Chapter-1: Electric Charges and Fields-Formulae

Property of charges:

- Quantization of charge, where n=integer multiple and e =-1.6x10^{-19} C

$$q=\pm ne$$

- Additivity of charges,

$$Q_{net} = \sum_{i=1}^{n} q_i$$

- Conservation of charges

$$Q_{net}\ (isolated) = Constant.$$

Coulomb's Force

- The below formula can be used when force on multiple individual charge is being calculated.

$$\vec{F_j} = \sum \vec{F}_{ij} = \frac{q_j}{4\pi\varepsilon}\sum_{i=1}^{n}\frac{q_i(\overrightarrow{r_{ij}})}{|\overrightarrow{r_{ij}}|^3}$$

- Coulomb's force between charge q_1 and q_2 separated by distance r and placed in vacuum, can also be expressed in vector form as-

$$\vec{F} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1 q_2}{r^2}\grave{r}$$

- In magnitude form, Coulomb force can be written as -

$$F = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1 q_2}{r^2} \text{ i. e. } F \propto \frac{1}{r^2} \text{ and } F \propto q_1 q_2$$

- Coulomb's force in vector form shows that it obeys Newton's third law of motion as-

$$\overrightarrow{F_{12}} = -\overrightarrow{F_{21}}$$

And we read this as ''force on charge q_2 due to charge q_1 = negative of force on charge q_1 due to q_2''.

$$\vec{F_j} = \frac{q_j}{4\pi\varepsilon}\sum_{i=1}^{n}\frac{q_i(\vec{r}-\vec{r_i})}{|\vec{r}-\vec{r_i}|^3}$$

The above formula can be used when force on any one charge is being calculated.
We read as force on charge j having position r due to charges q_1, q_2, q_3, ----having position r_1, r_2, r_3----placed in a medium. This is also called ''Principle of Superposition'' for force determination due to several charges.

Electric Field Intensity

- Electric field intensity E can be defined as

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

Where q0 is the test charge and is so small and not to disturb the strength of field produced by Source or point charge

- Electric field intensity at a point whose position is r due to other charges whose positions are r_1, r_2, r_3, -------can be expressed as- (Principle of Superposition for Electric field intensity)

$$\vec{E_J} = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{n}\frac{q_i(\vec{r}-\vec{r_i})}{|\vec{r}-\vec{r_i}|^3}$$

- Electric field intensity E at a point placed at distance r due to point or source charge q kept in free space is given by in vector form as-

$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{r^2}\ \grave{r}$$

- In magnitude form, Electric field intensity at any point in the region of electric field is given by-

$$E= \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{r^2} \text{ i.e. } E \propto \frac{1}{r^2} \text{ and } E \propto q$$

Dipole

- To find the net electric field intensity at Axial Point (A) at distance x from dipole centre
$\overrightarrow{E_A} = \frac{1}{4\pi\varepsilon}\frac{4aqx\acute{\imath}}{[x^2-a^2]^2}$
- Dipole moment (p) and its direction from –q to +q separated by 2a and placed in X-direction is given by –

$$\vec{p} = 2aq\grave{\imath}$$

- The net electric field intensity at Axial point If x>>a is given by-

$$\overrightarrow{E_A} = \frac{1}{4\pi\varepsilon}\frac{2\vec{p}}{x^3}$$

- In unit vector net field intensity in the direction X-axis at axial position is-

$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2p\grave{\imath}}{x^3}$$

$In\ m$agnitude form,

$$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2p}{x^3} \text{ This shows that } E \propto \frac{1}{x^3}$$

- To find the electric field intensity when dipole makes an angle θ with the horizontal x-axis

$$\overrightarrow{E_A} = \frac{1}{4\pi\varepsilon}\frac{4aqx(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})}{[x^2 - a^2]^2}$$

- To find the net field intensity at the equatorial point (B) of the dipole at distance x from the centre of dipole

$$\overrightarrow{E_B} = \frac{1}{4\pi\varepsilon}\frac{2aq(-\hat{\imath})}{[x^2 + a^2]^{\frac{3}{2}}}$$

- To find the field intensity at the equatorial point (B) of the dipole at distance x from the centre of dipole and when x>>a

$$\overrightarrow{E_B} = \frac{1}{4\pi\varepsilon}\frac{-\vec{p}}{x^3}$$

- In magnitude form,

$$\overrightarrow{|E_B|} = \frac{1}{4\pi\varepsilon}\frac{\overrightarrow{|p|}}{x^3}$$

- The ratio of net field intensity at axial point to the equatorial point is given by-

$$\frac{E_{axial}}{E_{equatorial}} = 2:1$$

- Torque in a dipole

$$\vec{\tau} = \vec{p}x\vec{E}$$

  Where $\vec{p} = p_x\hat{\imath} + p_y\hat{\jmath} = p\cos\theta\,\hat{\imath} + p\sin\theta\,\hat{\jmath}\ and\ \vec{E} = E\hat{\imath}$

  Therefore, $\vec{\tau} = pE\sin\theta\left(-\hat{k}\right)$ that is, the direction of torque is both dipole moment and electric field and pointing downward (going into the plane of paper) in clockwise acting from p to E at an angle θ between p and E.

- In magnitude form, Net torque

$$\tau = pE\sin\theta$$

- Potential energy in rotating a dipole at an angle θ is given by

$$U= - \vec{p}.\vec{E}$$

- In magnitude form, Potential in the dipole is

$$U=-pE\cos\theta$$

  The dipole is in stable equilibrium when $\theta=0^0$, U is minimum and equals to – pE. The dipole is in unstable equilibrium, when $\theta=180^0$, U is maximum and equals to +pE.

  In both stable and unstable equilibrium, $\tau_{net} = 0\ and\ F_{net} = 0$

Long straight charged conductor (infinite)

- Electric field intensity at distance r on x axis from the long charged conductor (infinite) having linear charge density λ is given by

$$E=\frac{\lambda}{2\pi\varepsilon_0 r}$$

  i.e. $E \propto \frac{1}{r}$ and the graph between E vs. r is rectangular hyperbola.

Field due to a uniform ring

- Electric field intensity due to a ring having radius R and linear charge density λ, at a distance x from the centre of the ring on axial position is given by-

$$E_x = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{qx}{\left((x^2+R^2)^{\frac{3}{2}}\right)}$$

And we see that $E_x \propto \frac{1}{x^2}$

- When x=0, field is zero at the centre of the ring.
- $E_x = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$ For x>>R when point is much farther from the ring, its field is same as that of a point charge.

It will be maximum when $\frac{dE_x}{dx}$=0 and $(E_x)_{max} = \frac{2}{\sqrt[3]{3}}\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{R^2}$

Electric Flux

- Total electric flux through a surface

$$d\varphi_E = \vec{E}.\overrightarrow{dS} \text{ or, } \varphi_E = \int \vec{E}.\overrightarrow{dS}$$

For uniform field, $\varphi_E = \vec{E}.\vec{S}$

Here, $\overrightarrow{dS}$ is a small area element vector perpendicular (normal) to a plane surface and pointing outward.

- In magnitude form,

$\varphi_E = ES\ \cos\theta$ where θ is the angle between Electric field $\vec{E}$ and $\overrightarrow{dS}$.

If any square or rectangular slab placed in co-ordinate axes , the field is uniform, the angle θ value is given by according to position of plane of the slab as mentioned below-

- If the plane is parallel to the electric field, $\theta=90^0$
- If the plane is perpendicular to the field, $\theta=0^0$
- If the plane makes an angle say 30^0, 60^0 etc, $\theta=90^0 - 30^0$ or 60^0

Gauss's Law

$$\varphi_E = \oint \vec{E}.\overrightarrow{dS} = \frac{q_{in}}{\varepsilon_0}$$

Electric field intensity due to infinite plane sheet (charge on one side of sheet) is given by-

$$E = \frac{\sigma}{2\varepsilon_0}$$

Electric field intensity due to a charged conductor –charged on both sides (near the conductor)

$$E = \frac{\sigma}{\varepsilon_0}$$

$$E_{in} = 0$$

Electric field intensity in between and outside two long parallel conductors having surface charge densities ±σ is as-

(i) Outside parallel conductor, E=0

(ii) Between parallel conductor, $E = \frac{\sigma}{\varepsilon_0}$

Electric field intensity in between and outside two long parallel conductors having surface charge densities +σ on each conductor is as-

(i) Outside parallel conductor, $E = \frac{\sigma}{\varepsilon_0}$

(ii) Between parallel conductor, E=0

Electric field intensity due to thin spherical shell or hollow shell having charge +q on its surface (surface charge density σ) and radius R.

(i) Inside the shell (r<R) E=0 as $q_{in} = 0$

(ii) Outside the shell (r>R), $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

(iii) On the surface of shell, r=R, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

- Electric Potential V =constant (inside the shell)
- Electric potential V = $\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ (on the surface)
- Electric Potential V = $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ (outside the sphere)

Electric field intensity due to solid spherical shell having charge +q distributed inside sphere (volume charge density ρ) and radius R

(i) Inside the shell (r<R) $E = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}$

(ii) Outside the shell (r>R), $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

(iii) On the surface of shell, r=R, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

For solid sphere,

- Electric potential V $= = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right]$ inside the solid sphere
- At the centre, r=0 and $V_c = \frac{3}{2}V_s$

(To find the electric potential, apply the formula dV=- Edr)