Chapter-1: Electric Charges and Fields

Static or frictional electricity:

Electricity produced due to friction of two bodies or rubbing each other. For example, (i) glass rod with silk clothe, (ii) a plastic comb with dry hair etc. In all these experiments, it is able to attract light objects.

Electricity:

The property of rubbed substances due to which they attract light objects is called electricity.

Electrically charged substances:

The substances after rubbing, gets electrified, is called electrically charged substances.

Electric charge:

‘’It is an intrinsic (inherent or essential) property of elementary particle of matter which gives rise to electric force between various objects’’.

If an object possesses electric charge, it is said to be electrified or charged. When it has no charge, it is said to be neutral.

Electric charge is a scale quantity.

Its SI unit is Coulomb (C). Its smaller units are milli-coulomb (mC, 1 mC = 10^{-3} C), micro-coulomb (μC, 1 μC = 10^{-6} C), nano-coulomb (nC, 1 nC =10^{-9}C), pico-coulomb (pC, 1 pC = 10^{-12} C) etc.

The CGS units are state coulomb and ab coulomb. 1Coulomb = 3 X 10^{9} stat coulomb or 1 C = 3 X 10^{9} esu (esu means electrostatic unit). 1 C = $\frac{1}{10}$ ab coulomb = $\frac{1}{10}$ emu (emu means electromagnetic unit.)

So, 1 ab coulomb = 3 X 10^{10} stat coulomb.

A proton has positive charge (+e) and electron has negative charge (-e). (where E = 1.6 x 10^{-19} C)

The dimensional formula of charge (q) is [q] = [AT]

Charge is:

1. **Transferrable: It can be transferred from one body to another.**
2. **Associated with mass: Charge can’t exist without mass but reverse is not true.**
3. **Conserved: It can neither be created nor be destroyed.**
4. **Invarient: It is independent of velocity of charged particle.**

Note: Electric charges produce electric field, magnetic field and electromagnetic radiation.

Charge is		
1. Stationary	Electric field is produced	Dealt in Ch-1 NCERT, Class 12 Physics
2. Charge is moving with uniform velocity	Both electric and magnetic field is produced	Dealt in Ch-4 NCERT, Class 12 Physics
3. Charge is moving with accelerated velocity	Electric field, magnetic field and electromagnetic radiations are produced.	Dealt in Ch-8 NCERT, Class 12 Physics

Matter is neutral if no. of electrons = number of protons

Kind of electric charges:

There are two types of charges.

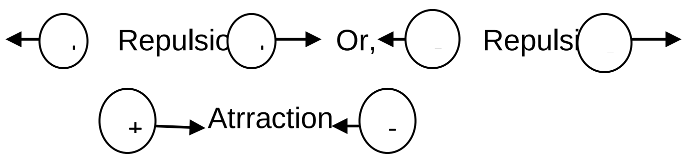
One is positive and another is negative. If a body gives electrons, it is said to be positively charged and its mass will decreases. (mass of electrons $m_e = 9.1 \times 10^{-31}$ kg) and if a body acquires electrons, it is said to be negatively charged and in this way, its mass increases.

In positively charged body, number of electrons is less than (<) number of protons, $(n_e < n_p)$.

In negatively charged body, number of electrons is greater than (>) number of protons, $(n_e > n_p)$.

In neutral body, the number of electrons = number of protons, $n_e = n_p$

Note: Two like charges (+ve and +ve or –ve and –ve) repel each other and two unlike charges (+ve and –ve attract each other. The direction of repulsive and attractive forces are shown as



Method of charging:

(i) By friction or rubbing (hand held),

(ii) By conduction or in contact and (iii) by induction or nearby but not in contact with body

By friction or rubbing:

'When two bodies are rubbed together, both positive and negative charges appear simultaneously due to the transfer of electrons from one body to the other bodies'. That is, one body acquires positive charge and other will acquire negative charge and of same magnitude as of first one.

There are few materials which are listed below in sequence and when first one is rubbed with another body which is coming after the first one, the the first body will be positively charged and another one will be negatively charged.

Note: The charge on body will appear when two bodies are held with insulated material in hand. These are:

(1) Fur (cat's eye), (2) Flannel (soft woollen cloth), (3) sealing box, (4) Glass, (5) cotton, (6) paper, (7) silk, (8) human body, (9) wood, (10) metals, (11) rubber, (12) resin, (13) amber, (14) sulphur, (15) ebonite, (16) gutta percha, For example, if glass rod is rubbed with silk cloth, the glass rod gets positive charge and silk cloth will be negatively charged. But if glass rod is rubbed with fur, the glass rod will possess negative charge and fur will be positively charged.

A table is also given below for two bodies to be rubbed together, which will have positive charge and which will have negative charge.

Positive charge	Negative charge
Glass rod	Silk
Fur	Ebonite rod
Comb	Dry hair
Wool	Amber

Note: Electric charges remain confined only to the rubbed portion of a non-conductor (insulator)but in case of a conductor, charges are spread up throughout the conductor.

By Conduction or in contact

If one charged body is brought in contact with another uncharged body of similar size (identical size), the charges will be shared equally by both bodies after separation. For example, body A has charge q and it is brought in contact with another charged body B, then after separation, body A will have charge q/2 and body B will also have charge q/2. Similarly if body A has charge –q and is brought in contact with body B which is uncharged and has identical size with that of body A, then after separation, body A will have charge –q/2 and body B will also have charge –q/2.

Note: (2) If both bodies are not identical in sizes, charges will be different. Then charges will be shared according to their radius. For example, the body A has charge q' and radius R_1 and is brought in contact with body B of size (radius) R_2, then after separation, the charge distribution will be as: $q \propto R$, so, $\frac{q_1}{q_2} = \frac{R_1}{R_2}$ then after separation, body A will have charge $q_1 = q\frac{R_1}{R_1+R_2}$ and $q_2 = q\frac{R_2}{R_1+R_2}$, q = q'

When made in contact of two bodies, the potential will be same of each body.

Property of charges:

(i) **Quantization of charge**, where n=integer multiple and e =-1.6x10^{-19} C. That is, q=±ne, where n = 1, 2, 3, -------, that is charge on any body exists as integral multiple of electronic charge. The charge on any body can never be $\frac{e}{3}$ or $1.5e$, etc.

Apart from charge, energy and angular momentum are also quantized. Quantum of energy is $E = nhf$, here n = number of photon and energy of one photon, $E = hf$ and quantum of angular momentum is $L = \frac{nh}{2\pi}$, here n is principle quantum number in Bohr's orbit or Bohr's orbital number.

(ii) **Additivity of charges**, $Q_{net} = \sum_{i=1}^{n} q_i$

(iii) **Conservation of charges** , $Q_{net}\ (isolated) = Constant$. That is, the charge can neither be created nor destroyed but it may simply be transferred from one body to the other. So, we can say that charges can be created or destroyed in equal and opposite pair. For example, in pair production process, $\gamma \rightarrow e^- \quad + e^+$

(electron) (positron)

(iv) Similar charges repel and dissimilar charges attract each other.

(v) A charged body attracts light uncharged bodies due to polarisation of uncharged bodies.

Conductors and Insulators:

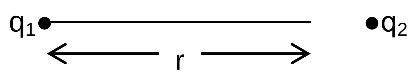
Conductors: The materials which allow electric charge to flow freely through them are called conductors. Metals are very good conductor of electricity. Silver, copper and aluminium are some of the best conductors of electricity. Our skin is also a conductor of electricity. Graphite is the only non-metal which is a conductor of electricity.

All metals, alloys and graphite have free electrons which can move freely through out the conductor. These free electrons make metals, alloys and graphite good conductor of electricity. Aqueous solutions of electrolytes are also conductor.

Insulators: The materials which do not allow electric charge to flow through them are called non-conductors or insulators. For example, plastic, rubber, non-metals (except graphite), dry wood, wax, mica, porcelain, dry air, etc. are insulators. Insulators can be charged but do not conduct electric charge. Insulators do not have free electrons that is why insulators do not conduct electricity. Induced charge can be lesser or equal to inducing charge and its mass value is given by $q' = -q\left[1 - \frac{1}{K}\right]$.

Coulomb's Force in electrostatics: Coulomb's law:

If two stationary point charges q_1 and q_2 are kept in space and separated by distance r, then magnitude of force of attraction or repulsion is directly proportional to product of two charges and inversely proportional to square of distance between two charges and this is given as



$F \propto q_1 q_2$ and $F \propto \frac{1}{r^2}$ or, $F = k\frac{|q_1||q_2|}{r^2}$ (**Coulomb's law in magnitude form)**

Here, k = 1 (for air) in CGS unit and $F = \frac{|q_1||q_2|}{r^2}$

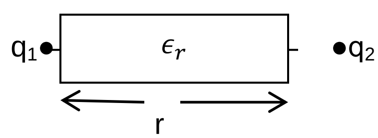
In S. I. unit, $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 = 9 \times 10^9$ N-m^{2}C^{-2} (for air or free space)

Here, $\epsilon_0 = 8.85 \times 10^{-12}$ C^{2}N^{-1}m^{-2} [or C^2/N-m^2 or Farad/m (Fm^{-1})]=absolute permittivity of air or free space.

Dimensional formula of permittivity of free space ($\boldsymbol{\epsilon_0}$) :

$$[\epsilon_0] = [M^{-1}L^{-3}T^4A^2]$$

Effect of medium:

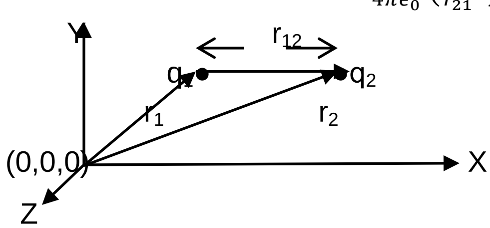


Then, (Here, medium thickness is t =r), $F_{medium} = \frac{F_{air}}{\epsilon_r} = \frac{1}{4\pi\epsilon_0\epsilon_r}\left(\frac{|q_1||q_2|}{r^2}\right)$

If If di-electric medium of constant $(K\ or\ \epsilon_r)$ and thickness ((t<r) is partially filled by two charges q_1 and q_2 which are separated by r, then the effective air separation between charges becomes $(r - t + t\sqrt{K})$ and the force between two charges is given by $F = \frac{1}{4\pi\epsilon_0}\left(\frac{|q_1||q_2|}{(r-t+t\sqrt{K})^2}\right)$

Coulomb's law in vector form:

Force on charge q_2 due to charge q_1 is given by $\vec{F}_{21}$ and force on charge $q_1$due to charge q_2 is given by $\vec{F}_{12}$ and these forces are $\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1q_2}{{r_{21}}^2}\right)\hat{r}_{21}\ and\ \vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1q_2}{{r_{12}}^2}\right)\hat{r}_{12}$



$\vec{r}_1 + \vec{r}_{12} = \vec{r}_2$

and $\hat{r}_{12} = \frac{\vec{r}_{12}}{r_{12}}$

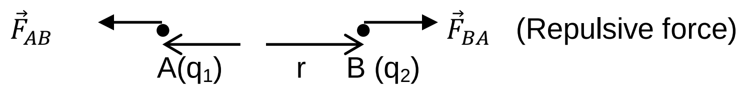
So, the **Coulomb's law in vector form** is, (in free space)

$$F = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1q_2}{r^2}\right)\hat{r} \quad (\boldsymbol{in\ Vector\ form})$$

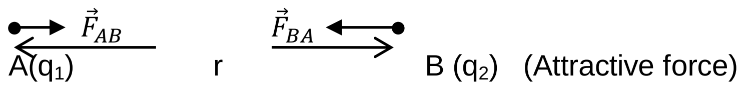
Here $k = \frac{1}{4\pi\epsilon_0\epsilon_r} = Coulomb's\ constant\ (+ve)$

Direction of force: Suppose the charge q_1 (placed at point A) and q_2 (placed at point B) and separated by distance r, then two situations arise and direction of force is given and shown as:

Situation-1: q_1 and q_2 are of similar sign (both +ve or both –ve)



Situation-2: q_1 and q_2 are of opposite sign (q_1 is +ve and q_2 is negative or q_1 is –ve and q_2 is +ve)

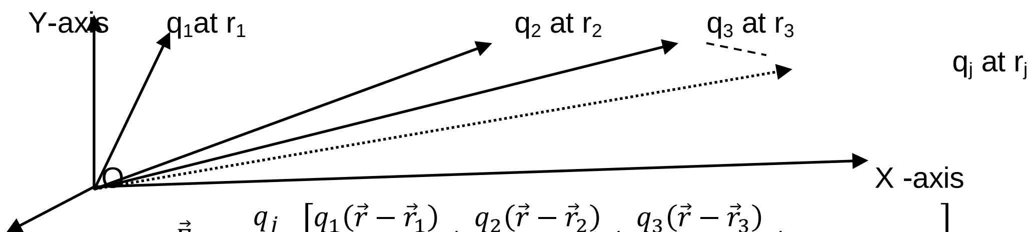


Superposition principle for Coulomb's force:

Suppose several charges q_1, q_2, q_3,---- are placed in free space and we have to find out force on any one charge, then resultant or net force acting on that charge will be the vector sum of all forces and is given as $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ----$

The below formula can be used when force on individual charge due several charges in space is being calculated.

$$\vec{F_j} = \sum_{i=1}^{n} \vec{F}_{ij} = \frac{q_j}{4\pi\varepsilon} \sum_{i=1}^{n} \frac{q_i(\overrightarrow{r_{ij}})}{\left|\overrightarrow{r_{ij}}\right|^3} Or,$$



$$\vec{F_j} = \frac{q_j}{4\pi\epsilon_0}\left[\frac{q_1(\vec{r}-\vec{r}_1)}{|\vec{r}-\overrightarrow{r_1}|^3} + \frac{q_2(\vec{r}-\vec{r}_2)}{|\vec{r}-\overrightarrow{r_2}|^3} + \frac{q_3(\vec{r}-\vec{r}_3)}{|\vec{r}-\overrightarrow{r_3}|^3} + -----\right]$$

Coulomb's force in vector form shows that it obeys Newton's third law of motion as-

$$\overrightarrow{F_{12}} = -\overrightarrow{F_{21}}$$

And we read this as ''force on charge q_2 due to charge q_1 = negative of force on charge q_1 due to q_2''.

Limitations of Coulomb's law:

(1) The electric charges must be at rest

(2) The electric charges must be point charges. (That is, extension of charges is much smaller than separation between charges.)

(3) The separation between charges must be greater than the nuclear size (10^{-15} m) and less than macroscopic distance (10^{18} m).

Inverse square is valid over this range of separation (mention in point 2 and point 3) to a high degree of accuracy.

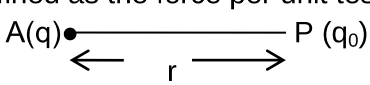
Electric field:

The region or space around a charge or group of charges in which its effects can be experienced. If the charge exists, there is field line and if it does not, no field exists.

The point on the electric field line shows the electric field strength or electric field intensity and the tangent drawn at that point gives the direction of electric field.

Electric Field Intensity:

It is defined as the force per unit test charge (or simply charge) at any point. That is,



Electric field intensity E can be expressed as $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$ $or\ simply$, $\vec{E} = \frac{\vec{F}}{q}$

The direction of electric field intensity depends on the sign of charge. That is, if charge is positive, the direction of electric field intensity is along the direction of force and if charge is negative, the direction of electric field intensity is opposite to the direction of force.

Where q_0 is the test charge (means it is so small and which do not disturb the strength of field produced by Source or point charge)

Unit of electric field intensity:

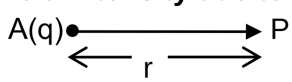
In SI unit, its unit is Newton /Coulomb or as N/C. Another unit of electric field intensity is Volt/metre or as V/m (because, $E = \frac{Newton}{Coulomb} = \frac{Newton \times metre}{Coulomb \times m} = \frac{Joule}{Coulomb\ metre} = \frac{volt}{metre}$)

Its CGS unit is dyne/stat coulomb.

Dimensional formula of electric field intensity:

$$[E] = [MLT^{-3}A^{-1}]$$

Electric field intensity due to a point charge:



Electric field intensity at point P due to a point charge q at distance r is given as:

$$\vec{E}_P = \frac{\vec{F}_{PA}}{q_0} = \frac{1}{4\pi\epsilon_0}\left(\frac{qq_0}{q_0 r^2}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\right)$$

So, the electric field intensity at any point P due to a point charge q at distance r can be expressed in vector form as:

$$\vec{E} = \frac{1}{4\pi\epsilon}\left(\frac{q}{r^2}\right)\hat{r}\ (\boldsymbol{in\ vector\ form})$$

If the charge or charges are placed in free space or in air, $\epsilon_r = 1$ hence, $\epsilon = \epsilon_0\epsilon_r = \epsilon_0$ and the equation (13) can be written as

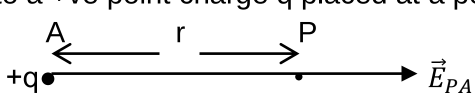
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\right)\hat{r}\ \ (in\ vector\ form)$$

In magnitude form, the electric field intensity at a point at distance r from the point charge in free space or in air can be expressed as

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{|q|}{r^2}\right)\ \ (\boldsymbol{in\ magnitude\ form})$$

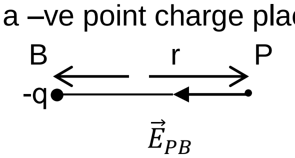
Direction of electric field intensity: Its direction is shown in the following figures.

(i) Due to a +ve point charge q placed at a point (say at point A) in free space or in air:

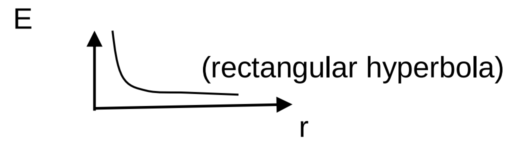


So, due to a positive point charge, the direction of electric field intensity at point (say P) is away from that point and along AP (that is, line joining point charge (+q) and the point P.). Its magnitude is $E_{PA} = \frac{1}{4\pi\epsilon_0}\left(\frac{|q|}{r^2}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\right)$

(ii) Due to a –ve point charge placed at a point (say at point B) in free space or in air:



So, due to a negative point charge, the direction of electric field intensity at point (say P) is towards negative charge from that point and along PB (that is, line joining point charge(-q) and the point P.)



Its magnitude is $E_{PB} = \frac{1}{4\pi\epsilon_0}\left(\frac{|-q|}{r^2}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\right)$

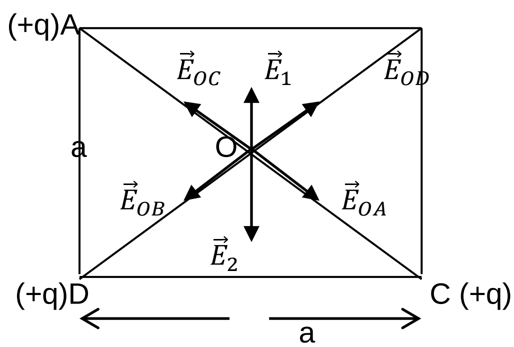
We see that $E \propto \frac{1}{r^2}$ due to a point charge.

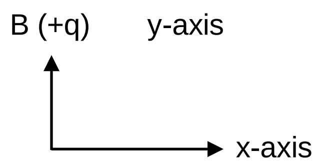
Superposition principle of electric field intensity at a point (say point j) due to several charges:

Electric field intensity at a point whose position is r due to other charges whose positions are r_1, r_2, r_3, -------can be expressed as- (Principle of Superposition for Electric field intensity)

$$\vec{E_J} = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{n}\frac{q_i(\vec{r}-\vec{r_i})}{|\vec{r}-\vec{r_i}|^3} ------(14)$$

Note: Electric field intensity at centre of square due to same charge and of same sign at each corner is equal to zero. Say, if each charge is positive as shown below.

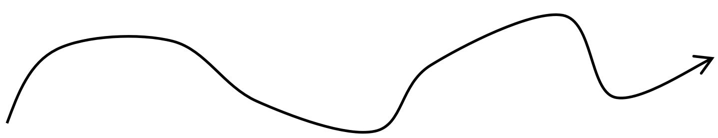




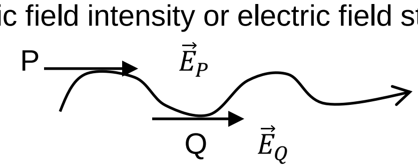
Since, OA =OC =OB = OC = $\frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$

Similarly, in an equilateral triangle also, at the centroid, the net electric field intensity is zero.

Electric field lines: These lines are imaginary lines (either straight lines or curved lines) as :



The tangent line drawn at any point shows the direction of electric field line and at that point, electric field intensity or electric field strength is calculated.

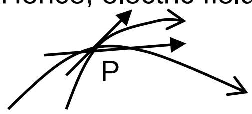


Properties of electric field lines:

(i) Electric field lines originate from positive charge (isolated +ve point charge) radially and terminate on infinity. Likewise, electric field lines terminate on –ve charge and originate from infinity.

(ii) Two electric field lines never cross each other. **Why? Explain**.

Answer: If they cross each other, there will be two tangents at the point of intersection and so, there will be two directions of electric field at the same point, which is not possible. Hence, electric field lines never cross each other.



(iii) Electric field lines never form a closed loop. **Why? Explain**.

Answer: Because electric field line never originates and terminates from the same charge.

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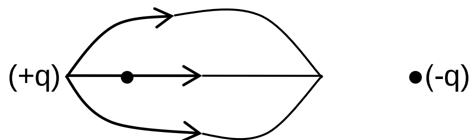


(iv) Electric field lines are perpendicular to the surface of conductor. **Why? Explain**.

Answer: If not, there will be horizontal component of electric field, it means current will flow on the surface of conductor but charges are stationary, so it is always perpendicular.

(v) Electric field lines are a continuous curve. **Why? Explain**.

If there is break in line, charge will jump but charge is static, so it is a continuous curve.

(vi) Magnitude of charge is directly proportional to number of field lines. That is,

$$q \propto N \ or \ \frac{q_A}{q_B} = \frac{N_A}{N_B}$$

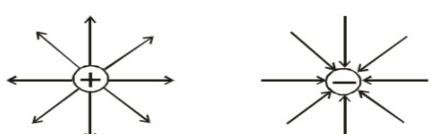
That is, if if $q_A = 2\mu C$ and $q_B = 1\mu C$ and 100 lines pass through q_A, hence 50 lines will pass through q_B.

(vii) Electric field lines do not exist inside a conductor because charge inside a conductor is zero.

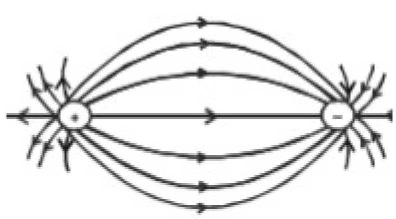
(viii) Electric field lines contract lengthwise indicating that unlike charges attract each other. It is cylindrically symmetric.

To draw electric field lines:

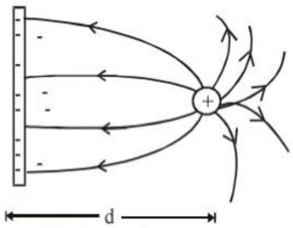
(i) Due to an isolated positive charge and negative charge. (Spherical symmetric)



(ii) Due to a dipole (same magnitude and opposite sign charge, separated by small distance (cylindrically symmetric)

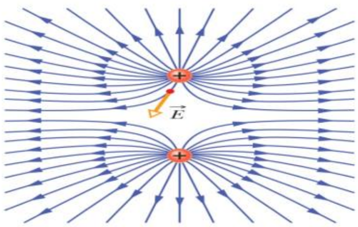


(iii) Due to infinite line charge and –ve point charge separated at distance d.



Fixed point charge
near infinite metal plate

(iv) Due to two similar and of same sign separated by small distance



Electric dipole and dipole moment

Electric dipole: When two static point charges, same in magnitude and opposite in sign, are placed small distance apart in the medium (say, air/free space/or any other medium), they form a dipole.

For example,

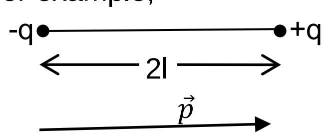
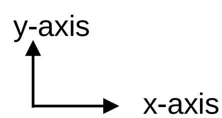


Fig-1 (one dipole)



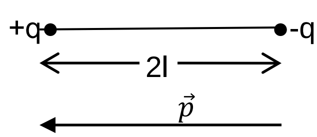
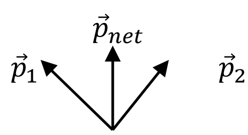


Fig-2 (one dipole)



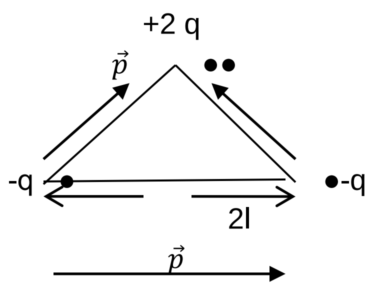


Fig-3 (two dipoles)

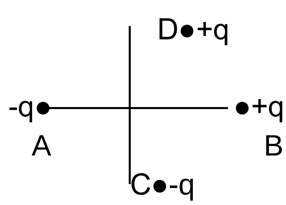


Fig-4 (two dipole)

Here, AB = CD = 2l (say)

Electric dipole moment ($\vec{p}$): It is defined as the product of magnitude of any of two charges and separation between two charges making dipole. Let two static point charges –q and +q are placed in free space and separated by 2l distance as shown in Fig-1 and Fig-2 above, then, magnitude of dipole moment is $p = |\vec{p}| = |q|(2l)$

Note: The electric dipole moment is a vector quantity.

Direction of electric dipole moment: Its direction is from –q charge to +q charge. The direction of electric dipole moment is shown in Fig-1 and Fig-2 above.

In vector form, the electric dipole moment is shown as $\vec{p} = q(2\vec{l})\ or, \vec{p} = |q|(2l)\hat{r}$

Important: As shown in Fig-1 above, the electric dipole moment in vector form is,

$$\vec{p} = |q|(2l)\hat{\imath} = q(2l)\hat{\imath}$$

And, in the Fig-2 above, the electric dipole moment in vector form is,

$$\vec{p} = |q|(2l)(-\hat{\imath}) = q(2l)(-\hat{\imath})$$

SI unit of electric dipole moment: Its SI unit is C-m (coulomb metre). Other very small unit is **Debye**. [1 Debye = 3.3 x 10^{-30} C-m]

Dimensional formula of electric dipole moment: $[p] = [M^0][L][T][A]$

Net electric force and torque in dipole:

(i) **When a dipole is placed in uniform electric field**:
The net force acting on dipole is equal to zero. The net torque acts on dipole. The net force is equals to zero and it is explained as below.

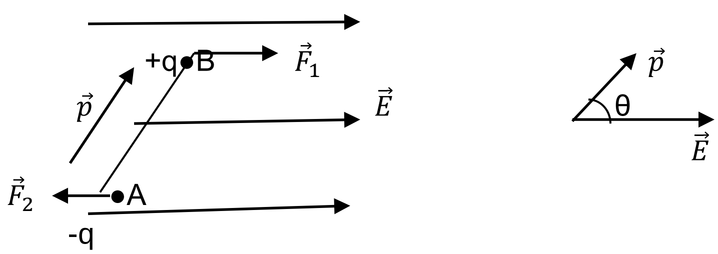


Fig-5

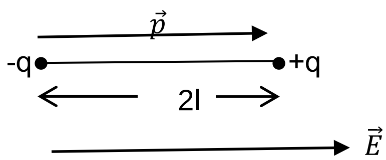
In the above figure (Fig-5), $\vec{F}_1 = q\vec{E} = qE\hat{\imath}$ and $\vec{F}_2 = -q\vec{E} = qE(-\hat{\imath})$, hence,
Net force acting on dipole is

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = qE(\hat{\imath}) + qE(-\hat{\imath}) = qE(1-1)(\hat{\imath}) = 0$$

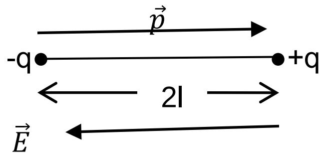
(ii) When a dipole is placed in non-uniform electric field:
The net electric force is not zero and also torque acts on dipole.

Stable and unstable equilibrium of dipole:

Stable equilibrium position: In this position, the direction of electric dipole moment and the direction of electric field are same. That is, the angle between $\vec{p}$ and $\vec{E}$ is zero. Further, it means, $\vec{p}$ ↑↑ $\vec{E}$ (both are parallel). It is shown below.



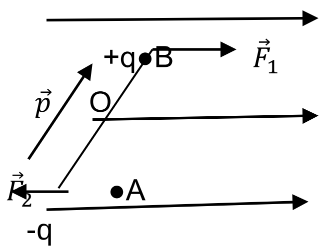
Unstable equilibrium position: In this position, the direction of electric dipole moment is opposite to the direction of electric field. That is, the angle between $\vec{p}$ and $\vec{E}$ is 180^0. Further, it means, $\vec{p}$ ↑↓ $\vec{E}$ (both are anti-parallel). It is shown below.



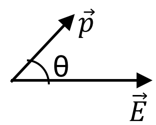
Electric torque due to dipole placed in uniform electric field:

Derivation # 1: To derive torque acting due to a dipole of charge $\pm q$, separated by 2l distance and placed in a uniform electric field

Answer: Let a charge –q is placed at point A and another charge +q is placed at point B and both charges are separated by 2l. Let O be the centre of dipole. The dipole is placed in uniform electric field $\vec{E}$. Here AB = 2l



(Fig-6)



(Fig-7)

The force acting at charge +q at point B is $\vec{F}_1$ and at point B at charge –q is $\vec{F}_2$. The torque acting due to force $\vec{F}_1$about the point O is $\vec{\tau}_1 = \overrightarrow{OB} \times \vec{F}_1 - - - -(i)$

The torque acting due to force $\vec{F}_2$about the point O is $\vec{\tau}_2 = \overrightarrow{OA} \times \vec{F}_2 - - - -(i)$

The net torque about point O is

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 = \overrightarrow{OB} \times \vec{F}_1 + \overrightarrow{OA} \times \vec{F}_2 = \overrightarrow{OB} \times \left(q\vec{E}\right) + \overrightarrow{OA} \times (-q)\vec{E} = \left(\overrightarrow{OB} - \overrightarrow{OA}\right) \times q\vec{E}$$

Or, $\vec{\tau} = q\left(\overrightarrow{OB} + \overrightarrow{AO}\right) \times \vec{E} = q\left(\overrightarrow{AO} + \overrightarrow{OB}\right) \times \vec{E} = q\left(2\vec{l}\right) \times \vec{E} = \vec{p} \times \vec{E}$

Here, electric dipole moment is $\vec{p} = q\left(2\vec{l}\right)$, hence, net torque acting about centre of dipole is

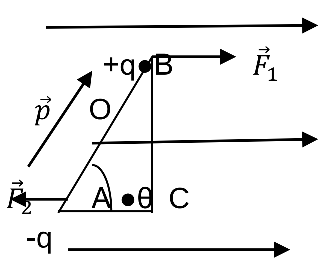
$\vec{\tau} = \vec{p} \times \vec{E} - - - - - - (17)$ *in Vector form*

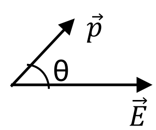
In magnitude form, net torque $(\vec{\tau})$acting due to dipole about O placed in uniform electric field $\left(\vec{E}\right)$ is given as $\tau = pE\sin\theta - - - - - - - - (18)$ *in magnitude form*

Here, θ is the angle between $\vec{p}$ (dipole moment) and $\vec{E}$ (electric field intensity). ■

Direction of Torque: It is taken from $\vec{p}$ to $\vec{E}$ by using Right hand Thumb Rule, in which Palm (with fingers) are rotated from $\vec{p}$ to $\vec{E}$ and thumb will point the direction of torque.

Another method: As shown in the below arrangement, let AB = 2l, The direction of dipole moment is shown which is from –q (at point A) to +q (at point B). Let $\vec{p}$ makes angle θ with $\vec{E}$ as shown in the below arrangement. Since, $BC = AB\sin\theta = 2l\sin\theta$ in right angle triangle ABC ($\angle$C = 90^0).





(Fig-7)

The net torque acting about O is

$$\tau = perpendicular\ distance\ BC \times Force = AB \sin\theta \times F = 2l \sin\theta \times qE$$

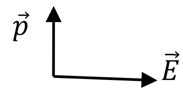
$= q \times 2l \times E \times \sin\theta = pE \sin\theta$ (where p = dipole moment = q x 2l)

So, $\tau = pE \sin\theta$ (**Derived**)

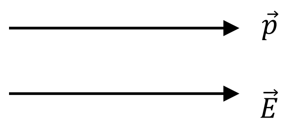
Dependence of Torque on angle θ: The torque acting on dipole is given as

$$\tau = pE \sin\theta$$

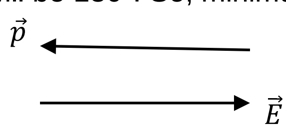
(i) Torque will be maximum when $\vec{p} \perp \vec{E}$ that is, angle between $\vec{p}$ and $\vec{E}$ will be 90^0.So, maximum torque $\tau_{max} = pE \sin 90^0 = pE$



(ii) **In stable equilibrium condition**, torque will be zero as $\vec{p} \uparrow\uparrow \vec{E}$, that is, angle between $\vec{p}$ and $\vec{E}$ will be 0^0. So, minimum torque, $\tau_{min} = pE \sin 0^0 = 0$



(iii) In the unstable equilibrium condition, torque will be zero as $\vec{p} \uparrow\downarrow \vec{E}$, that is, angle between $\vec{p}$ and $\vec{E}$ will be 180^0. So, minimum torque, $\tau_{min} = pE \sin 180^0 = 0$



Derivation # 2: To derive potential energy due to a dipole of charge $\pm q$, separated by 2l distance and placed in a uniform electric field.

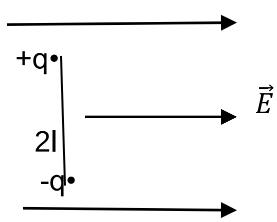


Fig-1

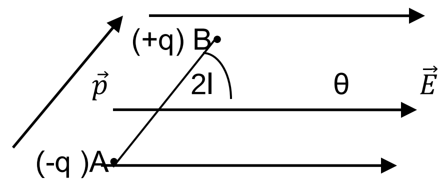


Fig-2

Answer: As shown in the Fig-1, the potential energy stored due to dipole is zero. {we will show it in the last.] In Fig-2, a dipole is placed in uniform electric field in which electric dipole moment $\vec{p}$ making angle θ with $\vec{E}$ as shown. The torque acting in this case is downward clockwise from perpendicular position of dipole shown in Fig-1.

So, small work done to rotate a dipole through small angle dθ is

$$dW = \tau d\theta - - - - - (i)$$

Since torque is acting opposite to work done so, putting $\tau = -pE \sin\theta$.

Now, putting the value of torque τ in equation (i), we get

$$dW = -pE \sin\theta\, d\theta - - - (ii)$$

Now, potential energy stored, $dU = -dW = pE \sin\theta\, d\theta$ (by the electric force), hence, energy stored due to dipole by rotating it from 90^0 to θ is given as

$$U = \int dU = \int_{90^0}^{\theta} pE \sin\theta \, d\theta = pE\,(-\cos\theta)_{90^0}^{\theta} = -pE\cos\theta$$

Hence, energy stored in dipole placed in uniform electric field is, $U = -pE\cos\theta — (19)$

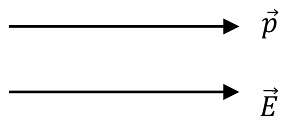
In vector form, energy stored =Potential energy in dipole = $-\vec{p}\cdot\vec{E} - -(20)$, **Derived**.

Now, we can check that when $\vec{p} \perp \vec{E}$ as in Fig-1, θ =90^0 and from equation (19), U = 0

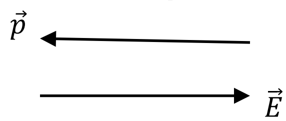
Dependence of potential energy on angle θ:

The potential energy depends on angle θ as from equation (19),

(i) **In stable equilibrium condition**, potential energy will be minimum as $\vec{p} \uparrow\uparrow \vec{E}$, that is, angle between $\vec{p}$ and $\vec{E}$ will be 0^0. So, minimum P. E. , $U_{min} = -pE\cos 0^0 = -pE$ $(since\ \cos 0^0 = 1)$



(ii) **In the unstable equilibrium condition**, Potential energy will be maximum as $\vec{p} \uparrow\downarrow \vec{E}$, that is, angle between $\vec{p}$ and $\vec{E}$ will be 180^0. So, maximum, $U_{max} = -pE\cos 180^0 = +pE$



(iii) Potential energy is zero when $\vec{p} \perp \vec{E}$ as in Fig-1, θ =90^0 and from equation (19), U = 0

Work done in rotating a dipole:

(i) Work done in rotating a dipole from angle $\theta_1\ to\ \theta_2$by the electric force is:

$$W = -\Delta U = -\left(U_{\theta_2} - U_{\theta_1}\right) = -[-pE\cos\theta_2 — (-pE\cos\theta_1) = pE(\cos\theta_2 - \cos\theta_1)$$

(ii) Work done in rotating a dipole from angle $\theta_1\ to\ \theta_2$against the electric force Or by external agent is,

$$W = \Delta U = \left(U_{\theta_2} - U_{\theta_1}\right) = [-pE\cos\theta_2 — (-pE\cos\theta_1) = pE(\cos\theta_1 - \cos\theta_2)$$

Note: If in a question, it is asked find the work done but nothing is mentioned by the electric force or against the electric force, then apply the formula by against the electric force as mentioned in equation (22) as given by $W = \ pE(\cos\theta_1 - \cos\theta_2)$

For example,

Find the work done to rotate a dipole from stable equilibrium to unstable equilibrium when the magnitude of electric dipole moment is p and magnitude of uniform electric field is E in the x-direction.

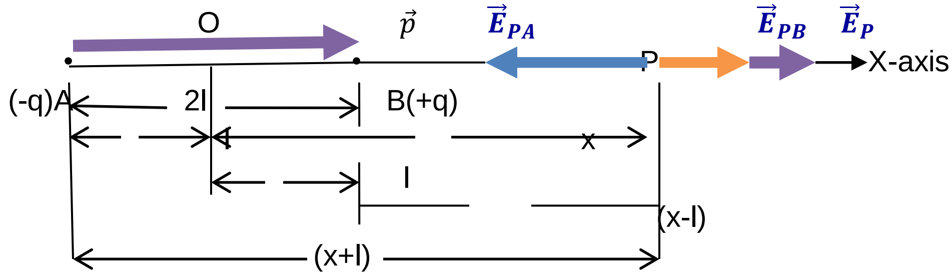
Solution: We know that work done in rotating a dipole from angle $\theta_1\ to\ \theta_2$ when dipole is placed in uniform electric field, is

$$W = \ pE(\cos\theta_1 - \cos\theta_2) - - - - - (i)$$

We also know that in stable equilibrium, $\theta_1 = 0^0$ and in unstable equilibrium, $\theta_2 = 180^0$. Hence from equation (i) above,

$W = \ pE(\cos\theta_1 - \cos\theta_2) = pE(\cos 0^0 - \cos 180^0) = pE[1 - (-1)] = 2pE$, **Ans.**

Derivation # 3: To derive the expression of electric field intensity due to a dipole of charge $\pm q$, separated by 2l distance at axial position when x >>l when placed in free space.



Answer: Let a dipole of charge $\pm q$ are separated by small distance 2 l and placed along x-axis as we have to derive the electric field strength or intensity in axial position. Let P be the where field intensity is being derived and point P is at distance x from the centre of dipole O as shown in the above figure. Let –q is placed at point A, +q at point B and distances of point P from point A and point B are also shown.

The direction of electric dipole moment is from –q charge to +q charge as shown in the figure.

Now, magnitude of electric field intensity at point P due to charge +q at point B is

$$E_{PB} = |\vec{E}_{PB}| = \frac{1}{4\pi\epsilon_0}\left(\frac{|q|}{(x-l)^2}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{(x-l)^2}\right) - -(i), away\ from\ P\ along\ BP$$

Magnitude of electric field intensity at point P due to charge -q at point A is

$$E_{PA} = |\vec{E}_{PA}| = \frac{1}{4\pi\epsilon_0}\left(\frac{|-q|}{(x+l)^2}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{(x+l)^2}\right) - -(ii), towards - q\ along\ PA$$

Since, $E_{PB} > E_{PA}(\ as(x-l) < (x+l)$, so, the net electric field intensity at point P due to both point charges –q and +q (making a dipole) is along AB and away from point P and its magnitude is given by,

$$E_{net} = E_{PB} - E_{PA} = \frac{1}{4\pi\epsilon_0}\left[\left(\frac{q}{(x-l)^2}\right) - \left(\frac{q}{(x+l)^2}\right)\right] = \frac{1}{4\pi\epsilon_0}\left[q\left\{\frac{(x+l)^2-(x-l)^2}{(x^2-l^2)^2}\right\}\right]$$

Or, $E_{net} = E_P = \frac{1}{4\pi\epsilon_0}\left[\frac{4qxl}{(x^2-l^2)^2}\right] = \frac{1}{4\pi\epsilon_0}\left[\frac{2px}{(x^2-l^2)^2}\right] - - - -(iii)$, where, p = electric dipole moment Or , p = q (2l), we also see that direction of net electric field intensity is along the direction of electric dipole moment (that is, the case of stable equilibrium).

If $x \gg l, then x^2 \gg l^2$, so, from equation (iii),

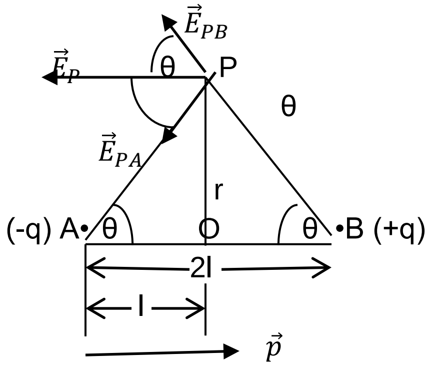
$$\boldsymbol{E_{net} = \frac{1}{4\pi\epsilon_0}\left[\frac{2px}{x^4}\right] = \frac{1}{4\pi\epsilon_0}\left(\frac{2p}{x^3}\right) \quad Derived}.$$

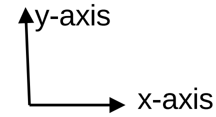
In the diagram above, if x is replaced by r, the distance of axial point P from the centre of dipole O, then,

$$\boldsymbol{E_{net} = \frac{1}{4\pi\epsilon_0}\left(\frac{2p}{r^3}\right)}$$

In vector form, the net electric field intensity at axial position at point P when $r \gg l$, x is replaced by r, is $\boldsymbol{\vec{E}_P = \frac{1}{4\pi\epsilon_0}\left(\frac{2p}{r^3}\right)(\hat{i})}$

Derivation # 4: To derive the expression of electric field intensity due to a dipole of charge $\pm q$, separated by 2l distance at equatorial position when r >>l when placed in free space.





Here direction of electric dipole moment ($\vec{p}$) is from –q to + q charge. The direction of electric field intensities due to charges –q and + q are shown as $\vec{E}_{PA}$ and $\vec{E}_{PB}$ respectively drawn from point P. Let OP be the r from centre of dipole O to point P which is at equatorial line (or broadside-on position). Here in right angle triangle AOP, $AP = \sqrt{OA^2 + OP^2} = \sqrt{l^2 + r^2}$

As shown in the diagram, the magnitude of electric field intensity at point P due to charge –q at point A is $E_{PA} = \frac{1}{4\pi\epsilon_0}\left(\frac{|-q|}{r^2+l^2}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2+l^2}\right) ----(i), towards - q\ charge$ along line PA and the magnitude of electric field intensity at point P due to charge +q at point B is $E_{PB} = \frac{1}{4\pi\epsilon_0}\left(\frac{|+q|}{r^2+l^2}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2+l^2}\right) ----(ii), away\ from\ P\ and$

along line PB. From the figure above shown, $\cos\theta = \frac{AO}{AP} = \frac{BO}{BP} = \frac{l}{\sqrt{r^2+l^2}}$

Now, angle between $\vec{E}_{PA}$ and $\vec{E}_{PB}$ is 2 θ, so, net electric field intensity at point P is

$$E_P = \sqrt{{E_{PA}}^2 + {E_{PB}}^2 + 2(E_{PA})(E_{PB})\cos 2\theta} = \sqrt{E^2 + E^2 + 2E^2\cos 2\theta} = \sqrt{2E^2 + 2E^2(2cos^2\theta - 1)}$$

$$= \sqrt{2E^2 + 4E^2cos^2\theta - 2E^2} = 2E\cos\theta$$

Here, $E_{PA} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2+l^2}\right) = E_{PB} = E\ (say)$ and $\cos 2\theta = 2cos^2\theta - 1$

Hence, net electric field intensity at point P at equatorial position is

$$E_P = 2\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q}{r^2+l^2}\right)\cos\theta = \frac{1}{4\pi\epsilon_0}\left(\frac{2q}{r^2+l^2}\right)\left(\frac{l}{\sqrt{r^2+l^2}}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{2ql}{(r^2+l^2)^{\frac{3}{2}}}\right), hence$$

$$\boldsymbol{E_P = \frac{1}{4\pi\epsilon_0}\left(\frac{2ql}{(r^2+l^2)^{\frac{3}{2}}}\right) --- (iii) here\ difference\ between\ r\ and\ l\ is\ not\ much.}$$

Since, magnitude of dipole moment is $p = 2ql$, so, when $r \gg l, then\ l^2$ can be neglected in comparison to r^2, therefore, net electric field intensity at point P at equatorial line is given as,

$\boldsymbol{E_P = \frac{1}{4\pi\epsilon_0}\left(\frac{p}{r^3}\right) ----(24) in - x\ direction\ antiparallel\ to\ \vec{p}}$ **Derived.**

In vector form, $\boldsymbol{\vec{E}_p = \frac{1}{4\pi\epsilon_0}\left(\frac{p}{r^3}\right)(-\hat{i})}$

Note-1: The ratio of electric field intensity at axial position to the equatorial position when $r \gg l,$ due to dipole $\pm q$ and separated ny 2l distance is, (from equation (23) and (24))

$$\frac{E_{axial}}{E_{equatorial}} = 2:1$$

Note-2: Electric field strength at a point P at distance r from the centre of dipole making angle θ from x-axis if dipole is $\pm q$ placed horizontal from –q to +q and separated by 2l is

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{P\sqrt{1+3cos^2\theta}}{r^3}\right)$$

From the formula above given in equation

(i) Electric field intensity at axial position: $\theta = 0^0$ (along$\vec{p}$) or $\theta = 180^0(antiparallel\ to\ \vec{p})$

$$E_{net} = \frac{1}{4\pi\epsilon_0}\left(\frac{2p}{r^3}\right)$$

(ii) Electric field intensity at equatorial position: $\theta = 90^0$

$$E_{net} = \frac{1}{4\pi\epsilon_0}\left(\frac{p}{r^3}\right)$$

Continuous Distribution of Charges: When charges are uniformly distributed over the surface of conductor, it is called continuous distribution of charges.

There are three types of distribution of charges:

(i) **Line charge distribution**: In this, charges are distributed on the line conductor or over circular ring. The charge density is called linear charge density.
Linear Charge density (λ): It is charge per unit length. Its unit is coulomb /meter. That is, λ= $\frac{q}{l} = \frac{dq}{dl}$

(ii) **Surface charge distribution:** In this, charges are distributed continuously over the surface of conductor. It is two dimensional. (x-y or y-z or z-x axis) Example: Charge distribution over surface of infinite plane sheet, cylinder and spherical shell (hollow spherical conductor), etc. The charge density is called surface charge density.
Surface charge density (σ): It is the ratio of charge to the surface area. That is, $= \frac{q}{S} = \frac{dq}{ds}$, where S is the surface area. Its unit is C m^{-2}. Surface area (S) of cylinder is $S = 2\pi rl$, where r = radius of cylinder, l = length of cylinder, surface area of spherical shell is $S = 4\pi r^2$, where r is the radius of spherical shell.

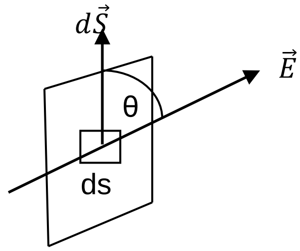
(iii) **Volume charge distribution**: It is in three dimensional and charge is distributed over its surface. For example, solid sphere. The charge density is called volume charge density. Its unit is C m^{-3}.
Volume charge density (ρ): It is defined as charge per unit volume. That is, $\rho = \frac{q}{V} = \frac{q}{\frac{4}{3}\pi r^3} = \frac{dq}{dV}$

Electric Flux:

It is defined as number of electric field lines passing through a surface.

[In another way, it is defined as number of field lines passing through per unit perpendicular area is directly proportional to electric field. That is, $\frac{N}{S} \propto E\ or\ N \propto ES$ and $ES = \varphi$ so, we can say,

number of field lines $N \propto \varphi$ (that is number of field lines passing through any surface is called flux.]
The electric flux φ through a surface is the amount of electric field that pierces the surface. It is expressed as $\varphi = ES\cos\theta$.where E = electric field intensity, S = surface area, θ = angle between area vector and electric field intensity. In **vector** form, $\varphi = \vec{E}.\vec{S}$
Here, $\vec{S}$ is the area vector that is perpendicular to the surface and has magnitude S, where S is the area of rectangle (S = length x breadth) or area of square (S = square of side) or area of circle ($S = \pi r^2$, here r is the radius of circle).
Electric flux is a scalar quantity.
Unit of electric Flux: (i) Its unit is Nm^2C^{-1} or (ii) V-m (volt metre).
Dimensional formula of electric flux: $[\varphi] = [ML^3T^{-3}A^{-1}]$
For a non-planar surface, the electric flux $d\varphi$ through a patch element with area vector $d\vec{S}$ is given by dot product as $d\varphi = \vec{E}\cdot d\vec{S} = EdS\cos\theta$



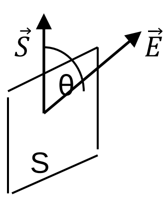
$$\boldsymbol{d\varphi = EdS\cos\theta} \qquad \boldsymbol{Or,}$$

$$\boldsymbol{\varphi = \int d\varphi = \int EdS\cos\theta = \int \vec{E}\cdot d\vec{S} - - - - (total\ flux)}$$

The net flux through a closed surface which is used in Gauss's law is given by

$$\boldsymbol{\varphi = \oint \vec{E}\cdot d\vec{S} - - - (net\ flux)}$$

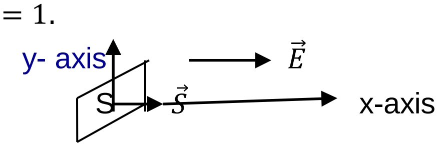
Where the integration is carried out over the entire surface.
For a planar surface,



$$\varphi = ES$$

Thus, we see that electric flux (φ) depends upon angle θ.
Here, θ may be defined as the angle normal to the plane of surface and electric field vector. **The following conditions arise**:

(i) If normal to the plane of surface is parallel to the electric field. Or, electric field is parallel to the normal to the plane of the surface, then $\theta = 0^0$ and flux $\varphi = ES$ (which is maximum) as $\cos 0^0 = 1$.



(ii) If plane of the surface is parallel to the electric field or electric field is parallel to the plane of the surface, then $\theta = 90^0$ and flux $\varphi = 0$ (which is minimum) as $\cos 90^0 = 0$.

(iii) If plane of the surface making angle x with the electric field, then $\theta = 90^0 - x$ and flux $\varphi = ES\cos(90^0 - x) = ES\sin x$

(iv) If normal to the plane of surface making angle x, $\theta = x^0$ and $\varphi = ES\cos x$

Now, we can observe that electric flux may be (i) positive when θ lies between 0 and 90^0 that is, $0^0 \leq \theta < 90^0$ (ii) negative when $90^0 < \theta \leq 180^0$, (iii) zero when θ = 90^0.

Area Vector: It is always perpendicular to the plane of closed surface and outward.

For example, a square of side 10 cm is placed in x-y plane and electric field is $\vec{E} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})\ \times 10^{-3}$ N/C. Calculate the flux. [Ans. $\varphi = \vec{E}\cdot\vec{S} = \left(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}\right) \times 10^{-3} \cdot \left(100 \times 10^{-4}\hat{k}\right) = 500 \times 10^{-7} = 5 \times 10^{-5}$ V-m]

Note-1:

(i) If the electric field vector pierces inward, while the area vector is outward, the electric flux will be negative or we can say when area vector and field vector are opposite in direction, then flux will be negative.

(ii) An outward piercing field is positive flux and

(iii) A skimming field is zero flux.

Note-2: In the above example, **Remember**, for calculating electric flux as $\varphi = \vec{E}\cdot\vec{S}$ where $\vec{E}$ is given as $\vec{E} = E\hat{\imath} + E\hat{\jmath} + E\hat{k}$ if a square or rectangular plane is placed in:

(i) x-y plane then area vector will be in z-direction and apply $\vec{S} = S\hat{k}$ where S is the area of square or rectangle.

(ii) y-z plane then area vector will be in x-direction and apply $\vec{S} = S\hat{\imath}$ where S is the area of square or rectangle.

(iii) x-z or z-x direction then area vector will be in y-direction and apply $\vec{S} = S\hat{\jmath}$ where S is the area of square or rectangle.

Gauss's law:

It is a tool of simplifying electric field calculation where there is symmetrical distribution of charge. Gaussian surface is an imaginary surface

It states that the flux of net electric field (surface integral of net electric field intensity) over a closed surface is equal to $\frac{1}{\epsilon_0}$ times charge enclosed by that surface. That is, mathematically,

$$\varphi = \oint_S \vec{E}\cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Here, q_{in} is the total charge enclosed by Gaussian surface. Here closed surface is the Gaussian surface.

Gauss's law relates the electric field at a point on a closed Gaussian surface to the net charge enclosed by that surface.

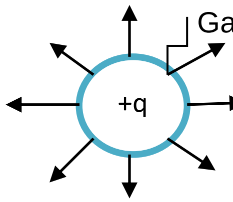
Note: The Gaussian surface is (i) cylinder for line charge, (ii) cylinder for infinite plane sheet, (iii) sphere for spherical shell, (iv) sphere for solid sphere.

In the equation (28), we see that electric flux $\varphi = \frac{q_{in}}{\epsilon_0}$ and flux depends on

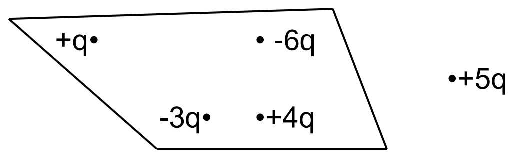
Charge enclosed by the Gaussian surface and medium.

Note: The Gaussian surface does not depend on radius of the sphere or we can say, size of the sphere. If the charge is outside the Gaussian surface, flux will not be dependent on that chat charge.

Here is Gaussian surface is shown below and a charge + q is enclosed. So, flux $\varphi = \frac{q}{\epsilon_0}$



Taking another example,



What will be the net electric flux?

Here answer will be total charge enclosed by Gaussian surface,

$q_{in} = [(+q) + (-6q) + (-3q) + (+4q)] = -4q$, hence, net flux $\varphi = \frac{q_{in}}{\epsilon_0} = \frac{-4q}{\epsilon_0}$

Here, +5 q charge is outside the Gaussian surface, so, flux does not depend on +5q charge.

Important Note to know electric flux (in the Gaussian surface):

(i) If charge q is at the centre of a cube, then flux passing through the cube is flux $\varphi = \frac{q}{\epsilon_0}$

(ii) If a charge q is placed at the centre of a cube of side l, then flux passing through each face of the cube, $\varphi = \left(\frac{1}{6} \times \frac{q}{\epsilon_0}\right) = \frac{q}{6\epsilon_0}$ (because there are 6 faces in a cube and flux passing through all six faces will be $\frac{q}{\epsilon_0}$).

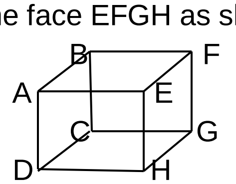
(iii) If a charge q is placed at the centre of a cube of side l, then flux passing through two opposite faces of the cube, $\varphi = \left(\frac{2}{6} \times \frac{q}{\epsilon_0}\right) = \frac{q}{3\epsilon_0}$

(iv) If the symmetrical closed body has n identical faces with point charge at its centre (in a cube, 6 faces, in a suare/rectangle, 4 faces, in a hexagon, 6 faces etc.), the flux linked with each face will be $\frac{q}{n\epsilon_0}$.

(v) If the charge q is placed inside a cube but not at the centre, then flux linked with all faces will be $\frac{q}{\epsilon_0}$ but flux linked with each face of the cube $\neq \frac{q}{6\epsilon_0}$. (that is it will change).

(vi) The electric flux passing through a cube of side 'a' which encloses an electric dipole will zero.

(vii) If the charge q is placed at one of corners (say at corner D) then flux passing through one of the face EFGH as shown in the figure below.



The answer will be flux passing through face EFGH, $\varphi = \frac{q}{24\epsilon_{\epsilon_0}}$ (as if we assume a large cube comprising 8 small cube, and a charge q is placed at the centre or at D, the total flux passing through this large cube will be $\frac{q}{\epsilon_0}$ and there are 24 faces, so, through each face, it will be $\frac{q}{24\epsilon_{\epsilon_0}}$.

Suppose in the figure above, what is the flux passing through face AEHD. Flux will be zero as field lines are tangential to this face.

Flux passing through the below figures is as:-

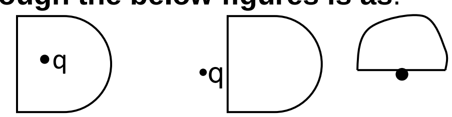
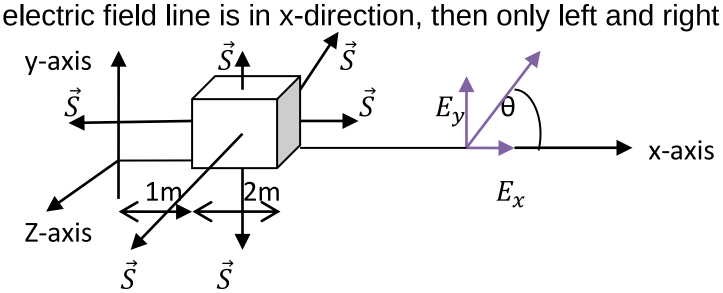


Fig-1 Fig-2 Fig-3

In Fig-1: $\varphi = \frac{q}{\epsilon_0}$, in Fig-2, $\varphi = 0$ ($charge\ is\ outside$), in Fig-3, $\varphi = \frac{q}{2\epsilon_0}$ (as charge lies at the centre on semicircle diameter.

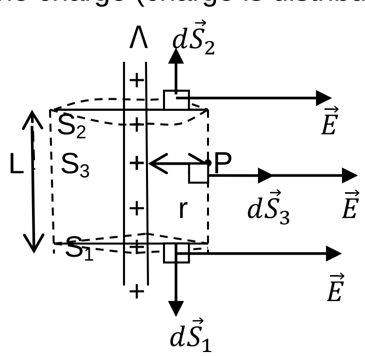
Here in the below figure, area vector $\vec{S}$ is shown at each face of the cube which points outward. If the electric field line is in x-direction, then only left and right face will contribute electric flux.



Note: (1) Gauss's law is based on the inverse square dependence on the distance. (2) When system is symmetrical, Gauss law is applicable. (3) Choose any surface as Gaussian surface but it should be symmetrical and Gaussian surface can pass through a continuous charge. (4) Gauss law is true for any closed surface, no matter what its shape or size. (5) Electric field at every point on the Gaussian surface is either perpendicular or tangential.

Applications of gauss's law: The followings can be derived using Gauss law:

Derivation -5: Using Gauss's law, derive electric field intensity at point r due to thin infinite (long) line charge (charge is distributed uniformly).of charge density λ C m^{-1}



Answer: As shown in the arrangement, for a line charge, Wwe know that Gaussian surface is cylinder which has surface area S_1 and S_2 (for flat faces) and S_3 (for curved faces) surface

For small surface area dS_1, dS_2 and dS_3, the area vectors are shown. Since electric field due to positive charge at point with distance r from line charge, points outward as shown..
Let L be the length of Gaussian cylinder with radius r. Hence, total surface area of Gaussian cylinder S = $2\pi rL$

Using Gauss's law, $\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0} - - - (i), here,$

$$\oint \vec{E} \cdot d\vec{S} = \oint \vec{E} \cdot d\vec{S}_1 + \oint \vec{E} \cdot d\vec{S}_2 + \oint \vec{E} \cdot d\vec{S}_3 = 0 + 0 + ES_3 - - - (ii)$$

Since, $\vec{E}$ and $d\vec{S}_1$ are perpendicular to each other, also $\vec{E}$ and $d\vec{S}_2$ are perpendicular to each other, so, flux passes through these surfaces is zero. But, $\vec{E}$ and $d\vec{S}_3$ are in the same direction, so, this surface will contribute flux. Moreover, surface area S_3 = S = $2\pi rL$ and charge enclosed by the cylinder is $q_{in} = q = \lambda L$, hence from equation (i) and (ii), we have,

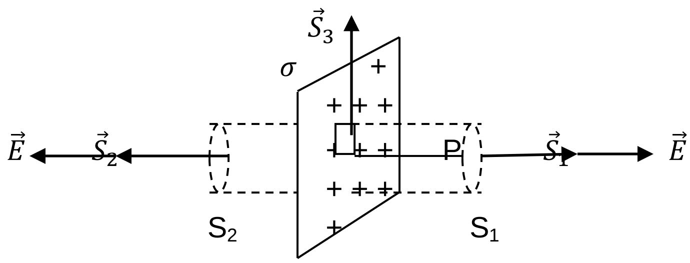
$ES = \frac{q}{\epsilon_0} = \frac{\lambda L}{\epsilon_0} \quad OR,\ E = \frac{\lambda L}{S\epsilon_0} = \frac{\lambda L}{2\pi rL\epsilon_0} = \frac{\lambda}{2\pi\epsilon_0 r}$

Net electric field intensity due to line charge at distance r is given as,

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \qquad \boldsymbol{Derived}.$$

Thus, we see that electric field due to a line charge at a point at distance r is inversely proportional to distance from line charge.

Derivation -6: To derive electric field intensity at point r due to thin infinite (long) sheet (charge is distributed uniformly).of charge density σC m^{-2}.



Answer: As shown in the above figure, Charge +q is uniformly distributed on the surface infinite plane thin sheet and surface surge density is σ. Two surface areas S_1 and S_2 are shown of flat surface and S_3 for curved surface. A cylindrical Gaussian surface (in dotted portion) is made to calculate electric field intensity at point P from the thin sheet.

The area vectors for these surfaces are shown in the figure. The surface charge density, $\sigma = \frac{q}{S}$ and charge enclosed by cylindrical Gaussian surface is $q_{in} = q = \sigma S$

For right face of the cylinder, the flux, $\varphi_R = \vec{E} \cdot \vec{S}_1 = E(i) \cdot S(i) = ES$

And, for left face of the cylinder, the flux, $\varphi_L = \vec{E} \cdot \vec{S}_2 = E(-\hat{\imath}) \cdot S(-\hat{\imath}) = ES$

The net flux, $\varphi_{net} = \varphi_L + \varphi_R = ES + ES = 2ES$

Using Gauss's law, $\oint \vec{E} \cdot d\vec{S} = \varphi_{net} = \frac{q_{in}}{\epsilon_0} = \frac{\sigma S}{\epsilon_0}$

Therefore, $2ES = \frac{\sigma S}{\epsilon_0} \ \ Or, E = \frac{\sigma}{2\epsilon_0}$

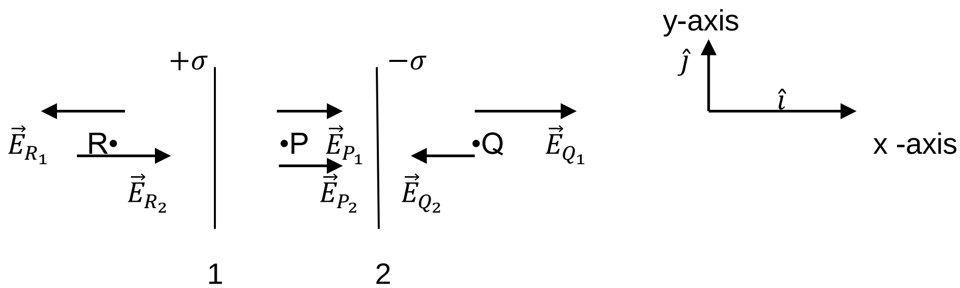
The electric field intensity due to an infinite (long) thin sheet at any point is

$$\boldsymbol{E = \frac{\sigma}{2\epsilon_0}} \qquad \boldsymbol{Derived}$$

Thus, we see that electric field intensity due to a thin plane sheet does not depend on distance of a point from thin sheet.

Derivation-7: To find electric field intensity at a point (1) between the long parallel thin sheet, (2) Right of thin sheet-2 and (3) left of thin sheet-1. If surface charge densities $+\sigma$ and $-\sigma$ are continuously distributed.

Answer:



As shown in the arrangement, the direction of electric fields at point P (inside to long sheet), at Q (outside plate 2) and at R (outside plate 1) are shown. So,

(1) Between the two thin long sheet: The net field at point P

$$\vec{E}_P = \vec{E}_{P_1} + \vec{E}_{P_2} = \frac{\sigma}{2\epsilon_0}(\hat{\imath}) + \frac{|-\sigma|}{2\epsilon_0}(\hat{\imath}) = \frac{2\sigma}{2\epsilon_0}(i) = \frac{\sigma}{\epsilon_0}(i)$$

(2) Right of long thin sheet-2: The net field at point Q:

$$\vec{E}_Q = \vec{E}_{Q_1} + \vec{E}_{Q_2} = \frac{\sigma}{2\epsilon_0}(\hat{\imath}) + \frac{|-\sigma|}{2\epsilon_0}(-\hat{\imath}) = \frac{\sigma-\sigma}{2\epsilon_0}(i) = 0$$

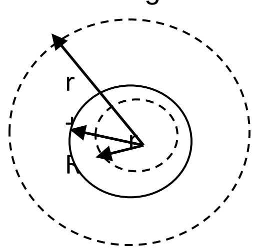
(3) Left of long thin sheet-1: The net field at point R:

$$\vec{E}_R = \vec{E}_{R_1} + \vec{E}_{R_2} = \frac{\sigma}{2\epsilon_0}(-\hat{\imath}) + \frac{|-\sigma|}{2\epsilon_0}(\hat{\imath}) = \frac{-\sigma+\sigma}{2\epsilon_0}(i) = 0$$

Derived

Derivation-8: Using Gauss's law, find electric field intensity at a point (1) inside the spherical shell at r < R, (2) outside the spherical shell at r > R and (3) at surface of spherical shell at r = R, if surface charge densities $+\sigma$ (or charge +q) is distributed uniformly on its surface of spherical shell of radius R.

Derivation: Taking spherical shell of radius R and charge +q is uniformly distributed on its surface as shown in the figure.



Here as shown in the spherical shell of radius R, a smaller Gaussian surface (dotted) of radius r is shown inside the sphere of radius R and a larger Gaussian spherical surface (dotted) of radius r is shown outside of spherical shell of radius R. Here considering a point which is at distance r (at which field intensity is being found out) from the centre of spherical shell of radius R and also at point outside spherical shell of radius R at distance r where also field intensity is being found out. So, a Gaussian sphere of radius r is drawn by dotted surface. Using Gauss's law,

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0} \text{ Or, } ES = \frac{q_{in}}{\epsilon_0} - - - - (i)$$

(1) At r < R, that is, inside the spherical shell,
Since, no charge is inside the spherical shell. Hence, charge enclosed by Gaussian surface inside = 0, therefore, electric field intensity inside a spherical shell everywhere is equal to zero.

(2) At r > R, that is, outside spherical shell,
The outer spherical Gaussian surface encloses the charge q_{in} = +q, and surface area of outer Gaussian surface of radius r is $S = 4\pi r^2$, hence, using equation (i),

$$E(4\pi r^2) = \frac{q}{\epsilon_0} \quad Or, E = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\right)$$

Hence, electric field intensity at a point outside the spherical shell at distance r from the centre of sphere is $\boldsymbol{E = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\right)}$

(3) At r = R, field intensity at the surface of spherical shell is, $\boldsymbol{E = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{R^2}\right)}$

To plot the Graph of it: