Chapter-7: Alternating Current Formulae

1. $V = V_m \sin(\omega t \pm \varphi)\ \ or\ V = V_m \cos(\omega t \pm \varphi)$
   Similarly, $i = I_m \sin(\omega t \pm \varphi)\ \ or\ i = I_m \cos(\omega t \pm \varphi)$
   Here V and i are instantaneous value of voltage and current respectively. The phase angle ϕ is between voltage and current.
2. $T = \frac{2\pi}{\omega}\ or\ \omega = 2\pi f$
3. Average value of current over positive half cycle if $i = I_m \sin \omega t$
   $$I_{av} =< i > = \frac{\int_0^{\frac{T}{2}} I_m \sin(\omega t)\, dt}{\int_0^{\frac{T}{2}} dt} = \frac{2I_m}{\pi} = 0.637 I_m$$
4. Average value of voltage over positive half cycle if $V = V_m \sin \omega t$
   $$V_{av} =< V > = \frac{\int_0^{\frac{T}{2}} \mathrm{V_m} \sin(\omega t)\, dt}{\int_0^{\frac{T}{2}} dt} = \frac{2V_m}{\pi} = 0.637 V_m$$
5. RMS value: It is the square root of the mean of the squares of the instantaneous current or voltage.
6. Root mean square (rms) value of current over one full cycle,
   $$I_{rms} = \sqrt{< i^2 >} = \frac{I_m}{\sqrt{2}} = 0.707\ I_m$$
7. Root mean square (rms) value of voltage over one full cycle
   $$V_{rms} = \sqrt{< V^2 >} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$
8. The average value of sin ωt, cos ωt, sin 2ωt and cos 2ωt over one complete cycle is zero.
9. The average value of $sin^2 \omega t\ and\ cos^2 \omega t$ over one cycle is 1/2.
10. Unless specified in the equation, take rms values of voltage or current.
11. Resistance of the circuit = R
12. Inductive reactance is $X_L = \omega L = 2\pi f L$
13. Capacitive reactance is $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$
14. Impedance of the circuit is Z.
15. The unit of resistance, inductive reactance, capacitive reactance and impedance is Ohm.
16. The reciprocal of reactance is susceptance and the reciprocal of impedance is admittance (Y). Its unit is Ω^{-1}.
17. $Power\ factor = \frac{Actual\ Power}{Apparent\ Power} = \cos \varphi = \frac{R}{Z}$
18. In purely resistive circuit, take $V = V_m \sin \omega t$ then $i = I_m \sin \omega t$
    The voltage and current are in same phase. The phase difference (ϕ) is zero and the power factor (cos ϕ) is 1.
19. In purely inductive circuit, take $V = V_m \sin \omega t$ then $i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$
    The instantaneous current lags the instantaneous voltage by 90^0 (or voltage leads the current by 90^0). The phase angle is + 90^0 and hence power factor = cos 90^0 = 0

20. In purely capacitive circuit, take $V = V_m \sin \omega t$ then $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$
    The instantaneous current leads the instantaneous voltage by 90^0 (or voltage lags the current by 90^0). The phase angle is - 90^0 and hence power factor = cos 90^0 = 0
21. In R-L circuit, take $V = V_m \sin \omega t$ then $i = I_m \sin(\omega t - \varphi)$. The current lags the voltage by phase difference ϕ. The power factor $PF = \cos\varphi = \frac{R}{Z}$ here $z = \sqrt{R^2 + \omega^2 L^2}$
22. In R-C circuit, take $V = V_m \sin \omega t$ then $i = I_m \sin(\omega t + \varphi)$. The current leads the voltage by phase difference ϕ. The power factor $PF = \cos\varphi = \frac{R}{Z}$ here $z = \sqrt{R^2 + {X_C}^2}$
23. The tangent of phase difference in R-L circuit is $\tan\varphi = \frac{X_L}{R} = \frac{\omega L}{R}$
24. The tangent of phase difference in R-C circuit is $\tan\varphi = \frac{X_C}{R} = \frac{1}{\omega CR}$
25. In phasor representation in R-L circuit (inductive circuit), $\bar{Z} = R + jX_L \ or$
    $Z = \sqrt{R^2 + {X_L}^2}$ or $\bar{V} = V_R + jV_L \ or \ V = \sqrt{{V_R}^2 + {V_L}^2}$
26. In phasor representation in R-C circuit (capacitive), $\bar{Z} = R - jX_C \ or$
    $Z = \sqrt{R^2 + {X_C}^2}$ or $\bar{V} = V_R - jV_C \ or \ V = \sqrt{{V_R}^2 + {V_C}^2}$
27. Here V is the resultant voltage, V_R is rms value of voltage across resistance, V_L is the rms value of voltage across inductance and V_C is the rms value of voltage across capacitance.
28. In inductive circuit, V_L leads the V_R by 90^0 by 90^0 phase difference.
29. In capacitive circuit, V_C lags the V_R by 90^0 phase difference.
30. If $\bar{Z} = Z\angle\phi$ it means current lags the voltage by phase difference ϕ and further means that the circuit is inductive which means that power factor is lagging.
31. If $\bar{Z} = Z\angle-\phi$ it means current leads the voltage by phase difference ϕ and further means that the circuit is capacitive which means that power factor is leading..
32. In L-C-R series circuit, ϕ = phase difference between instantaneous current and instantaneous voltage = $cos^{-1}\frac{R}{Z} = tan^{-1}\frac{X_L - X_C}{R}$
33. If $X_L > X_C$ then current lags the voltage and if $X_C > X_L$ then current leads the voltage.
34. Average power consumed in one cycle $P_{av} = <p> = \frac{\int_0^T p dt}{\int_0^T dt} = V_{rms} I_{rms} \cos\phi = I^2 R$
35. $I_{max} = \frac{V_{max}}{Z}$ and $I_{rms} = \frac{V_{rms}}{Z}$
36. RMS values of
    a) $V_R = I_{rms} R, V_L = I_{rms} X_L$ and $V_C = I_{rms} X_C$
    b) $V_{applied} = I_{rms} Z = \sqrt{{V_R}^2 + (V_L - V_C)^2}$
    c) $\omega = \omega_r = \frac{1}{\sqrt{LC}}$=Resonant frequency
37. At $\omega = \omega_r$
    a) $X_L = X_C$
    b) Z = minimum value = R
    c) I_{rms} = maximum value = $\frac{V_{rms}}{Z} = \frac{V_{rms}}{R}$
    d) $I_m = maximum\ value = \frac{V_m}{Z} = \frac{V_m}{R}$

e) Power factor $= \cos \phi = 1$

38. In one complete cycle, power is consumed only by resistance. No power is consumed by a capacitor or an inductor.
39. Regarding graph between resistance, reactance and impedance vs. angular frequency
    a) $X_L = \omega L \ or \ X_L \propto \omega \ \ and \ X_C = \frac{1}{\omega C} \ or \ X_C \propto \frac{1}{\omega}$
    b) R does not depend upon ω and it is constant. At $\omega = \omega_r : X_L = X_C \ and \ Z = Z_{min} = R$
40. In view of Euler's identity, $e^{j\theta} = \cos\theta + j\sin\theta$
41. There are three equivalent notations for a phasors.
    a) Polar form: $\vec{V} = V\angle\theta$
    b) Rectangular form: $\vec{V} = V(\cos\theta + j\sin\theta)$
    c) Exponential form: $\vec{V} = Ve^{j\theta}$
42. Impedance (Z): If impedance is given as $Z = \frac{V\angle\varphi}{I\angle\theta}$
    a) If impedance angle is positive ($\phi > \theta$), current lags the voltage and circuit is R-L circuit.
    b) If impedance angle is negative ($\phi < \theta$), current leads the voltage and circuit is R-C circuit.
    c) If impedance angle is 0^0, purely resistive circuit
    d) If impedance angle is $\pi/2$, current lags the voltage by $\pi/2$, the circuit is purely inductive.
    e) If impedance angle is $-\pi/2$, current leads the voltage by $\pi/2$, the circuit is purely capacitive.
43. **Leads/Lags mean**:
    a) If voltage maximum comes first than that of current at given time, voltage leads current. The circuit is R-L circuit.
    b) If current maximum comes first than that of voltage, current leads the voltage. The circuit is purely capacitive.
    c) If voltage and current maximum comes at same time, voltage is in phase with current and circuit is purely resistive.
44. Quality factor Q is defined as $Q = \frac{Resonant\ frequency}{Bandwidth} = \frac{\omega_r}{2\Delta\omega} = \frac{\omega_r}{\omega_2 - \omega_1}$
45. Quality factor at resonance is $Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r L}{R}$
46. Resonant frequency is, $\omega_r = \frac{1}{\sqrt{LC}}$
47. Quality factor can be in another form as, $Q = \frac{1}{R}\left(\sqrt{\frac{L}{C}}\right)$
48. Q is large if R is small or L is large, this means resonance is sharp or series resonance circuit is more selective.
49. Wattless Current: The current is ac circuit is said to be wattless if the average power consumed in the circuit is zero.
50. Choke coil: It is simply an inductor with large inductance which is used to reduce current in ac circuit without much loss of energy.
51. **L-C oscillations: Qualitative approach**

a) When switch is OFF: Initially the charge on capacitor is maximum (switch S is open) and electrical energy stored in the capacitor is $U_E = \frac{1}{2}\frac{{q_0}^2}{C}$

b) The energy in the inductor is zero.

c) When switch S is closed: The magnetic energy stored in the inductor for a maximum current is, $U_B = \frac{1}{2}L{I_0}^2$

d) Energy stored by a capacitor is, $U_E = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{{q_m}^2}{C}cos^2\omega t,$

e) Energy stored by inductor is, $U_B = \frac{1}{2}Li^2 = \frac{1}{2}L{q_m}^2\omega^2 sin^2\omega t = \frac{{q_m}^2}{2C}sin^2\omega t$, Since, at resonance,

f) $\omega^2 = \frac{1}{LC}$, so total energy $E = \frac{{q_m}^2}{C}$

52. **Transformer**:

a) It is a device (electrostatic) which is used to decrease (step down) or increase (step up) the voltage in ac circuit through mutual induction. A transformer consists of two coils wound over the same core.

$$\frac{\varphi_p}{N_p} = \frac{\varphi_s}{N_s} \Rightarrow \frac{1}{N_p}\frac{d\varphi_p}{dt} = \frac{1}{N_s}\frac{d\varphi_s}{dt} \Rightarrow \frac{e_p}{N_p} = \frac{e_s}{N_s} \Rightarrow \frac{e_s}{e_p} = \frac{N_s}{N_p}$$

b) In an ideal transformer, there is no loss of power. That is, $ei = constant, \frac{e_s}{e_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s}$

53. Regarding a transformer, **followings are few important points**:

a) In step up transformer, $N_s > N_p$. It increases voltage and reduces current
b) In step down transformer, $N_p > N_s$. It reduces voltage and increase current
c) It works only on AC.
d) A transformer cannot increase or decrease voltage or current simultaneously as ei = constt.
e) Some power is always lost due to eddy current or hysteresis etc.

$Output\ Power\ of\ transformer = Input\ Power\ x\ Efficiency\ x\ PF = VI \cos\varphi\ \eta$