Ch-10:Wave Optics Formula sheet

1. Expression of resultant amplitude and resultant intensity of two waves in interference:

$$A^2 = {A_1}^2 + {A_2}^2 + 2A_1A_2\cos\varphi$$

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\varphi$$

Where,φ is the phase difference between two waves in interference, $A_1\ and\ A_2$ are respective amplitudes and $I_1\ and\ I_2$ are respective intensity.

2. For constructive interference, $\cos\varphi = 1$, (Maxima or bright frimges) or in general, Phase difference $\varphi = 2n\pi$, path difference, $\Delta x = d\sin\theta = n\lambda$; where, n = 0, $\pm1, \pm2, \pm3, -----$
For Central maxima, n = 0, for first bright, n = 1
3. For destructive interference, $\cos\varphi = -1$ (Minima or dark fringes); or in general, phase difference, $\varphi = (2n-1)\pi$ and path difference, $\Delta x = d\sin\theta = (2n-1)\lambda/2$,where n = $\pm1, \pm2, \pm3, -----$
4. ***Comparison of intensities at maxima and minima***:
$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \frac{(A_1+A_2)^2}{(A_1-A_2)^2}$$
5. Resultant intensity when $I_1 = I_2 = I$,
$$I_r = 4Icos^2\left(\frac{\varphi}{2}\right) = I_0cos^2\left(\frac{\varphi}{2}\right)$$
Where, I_0 is the maximum intensity at central maxima, at central maxima, $\varphi = 0^0$
6. Relation between phase difference(φ) and path difference (Δx),
$$\varphi = \frac{2\pi}{\lambda}\times\Delta x$$
7. In YDSE, if $w_1\ and\ w_2$ are slit widths, then,
$$\frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{{A_1}^2}{{A_2}^2}; \qquad (I \propto A^2)$$
8. Position of bright fringes from Central maxima in YDSE:
(i) n^{th} distance of bright fringes from central maxima,
$$(y_n)_{bright} = \frac{nD\lambda}{d}, where\ n = 0, \pm1, \pm2, ----$$
(ii) n^{th} distance of dark fringes from central maxima;
$$(y_n)_{dark} = \frac{(2n-1)D\lambda}{2d}, where\ n = \ \pm1, \pm2, ----$$

Where, D = separation between slits and screen, d = slits separation, λ is wavelength of light.

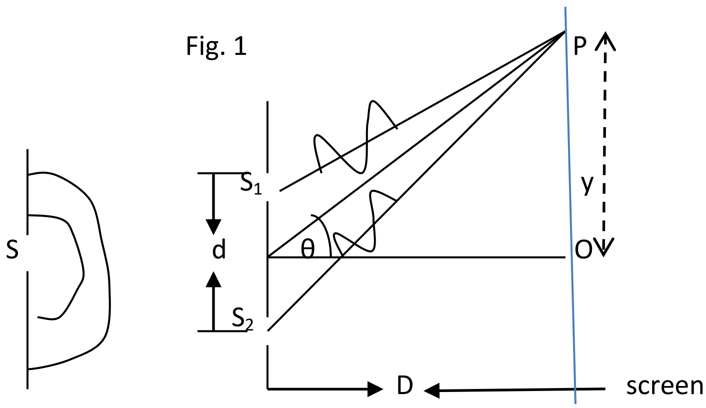
9. **Fringe width (β):It is the difference between two successive bright or two successive dark fringes.**

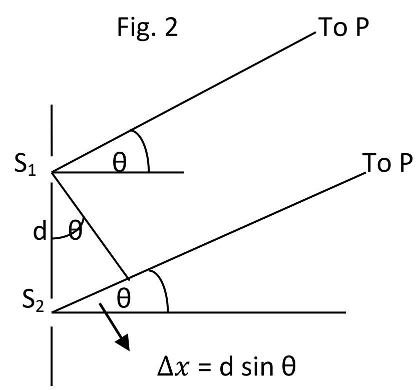
$$\beta = \frac{D\lambda}{d}$$

10. **Angular fringe width**: $\alpha = \frac{\beta}{d} = \frac{\lambda}{d}$
11. If YDSE apparatus is immersed in liquid of refractive index μ the wavelength of light and hence fringe width decreases μ times.

$$\mu\beta = constant$$

12. Two assumptions: If D >> d, then two waves becomes parallel. If d >> λ, so,angle becomes small and then $\tan\theta \approx \sin\theta = \frac{y}{D} = \frac{n\lambda}{d}$

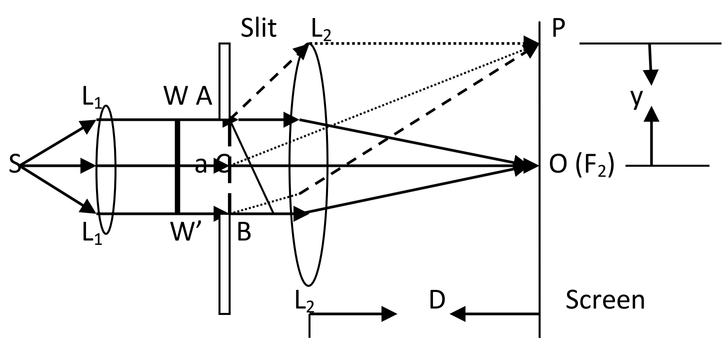




Diffraction of light:

Diffraction occurs on account of mutual interference of secondary wavelets which are not blocked by obstacle or from portions of wave fronts which are allowed to pass through the aperture.

Diffraction at single slit:



Fraunhofer Diffraction

13. **Position of n^th secondary minima**:

The general condition for destructive interference, $path\ difference = a\ \sin\theta = n\lambda$ or,

$\sin\theta = \dfrac{n\lambda}{a}, n = \pm1, \pm2, \pm3$___

Position of n^th minima from the centre of the screen:

$Since, \sin\theta = \theta = \tan\theta = \dfrac{y}{D} = \dfrac{n\lambda}{a}$

$$\therefore\ \boldsymbol{y_n = \frac{nD\lambda}{a}} - - - (1)$$

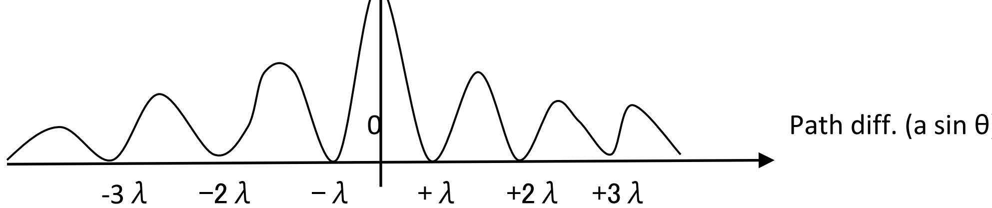
14. **Position of n[th] secondary maxima**: $Path\ difference\ BN = a\sin\theta = (2n+1)\frac{\lambda}{2}, n = \pm 1, \pm 2, \_\_$

    Position of n[th] maxima from the centre of screen:

$$Since, \sin\theta = \theta = \tan\theta = \frac{y}{D} = (2n+1)\frac{\lambda}{2a}$$

$$\therefore\ \boldsymbol{y_n = \frac{(2n+1)D\lambda}{2a}} - - - (2)$$

Note: Mathematical expression for diffraction minima and maxima are exactly reverse of mathematical expressions for interference minima and maxima.



$$\boldsymbol{Width\ of\ central\ maximum = 2y = \frac{2D\lambda}{a} = \frac{2f\lambda}{a}}$$

Where, f = focal length of lens L_2 which is held very close to the slit.

So, we find that the width of central maximum decreases when the slit width increases.

Angular width of central maximum: sin θ = θ when θ in radian and small, therefore,

$$\sin\theta = \theta = \frac{\lambda}{a}, hence$$

$$\boldsymbol{Angular\ width\ of\ central\ maximum\ (2\theta)} = \frac{2\lambda}{a}, hence$$

$$\boldsymbol{Width\ of\ central\ maximum = D(angular\ width\ of\ central\ maximum)}$$

Where D = distance between slit and screen

Law of Malus: In 1809, ''E. N. Malus discovered that when a beam of completely plane polarised light is passed through analyser, the intensity of transmitted light varies directly as the square of the cosine of angle θ between the transmission direction (pass axis) of polariser and analyser.'' This statement is known as law of Malus. Mathematically,

$$I \propto cos^2\theta\ or\ I = I_0 cos^2\theta$$