Ch-8: Electromagnetic wave

1. Displacement Current (i_d) : It is current inside a capacitor plates when it is getting charged. In the expression, ϵ_0 is permittivity of free space, φ is the flux due to electric field.

$$i_d = \epsilon_0 \frac{d\varphi}{dt}$$

2. Modified Ampere-circuital law: In this expression, i_c is the conduction current in the wire to which a capacitor is connected.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 (i_c + i_d)$$

3. Maxwell's four set of equations:

   Gauss law of electrostatics-

   This law tells that electric flux through a closed surface (or surface integral of electric field around a closed surface) is 1/Ɛ0 times charge enclosed by that surface.

$$\varphi = \oint \vec{E} \cdot \vec{dS} = \frac{q}{\epsilon_0}$$

   Charge on an insulated conductor resides only on its outer surface.

   Gauss law of magnetism-

   This law tells that magnetic flux through any closed surface is zero.

   $\varphi = \oint \vec{B} \cdot \vec{dS}$ =0

   It explains that isolated magnetic poles do not exist.

   Faraday's law of electromagnetic induction-

   This law tells that a changing magnetic field induces an electric field. The induced emf set up in a closed circuit is equal to the rate of change of magnetic flux (φ)linked with that circuit.

$$\boldsymbol{e = -\frac{d\varphi}{dt} = \frac{d}{dt}\oint \vec{B} \cdot \vec{dS} = \oint \vec{E} \cdot \vec{dl}}$$

   Modified Ampere's law-

   This law states that line integral of magnetic field around any closed circuit is equal to µ0 times the total current (the sum of conduction and displacement current) threading the closed circuit.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 (i_c + i_d) = \mu_0 \left(i_c + \epsilon_0 \frac{d\varphi}{dt} \right)$$

4. Speed of electromagnetic wave in vacuum is:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 m/s$$

5. Maxwell's prediction of electromagnetic wave:
   - A time varying magnetic field is a source of changing electric field
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6. The expression of electromagnetic wave is given as:

$$E = E_0 \sin(kx - \omega t) \; and \; B = B_{\_} \sin(kx - \omega t)$$

In the above expression, the negative sign between $kx \; and \; \omega t$ shows that waves are travelling in positive x −direction. If it were positive, it means the wave is travelling in negative x-direction.

The $x-, y-, z-$ direction shows by which one ($x \; or \; y \; or \; z$) is multiplied with k. Here k is wave number and ω is angular frequency.

7. Wave number k is $k = 2\pi/\lambda$, where λis the wavelength.
8. Angular frequency $\omega = 2\pi f = \frac{2\pi}{T}$,where f and T are frequency and time period respectively of the wave.
9. Speed of light in vacuum is $c = E_0/B_0$,where E_0 (in N/C) is maximum value of electric field intensity and B_0 (in Tesla) is maximum value of magnetic field induction.
10. If the wave is travelling in positive $x - direction$,then direction of electric field and magnetic field oscillations can be determined by the following expression as:

$$\vec{E} \times \vec{B} = \vec{v}$$

Suppose, EM wave is propagating with velocity($\vec{v}$) in +x-direction, then, it is in unit vector $\hat{\imath}$,so,$\hat{\jmath} \times \hat{k} = \hat{\imath}$,hence electric field oscillates in y- direction and magnetic field will oscillate in z-direction. Suppose, wave is propagating in + z-direction and electric field is in y-direction, the then direction of magnetic field oscillation will be as:

$$\hat{\jmath} \times (-\hat{\imath}) = \hat{k}$$

So, magnetic field will oscillate in negative x-direction.