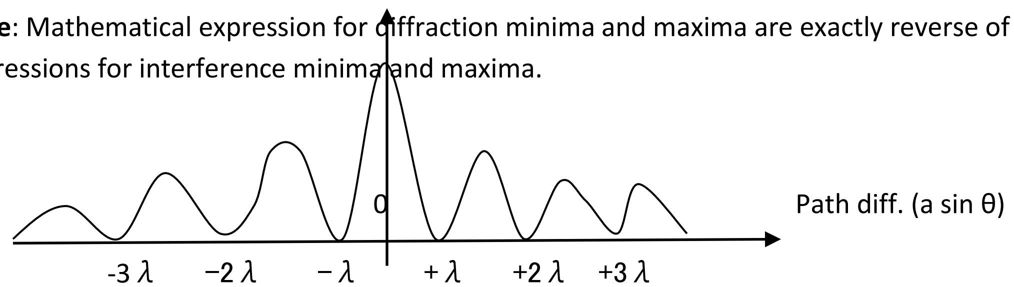
14. **Position of n[th] secondary maxima**: $Path\ difference\ BN = a\sin\theta = (2n+1)\frac{\lambda}{2}, n = \pm 1, \pm 2, \_\_$

    Position of n[th] maxima from the centre of screen:

$$Since, \sin\theta = \theta = \tan\theta = \frac{y}{D} = (2n+1)\frac{\lambda}{2a}$$

$$\therefore\ \boldsymbol{y_n = \frac{(2n+1)D\lambda}{2a}} - - - (2)$$

Note: Mathematical expression for diffraction minima and maxima are exactly reverse of mathematical expressions for interference minima and maxima.



$$\boldsymbol{Width\ of\ central\ maximum = 2y = \frac{2D\lambda}{a} = \frac{2f\lambda}{a}}$$

Where, f = focal length of lens L_2 which is held very close to the slit.

So, we find that the width of central maximum decreases when the slit width increases.

Angular width of central maximum: sin θ = θ when θ in radian and small, therefore,

$$\sin\theta = \theta = \frac{\lambda}{a}, hence$$

$$\boldsymbol{Angular\ width\ of\ central\ maximum\ (2\theta)} = \frac{2\lambda}{a}, hence$$

$$\boldsymbol{Width\ of\ central\ maximum = D(angular\ width\ of\ central\ maximum)}$$

Where D = distance between slit and screen

Law of Malus: In 1809, ''E. N. Malus discovered that when a beam of completely plane polarised light is passed through analyser, the intensity of transmitted light varies directly as the square of the cosine of angle θ between the transmission direction (pass axis) of polariser and analyser.'' This statement is known as law of Malus. Mathematically,

$$I \propto cos^2\theta\ or\ I = I_0 cos^2\theta$$

Chapter-11: Dual Nature of Radiation and Matter

1. **For photoemission to take place, either of the following conditions must be satisfied:**

$$\boldsymbol{E \geq W \;\; or \;\; f \geq f_0 \;\;\; or \;\; \lambda \leq \lambda_0}$$

2. **Effect of intensity of light on photoelectric current**

   Since the photoelectric current is directly proportional to the number of photoelectrons emitted per second which is also proportional to the intensity of incident radiation. Intensity ∝ photoelectric current ∝ number of photo electrons emitted per sec ∝ $\frac{1}{distance^2}$

3. **Maximum kinetic energy of emitted electrons is given as $\boldsymbol{K_{max} = eV_0 = \frac{1}{2}mv_{max}{}^2}$. Where m is the mass of photoelectron and V_{max} is the maximum velocity of photoelectrons. $\boldsymbol{V_0}$ is stopping potential or cut-off potential.**

4. **Einstein's photoelectric equation**

   When a photon of energy hf falls on a metal surface, the energy of the photon is absorbed by the electron and is used in the following two ways.

   (i)A part of energy is utilised to overcome the surface barrier and come out of the metal surface. This part of energy is called work function. W = hf_0

   (ii)The remaining part of energy is used in giving a velocity v to the emitted photoelectron. This is equal to maximum K. E. of the photoelectron. $K_{max} = ½\, mv_{max}{}^2$.

   According to the law of conservation of energy, $hf = W + \frac{1}{2}mv_{max}{}^2 = W + K_{max}$

   So, **Einstein photoelectric equation** is, $K_{max} = hf - W$

5. **Relation between stopping potential and frequency**

   $K_{max} = h(f - f_0)$. If stopping potential is V_0 then $K_{max} = eV_0$

   Therefore, $eV_0 = h(f - f_0)$, (here $f = \frac{c}{\lambda}$ and $f_0 = \frac{c}{\lambda_0}$)

   Then, $V_0 = \frac{hf}{e} - \frac{hf_0}{e} = \frac{h}{e}(f - f_0)$

6. **De-Broglie wave equation**:

   Considering photons as an em wave of frequency f, its energy from Planck's quantum theory is given by $E = hf - - - -(1) where\ h\ is\ Planck's constant$

   Considering photon as a particle of mass m, the energy associated with it is given by Einstein's mass energy relationship as $E = mc^2 - - - -(2)$

   From equation (1) and (2), we get $hf = mc^2 \quad or, \frac{hc}{\lambda} = mc^2 \quad or, \lambda = \frac{h}{mc} = \frac{h}{p} - - - -(3)$

   Where p (=mc) is the momentum of the photon. The above equation (3) has been derived for a photon of radiation.

   According to de-Broglie hypothesis, it must be true for material particles like electrons, protons, neutrons etc. Hence a particle of mass m moving with velocity v must be associated with a matter wave of wavelength λ , given by $\lambda = \frac{h}{p} = \frac{h}{mv} - - - -(4)$

   This equation (4) is the de-Broglie wave equation for material particles.

   It explains the dual nature of matter as it connects the wave characteristic 'λ' with particle characteristic 'p'.

From de-Broglie's equation, we find that The wavelength of moving particle is inversely proportional to its momentum, that is, $\lambda \propto \frac{1}{p}$

7. **De-Broglie wavelength of an electron**

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}}$$

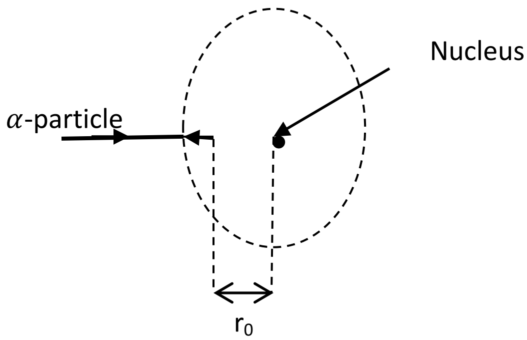
8. **For any charge particle,**

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

9. Similarly, de-Broglie wavelength for other charged particle can be given by
For proton, $\lambda = \frac{0.286}{\sqrt{V}}$ (in Å) , For deuteron, $\lambda = \frac{0.2o2}{\sqrt{V}}$ (in Å), For α-particle, $\lambda = \frac{0.101}{\sqrt{V}}$ (in Å)
For electrons, λ = $\frac{12.27}{\sqrt{V}}$ **(in Å)**

Ch-12:Atoms

1. **Distance of closest approach-Estimation of nuclear size**



The minimum distance from the nucleus up to which, α particle approaches and retraces its path, is called the distance of closest approach.
The charge on the α-particle is q_1 = + Z e and charge on nucleus, q_2 = + Ze where Z is the atomic number of foil atoms. So electrostatic potential energy of α-particle and nucleus at distance r_0 is

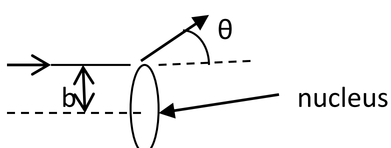
$$U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{2Ze^2}{r_0}\right)$$

This potential energy is equal to initial kinetic energy, so, initial K. E. of α-particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0}\left(\frac{2Ze^2}{r_0}\right) \Rightarrow r_0 = \frac{Ze^2}{m\pi\epsilon_0 v^2}$$

2. **Impact parameter**:

The scattering of α particle from a nucleus depends upon the closest approach to the nucleus.
''It is defined as the perpendicular distance of velocity vector of the α-particle from the central line of the nucleus when it is far away from the atom''.
The shape of the trajectory of scattered α-particle depends on the (i) impact parameter b and (ii) the nature of the potential field.



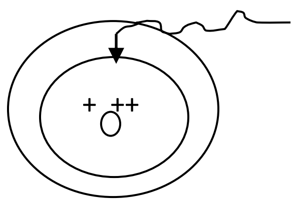
Rutherford deduced the following relationship between the impact parameter b and the scattering angle θ as

$$b = \frac{1}{4\pi\epsilon_0}\left(\frac{Ze^2 cot\frac{\theta}{2}}{K}\right), where\ K = \frac{1}{2}mv^2$$

For a head-on collision, the impact parameter b= 0, scattering angle θ = 180^0. That ie, α-particle is reversed back along its original path.

$$Impact\ parameter, b \propto \cot\left(\frac{\theta}{2}\right)$$

3. **Bohr's atomic model: Postulates of Bohr's theory of hydrogen atom:** Bohr's model based on the following **three postulates**.
Quantum condition: The electrons can revolve only in those orbits in which the angular momentum is the integral multiple of h/2 π. That is,
$L_n = mvr = \frac{nh}{2\pi}, where\ n = 1, 2, 3,$ are principal quantum number.
Stationary orbit: An electron in atom can move around nucleus in certain circular stable orbits without emitting radiations, is called stationary orbit.
Frequency condition: When the atom receives the energy from outside, then one (or more) of its outer electrons leaves its orbit and goes to some higher orbit. This state of atom is called **excited state**. The electron in the higher orbit stays only for 10^{-8} s and returns back to lower orbit. While returning back, the electron radiates energy in the form of electromagnetic waves



If the energy of the electron in the higher orbit be E_2 (here is E_i) and that in the lower orbit be E_1 (here is E_f) So, frequency of the emitted radiation is given by

$$f = \frac{E_2 - E_1}{h}, where, h = Planck's\ constant$$

This equation is called Bohr's frequency condition.

4. **Radius of electron: n^{th} radius of electron;**

$$r_n = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2} \quad Or, r_n \propto \frac{n^2}{Z}$$

Where, n = no.of orbit, Z = atomic number, m=mass of one electron,e = charge of one electron,h = Plancks constant, ϵ_0 = Permittivity of free space
For, H-atom, Z = 1; For Hydrozen like atom: For He^+,Z = 2, for Li^{++},Z = 2, For Be^{+++},Z = 4, Now, for Hydrogen atom, n^{th} radius is,

$$r_n = r_1 n^2\ or\ r_n \propto n^2$$

$$It\ means, ratio\ are: r_1 : r_2 : r_3 - - -= 1 : 4 : 9 - - -$$

Where r_1 = = 0.53 Ă (1 Ă= 10^{-10} m)= $\frac{\epsilon_0 h^2}{\pi m e^2}$

5. **Speed of nth orbit in Bohr's model of hydrogen atom**

$$v_n = \frac{e^2}{2\epsilon_0 nh}\ and\ v_n = \frac{v_1}{n}\ or\ v_1 \propto \frac{1}{n};\ v_1 = \frac{c}{137} = 2.19 \times \frac{10^6 m}{s}; c = speed\ of\ light\ in\ vacuum$$

6. **Speed of nth orbit in Bohr's model of hydrogen like atom**

$$v_n = \frac{Ze^2}{2\epsilon_0 nh}$$

7. **Fine structure constant** $\alpha = \frac{e^2}{2\epsilon_0 ch}, so\ v = \alpha\frac{c}{n}$

8. **Energy of the electron of H atom**:

The total energy in the nth orbit is the sum of kinetic energy and potential

$$E_n = -\frac{me^4}{8{\epsilon_0}^2 n^2 h^2}$$

$$E_n = \frac{E_1}{n^2} = \frac{-13.6}{n^2} = \frac{-Rhc}{n^2}\ \ (in\ eV)$$

9. **Some common terms used:**

(i) **Absorption of energy**: When the electron in the atom can jump from lower orbit to higher orbit, it absorbs energy.

(ii) **Emission of energy**: When an electron jumps from higher to lower orbit, it emits energy.

(iii) Energy of photon is equal to the difference of two orbits. $[E_i - E_f = hf]$

(iv) **Ground state and ground state energy**: The lowest state of an atom (n =1) is called ground state and energy associated with ground state is called ground state energy.

(v) **Ionization energy**: Energy required to ionise an atom is called ionisation energy. It is the energy required to make the electron jump from present orbit to the infinite orbit.

$$E_{ionisation} = E_\infty - E_n = 0 - (E_n) = 13.6\ eV$$

10. **Energy needed to remove the electron from any energy level means Ionisation Energy**.
(Minimum energy required to free the electron from ground state of the hydrogen atom is 13.6 eV, it is called ionization energy.)

11. **Ionization Potential** : The potential difference through which an electron should be accelerated to get ionization energy is called ionisation potential.

12. **Excitation energy**: The energy required to take an electron from lower state to higher state is known as excitation energy. If lower state energy is E_{n_1} and higher state energy is E_{n_2}, then excitation energy is $\Delta E_{n_1 n_2} = E_{n_2} - E_{n_2}$

13. **Wavelength of photon emitted in De-excitation**:
According to Bohr, when an atom makes a transition from one energy level to a lower level, it emits a photon with energy equals to the energy difference between the initial and final level. If E_i is the initial energy of the atom before such a transition, E_f is its final energy after the transition. And the photon energy is $hf = \frac{hc}{\lambda}$, then conservation of energy gives

$$E = hf = \frac{hc}{\lambda} = E_i - E_f energy\ of\ emitted\ photon$$

$$E_n = -\frac{Rhc}{n^2} = -\frac{13.6}{n^2}, where\ Rhc = 13.6\ eV$$

14. **Rydberg formula:** $\frac{1}{\lambda} = Z^2 R\left[\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right]$

Energy of the emitted radiation(H atom), $E = Rhc\left[\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right]$, here Rhc = -13.6 eV

Chapter-13: Nuclei

1. **Composition of nucleus**:
Nucleus is composed of two types of particles-(i) proton (positively charged particles) and (ii) neutrons, (neutral particles
Mass of neutron is slightly greater than the mass of proton.
Mass of proton, $m_p \approx 1840 m_e$ where m_e = mass of electron = 9.1 X 10^{-31} kg $\approx 0.0009 \times 10^{-27}$kg
Mass of proton $m_n = 1.6725 \times 10^{-27}$kg, (mass of neutron $m_n = 1.6747 \times 10^{-27}$kg)

2. **Atomic number**
Atomic number of an element is the number of proton inside the nucleus of a neutral atom. It is also equal to number of electrons revolving in various orbits around the nucleus of the atom. It is represented by Z. So, **Z = no. of proton = no. of electron**

3. **Mass number (A):**
The total number of neutrons and protons is called mass number. A chemical symbol or element is represented as ${}_Z^A X$ where X is any element. Total charge of nucleus is Ze, where e the charge on proton.

4. **Nuclear size**:
The volume of a nucleus is proportional to it mass number.$\frac{4}{3}\pi R^3 \propto A \quad (R\ is\ the\ radius\ of\ nucleus) \therefore \quad R \propto A^{\frac{1}{3}}\ or\ R = R_0 A^{\frac{1}{3}}$

R_0 is a constant= range of nuclear size. It is known as nuclear unit radius and its value is 1.2 x 10^{-15} m (1.2 fm).Hence, different nuclei of different elements have different size.

5. **Nuclear Density:**
It is the ratio of mass of nucleus to the volume of nucleus.
Let m be the average mass of nucleons (1.67 X 10^{-27} kg) and R is the radius of nucleus. If 'A' is the mass number of element, then $\rho = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3m}{4\pi {R_0}^3} \quad (R^3 = {R_0}^3 A)$
Thus, the density of a nucleus is constant and is independent of mass number A for all nuclei and ρ is found to be 2.3 x 10^{17} kg/m^3. Hence, matter in the nucleus is very densely packed.

6. **Nuclide**: It is a specific nucleus of an atom which is characterised by its atomic number Z and mass number A. example, ${}_{16}^{32}S$ together taken as one nuclide.

7. **Isotopes, Isobars and Isotones**
Isotopes: Atoms having same atomic number (Z) but different mass number (A).
For example, ${}_{92}^{235}U, {}_{92}^{238}U.\ {}_1^1H,\ {}_1^2H,\ {}_1^3H,\ {}_8^{16}O, {}_8^{17}O,\ {}_8^{18}O\ ;\ {}_2^3He,\ {}_2^4He$
Isobars: Atoms having same mass number (A) but different atomic number (Z)
For example,
Isotones: Nuclei containing same number of neutrons are called isotones. ${}_{17}^{37}Cl, {}_{19}^{39}K$.

8. **Atomic mass unit and energy equivalent to it**
A unit in which atomic and nuclear masses are measured is called atomic mass unit (u).
1 u = $\frac{1}{12}th$ of mass of an atom of ${}_6^{12}C$ isotope of carbon. 1 u (or 1 amu) = 1.66 x 10^{-27} kg.
1 u (or amu) represents the average mass of a nucleon.
1 eV = 1.602 x 10^{-19} J, 1u mass has energy 931.5 MeV.

9. **Mass defect** (Δm) = (sum of masses of neutrons and protons, or mass of nucleon) – actual mass of the nucleus
Certain mass disappears in the formation of a nucleus in the form of mass defect. That is,

$Zm_p + (A - Z)m_n - m_N$= mass defect. This mass loss appears in the form of energy (Einstein theory of mass-energy equivalence) called Binding Energy.

10. **Binding energy:**. Thus, binding energy is the minimum energy required to separate its nucleon and place them at rest at infinite distance apart. Or, it is defined as the energy equivalent to the mass defect of the nucleus.

$Binding\ energy = \Delta mc^2 = \left[Zm_p + (A - Z)m_n - m_N\right]c^2$. This binding energy is expressed in joule when Δm is in kg.

If Δm is measured in amu, then binding energy, $E_b = \Delta m\ amu = \{Zm_p + (A - Z)m_n - m_N\}$ = $\Delta m \times 931$MeV$where, Mass\ of\ nucleus\ or\ mass\ of\ atom, m_N = [m({}_Z^AX)]$

Here, m_p is the mass of proton and m_n is the mass of neutron.

$$Binding\ energy\ per\ nucleon, E_{be} = \frac{Total\ binding\ energy, E_b}{Mass\ number, A}$$

So, the higher the binding energy per nucleon, the larger will be stability.