# Chapter-6: Electromagnetic Induction –Formulae

1. Magnetic flux $\varphi = \int \vec{B}.\overrightarrow{dS}\ or\ \varphi = BS\cos\theta$ when $\vec{B}$ is uniform over a plane surface with total area S.
2. The following points regarding magnetic flux:
   a) The magnetic flux is a scalar quantity like electric flux
   b) The SI unit of magnetic flux is T-m$^2$ or weber (Wb).
   c) 1 Wb = 1 T-m$^2$ = 1 N-m/A
   d) In the special case in which $\vec{B}$ is uniform over a plane surface with total area S,
   $$\varphi = BS\cos\theta$$
   e) If $\vec{B}$ is perpendicular to the surface, θ = 0$^0$ and $\varphi = BS$
3. Gauss's law of magnetism $\oint \vec{B}.\overrightarrow{dS} = 0$ as there is no monopole in magnetism.
4. Faraday's law of electromagnetic induction $e = -\frac{d\varphi}{dt}$
5. Neumann's law could be stated as '' when the magnetic flux linked with a coil (or circuit) is changed in any manner, then an emf is set up in the circuit such that the produced emf is proportional to the rate of change of the flux linkage with the circuit.''
   The induced emf around the closed path is given by the line integral of the induced electric field E around the closed path. $e = \oint \vec{E}.\overrightarrow{dl} \Rightarrow -\frac{d\varphi}{dt} = \oint \vec{E}.\overrightarrow{dl}\ \ or\ \ -\frac{d}{dt}\oint \vec{B}.\overrightarrow{dS}$
   Hence, we can say that the electric fields associated with changing magnetic fields are non-conservative while the fields associated with stationary charges are conservative.
6. Thus the Faraday's law can be stated as, 'The line integral of electric field intensity around any closed path is equal to the time rate in decrease of magnetic flux induction through the surface enclosed by that curve''.
7. If a circuit in coil contains N loops or turns, in the same area and if $\varphi$ is the flux through one loop, an emf is induced in every loop, thus the total induced emf in the coil is given by $e = -N\frac{d\varphi}{dt}$
8. Induced emf is produced only when there is a change in a magnetic flux passing through a loop. The flux passing through the loop is given by $\varphi = BS\cos\theta$
9. Flux can be changed only when
   a) The magnitude of $\vec{B}$ can change with time, that is, $B = B(t)$
   b) The current producing in magnetic field can change with time. That is, I = i(t)
   c) The area enclosed by the loop can change with time, that is, S = S (t).This can be done by pulling a loop inside (or outside) a magnetic field.
   d) The angle θ between $\vec{B}$ and normal to the loop can change with time. This can be done by rotating a loop in a magnetic field.
10. When the magnetic flux passing through a loop is changed, an induced emf and hence, an induced current is produced in the circuit. If R is the resistance of the circuit, then induced current is given by $i = \frac{e}{R} = \frac{1}{R}\left(\frac{-d\varphi}{dt}\right)$
11. Current starts flowing in the circuit means flow of charge takes place. Charge flows in the circuit is given by $dq = idt = -\frac{d\varphi}{R}$

12. For a time interval $\Delta t$, $e = -\frac{\Delta\varphi}{\Delta t}, i = \frac{1}{R}\left(-\frac{\Delta\varphi}{\Delta t}\right)$ $and$ $\Delta q = \frac{1}{R}(\Delta\varphi)$

    We can see that e and i are inversely proportional to $\Delta t$ while $\Delta q$ is independent of $\Delta t$.

13. Faraday's experiment
    a) **Experiment 1**: Induced emf with a stationary coil and moving magnet
        i. When a north pole of the magnet is brought closer to the coil, its face towards the magnet develops north polarity (direction of current in coil is anticlockwise).
        ii. Similarly, when north pole the magnet is moved away from the coil, its face towards magnet develops south polarity (direction of current in the coil is clockwise)
        iii. When a south pole of the magnet is brought closer to the coil, its face towards the magnet develops south polarity and
        iv. When south pole the magnet is moved away from the coil, its face towards magnet develops north polarity
    b) **Experiment 2**: Induced emf with a stationary magnet and moving coil:

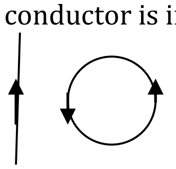
        When the relative motion between the coil and the magnet is fast, the deflection in the galvanometer is large and when the relative motion is slow, the galvanometer deflection is small.

14. Lenz's law: ''It states that the direction of induced current in a circuit is such that it opposes the cause or the change which produces it.''

15. **To know the direction of induced current in a loop, apply**

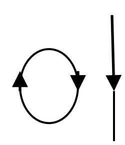
    RIN: R stands for right, I stands for increasing and N for north pole (anticlockwise). It means if a loop is placed on the right side of a straight current carrying conductor and the current I in the conductor is increasing, then induced current in the loop is anticlockwise.

I increasing



(Current in the loop is anticlockwise)

I decreasing



(Current in the loop is clockwise)

16. ⊗ IN: Suppose the magnetic field in the loop is perpendicular to paper inwards (⊗) and this field is increasing, then induced current in the loop is anticlockwise.

    ⊙ IN: If field increasing, induced current in the loop will be clockwise

    While in magnetic field, ⊗ (cross) means South Pole so magnetic field will be in clockwise and ⊙ (dot) means North Pole so magnetic field will be in anticlockwise direction as in case of solenoid.

17. Fleming Right hand Rule: ''If we stretch thumb and two fingers of our right hand in mutually perpendicular direction and if the forefinger points in the direction of magnetic field, thumb in the direction of motion of the conductor; then the central fingers points in the direction of current induced in the conductor.''

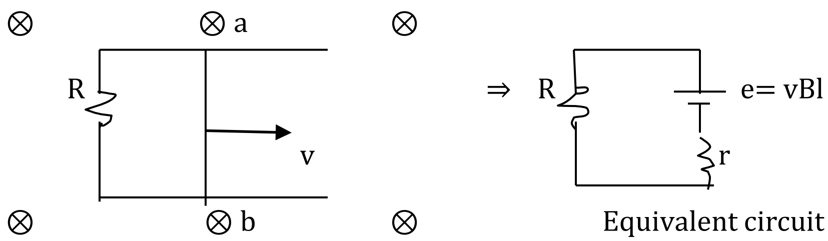
18. Motional emf
    a) On a straight conducting wire, $e = vBl$ = motional emf = $\Delta V$
    b) On a rotating conducting wire about one end, $e = \frac{B\omega l^2}{2}$ where v = ωl

19. Current induced in the loop: $i = \frac{e}{R} = \frac{vBl}{R}$

20. Force experienced in the movable conductor of length l carrying current I, is given by

$$F = Ilb \sin 90^0 = IlB = \frac{vBl}{R}(lB) = \frac{B^2 l^2 v}{R}$$

This force (due to induced current) acts in the outward direction (By Fleming Left hand Rule) opposite to the velocity of the arm in accordance with Lenz's law. Hence to move the arm with a constant velocity v, it should be pulled with a constant force F.




(From magnetic circuit to electric circuit)

The moving rod ab is replaced by a battery of emf vBl with the positive terminal at a and the negative terminal at b. The resistance r of the rod ab may be treated as the internal resistance of the battery. Hence, the current in the circuit is, $i = \frac{e}{R+r} = \frac{vBl}{R+r}$

21. Power delivered by the external force: $P_{del} = Fv = \frac{B^2l^2v}{R}v = \frac{B^2l^2v^2}{R}$
22. Power dissipated as Joule loss: $P_{dissipated} = i^2R = \frac{B^2l^2v^2}{R^2}R = \frac{B^2l^2v^2}{R}$

Note 1: $e = -\frac{d\varphi}{dt}$ is best applied to the problem in which there is a changing flux through a closed loop while e= Blv is applied to problem in which conductor moves through a magnetic field (fixed)

Note 2: If conductor is moving in a magnetic field but circuit is not closed, then only P.D. will be asked between two points of the conductor. If the circuit is closed, then current will be asked in the circuit.

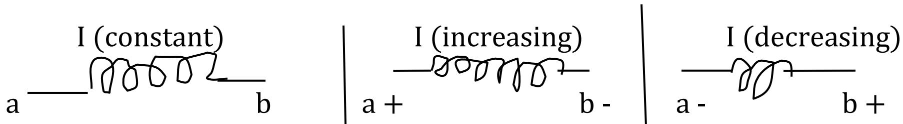
**23. Self Inductance: Definition**

a) Total flux linked with the coil is directly proportional to current in the coil. That is, $N\varphi \propto i \text{ or } N\varphi = Li \Rightarrow L = \frac{N\varphi}{i}$
b) it is total flux linkage per unit current. The SI unit of self inductance is Henry (H).

c) Hence, coefficient of self induction (or self Inductance) $L = \frac{N\varphi}{i} = \left|\frac{-e}{\frac{di}{dt}}\right|$

24. An inductor is a circuit element which opposes the change in current through it. It may be a circular coil, solenoid etc.

25. Potential difference across an inductor:




$\frac{di}{dt} = 0$, e =0, $V_{ab}$ =0 | $\frac{di}{dt} > 0$, $e_{ab} < 0$, $V_{ab} > 0$ | $\frac{di}{dt} < 0$, $e_{ab} > 0$, $V_{ab} < 0$

26. Assume that inductor has negligible resistance, so that PD $V_{ab} = V_a - V_b$. Here, $V_{ab} = -e = L\frac{di}{dt}$
27. Difference between Resistor and inductor: A resistor opposes the current i while inductor opposes the change in current (di/dt).
28. Kirchhoff's Second law with a inductor connected in series with resistor R and both are connected with a battery supplying current i, $E - iR - L\frac{di}{dt} = 0$
29. **Methods of finding self inductance of a circuit**:

a) Assume that current i flowing through the circuit
b) Determine the magnetic field B produced by the current
c) Obtain the magnetic flux
d) With the flux known, self inductance can be found from $L = \frac{N\varphi}{i}$

30. Self inductance of the solenoid $L = \frac{\mu_0 N^2 A}{l}$
31. This shows that L depends upon as: $L \propto N^2, L \propto A \text{ and } L \propto \frac{1}{l}$
32. Energy (potential energy) stored in an inductor in magnetic field, $U = \frac{1}{2}Li^2$
33. Energy density in magnetic field, $u = \frac{U}{V} = \frac{1}{2}(\frac{B^2}{\mu_0})$
34. This expression is similar to energy density in electric field as $u = \frac{1}{2}\epsilon_0 E^2$.
35. A change in current in one circuit can induce a current in a circuit, is called mutual induction.
36. Mutual Inductance (M): $M = \frac{N_2\varphi_2}{I_1}$ $or\ e_2 = -M\frac{di_1}{dt}$
37. Coefficient of mutual inductance of two closely wound circular coils, $M \propto N_1 N_2$
38. **The following steps worth noting for calculating the mutual inductance of two circuits**:

a) Assume any one of the circuits as primary (first) and the other as secondary (second).
b) Suppose a current $i_1$ flows through the primary circuit
c) Determine the magnetic field $\vec{B}$ produced by current $i_1$
d) Obtain the magnetic flux $\phi_2$ (in the secondary circuit) as $\varphi_2 = B_1 A_2 = \frac{\mu_0 N_1 i_1}{l} A_2$ (Incase of two solenoids placed co-axially)
e) With the flux known, mutual inductance can be found from, $M = \frac{N_2\varphi_2}{i_1}$

39. Coefficient of coupling, $M = K\sqrt{L_1 L_2}$

    Where, $K$ = Coefficient of coupling and is a number depending on the geometry of the coils and their relative closeness, having value between 0 and 1 ($K \leq 1$).

40. Combination of inductances (when spacing between two coils is more)

a) Inductance in series, $L_{eqvt} = L_1 + L_2$

b) Inductances in parallel, $\frac{1}{L_{eqvt}} = \frac{1}{L_1} + \frac{1}{L_2}$ $or\ L_{eqvt} = \frac{L_1 L_2}{L_1 + L_2}$

41. Combination of inductances (when two coils are closely spaced) : Series combination only

a) Let the series connection be such that the current flows in the same sense in the two coils.

$$L_{eq} = L_1 + L_2 + 2M$$

b) Let the series combination be such that flows in opposite senses in the two coils

$$L_{eq} = L_1 + L_2 - 2M$$

42. Current growth in an L-R circuit, $i = i_0\left(1 - e^{-\frac{t}{\tau}}\right)$

43. Current decay in L-R circuit, $i = i_0 e^{-\frac{t}{\tau}}$

44. $\tau = Time\ constant = \frac{L}{R}$ and its unit is second.

45. Oscillations in L-C circuit: In L-C circuit: charge, current and rate of change of current oscillate simple harmonically. These oscillations are similar to oscillations of spring-block system. $\omega = \sqrt{\frac{1}{LC}}$

46. Comparison of an oscillation of a mass spring system and an LC circuit:

| Mass Spring system | L-C circuit |
|---|---|
| Displacement x | Charge q |
| Velocity v | Current i |
| Acceleration a | Rate of change of current di/dt |
| $\frac{d^2x}{dt^2} = -\omega^2 x$ where $\omega = \sqrt{\frac{K}{m}}$ | $\frac{d^2q}{dt^2} = -\omega^2 q$ where $\omega = \frac{1}{\sqrt{LC}}$ |
| $x = A\sin(\omega t \pm \varphi)\ or\ x = A\cos(\omega t \pm \varphi)$ | $q = q_0 \sin(\omega t \pm \varphi)\ or\ q = q_0 \cos(\omega t \pm \varphi)$ |
| $\frac{1}{K}$ | Capacitance C |
| Mass m | Inductance L |
| $\lvert v_{max} \rvert = A\omega$ | $i_{max} = q_0 \omega$ |
| Kinetic energy $\frac{1}{2}mv^2$ | Magnetic energy $\frac{1}{2}Li^2$ |
| Potential energy $\frac{1}{2}kx^2$ | Potential energy $\frac{1}{2}\frac{q^2}{C}$ |

47. Difference between capacitance and inductance keeping in view in the following circuit:

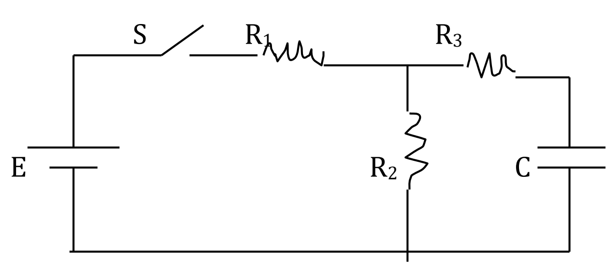



Figure-1

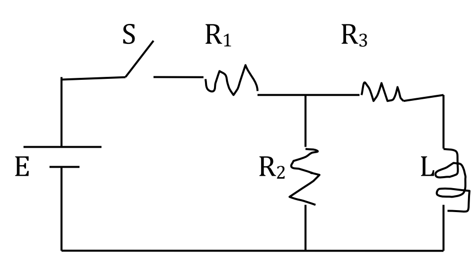



Figure-2

In figure-1, resistance offers by capacitor immediately after switch is closed, is zero (that is capacitor is shorted).

In figure-2, resistance offers by inductor immediately after switch is closed, is infinite (that is inductor is open).

In figure-1, resistance offers by capacitor after long time of closing the switch (in steady state condition), is infinite that is, capacitor is open circuited.

In figure-2, resistance offers by inductor after long time of closing the switch (in steady state condition), is zero that is, inductor is short circuited.

In figure-2, A long time after switch is reopened, current through all resistor will be zero.