Ch-9: Ray Optics Formulae

Plane Mirror

1. Suppose a mirror is rotated by angle θ (say anticlockwise), keeping the incident ray fixed, then the reflected ray rotates by 2θ along the same sense (anticlockwise).
2. Suppose the incident ray makes an angle i, after rotating the mirror by angle θ CCW, the reflected ray angle i - 2θ from the Y-axis.
3. The minimum length of plane mirror to see one's full height (H) in it is H/2.
4. A man is standing exactly mid-way between a wall and a mirror and he wants to see the full height of the wall (behind him) in a plane mirror in front of him. The minimum length of mirror in this case is H/3.
5. Focal length as well as the radius of curvature of a plane mirror is infinity.
6. Power of a plane mirror is zero.
7. When two plane mirrors are kept facing to each other at an angle θ and an object is placed between them, then
    i. Number of images, $n = \left(\frac{360}{\theta} - 1\right)$, if $\frac{360^0}{\theta}$ is even.
    ii. Number of images, $n = \frac{360^0}{\theta}$, if 360/θ is odd.

    Or, number of images formed is $n = \frac{360}{\theta} - 1$ if θ is a sub multiple of 180^0. If θ is not sub multiple of 180, then number of images formed is the integers next higher to it.

    Example-Let θ = 72^0, then 360/72 = 5 (odd), so number of images is 5. If θ = 90^0, the number of images formed is 3.
8. A man approaches a vertical plane at a speed of 2 m/s; at what rate does he approach his image? [Ans. 4m/s] or a plane mirror is approaching you at a speed of 10 m/s, at what speed will your image approach you.

Spherical Mirror Formula

9. Mirror formula (for all types of Mirrors):

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$where\ v$ is distance of image. u is distance of object and f is focal length from the pole of the mirror. For a concave mirror(converging mirror) , f is negative, and for a convex mirror (diverging mirror), f is positive.

10. Magnification formula in mirror: Linear magnification or transverse magnification or lateral magnification of a mirror is given by:

$$m = \frac{h_I}{h_O} = -\frac{v}{u}$$

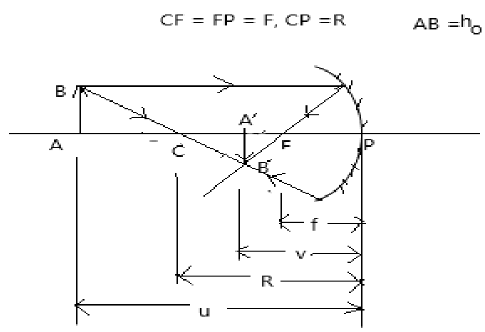
Where, $h_I = height\ of\ image, h_O = height\ of\ object$

Sign Convension in Mirror

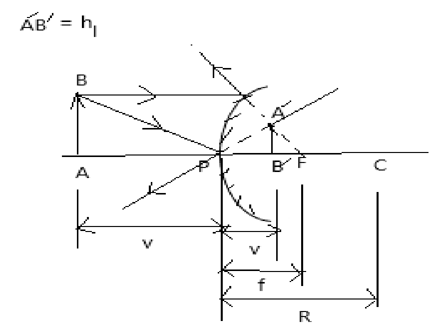
11. All distances are measured from the pole of the mirror.
12. Distance measured along the direction of incident ray from the pole of the mirror are taken positive and distance measured opposite to the direction of incident ray is taken negative.
13. Height above the principal axis is taken positive and height below the principal axis is taken negative.

Other information about the mirror:

14. For real image, magnification (m) is negative and for virtual image, magnification (m) is positive.



Concave Mirror
Fig-1



Convex Mirror
Fig-2

Position of Image for different Position of Object:

For concave Mirror:

Ser. No.	Position of Object	Details of Image
1	At ∞	At F, real, inverted, $\|m\| \ll 1$
2	Between C and ∞	Between F and C, real, inverted, $\|m\| < 1$
3	At C	At C, real, inverted, $\|m\| = 1$
4	Between F and C	Between C and ∞, real, inverted, $\|m\| > 1$
5	At F	At infinity, real, inverted, $\|m\| \gg 1$
6	Between F and P	Behind the mirror, virtual, erect, $\|m\| > 1$

For Convex Mirror:

Ser. No.	Position of Object	Details of Image
1	At ∞	At F, virtual, erect, $\|m\| \ll 1$
2	In front of mirror	Between P and CF, virtual, erect, $\|m\| < 1$

15. Two laws of reflection
    a) ∠i = ∠r
    b) Incident ray, reflected ray and normal lie on the same plane
16. Reflection from plane surface (or from plane mirror
    a) If object is real, its image from a plane mirror is virtual, erect, same size and at same perpendicular distance from the mirror.

b) F = ∞, P = 0

17. Reflection from spherical surface (concave or convex mirror)
    a) $f = \frac{R}{2}$
    b) $P\ (dioptre) = \frac{-1}{f\ (metre)}$
    c) Image speed $= (m^2)\ Object\ speed$
    They travel in the opposite directions.

Spherical Lens

18. Lens formula:
    For all type of Lenses, lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Where, u =distance of object from optical centre of lens, v = distance of image from optical centre, f = focal length of lens from optical centre oa a lens

19. Lens magnification formula:

$$m = \frac{h_I}{h_0} = \frac{v}{u}$$

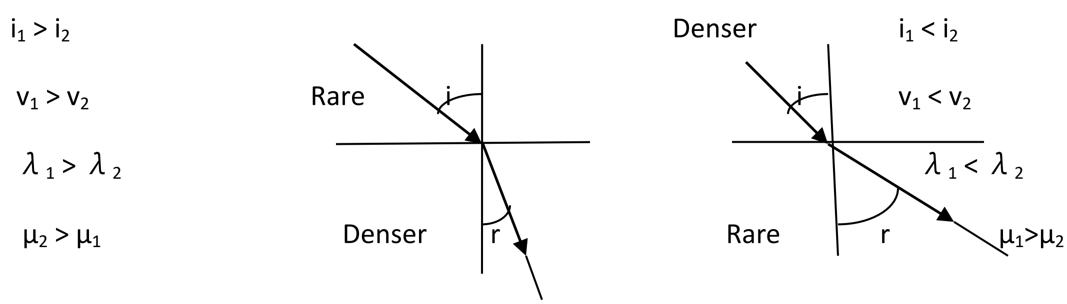
20. Here also in lenses, for all real image, magnification is negative and for virtual image, magnification is positive.
21. Convex lens is converging lens and has positive focal length f (Second Focus F_2),while concave lens is diverging lens, and has focal length negative (First Focus F_1).
22. Sign convention in lens is same as that in mirror.
23. **Snell's Law**

For two particular media, the ratio of sine of the angle of incidence to the sine of angle of refraction is constant. That is,

$$\frac{\sin i}{\sin r} = Constant = 1\mu_2 = \frac{\mu_2}{\mu_1}$$

Equation (1) is known as Snell's law.

The incident ray, the normal and the refracted ray all lie in the same plane.



The constant ratio is called the refractive index (Equation 1 above) for light passing from the first to the second medium (or refractive index of medium 2 with respect to medium 1). It is denoted by $1\mu_2$.

For two media,

$$\mu_1 \sin i = \mu_2 \sin r$$

If medium 1 is vacuum (say air), we refer $1\mu_2$ as μ_2 or simply μ. (($\mu_{air} = 1$).

If $\mu_2 > \mu_1$, $i > r$, if a ray of light passes from rare to denser medium, it bends towards normal and vice-versa. Again,

$$_1\mu_2 = \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$$

Here, v_1 is the speed of light in medium 1 and v_2 is the speed of light in medium 2. Correspondingly, λ_1 *and* λ_2 are wave lengths in medium 1 and medium 2 respectively.

If $\mu_2 > \mu_1$ then $v_1 > v_2$ and $\lambda_1 > \lambda_2$, i. e. in a rare medium, speed and wavelength of light is more.

In general, speed of light in any medium is less than its speed in vacuum.

So, Definition of refractive index (μ) is as

$$\mu = \frac{speed\ of\ light\ in\ vacuum}{speed\ of\ light\ in\ medium} = \frac{c}{v}$$

As ray of light moves from medium 1 to medium 2, its wave length changes but its frequency remains constant.

Principle of reversibility: This principle states that if the final path of a ray of light after it has suffered several reflections and refractions, is reversed, it retraces its path exactly.

24. Expression for lateral displacement

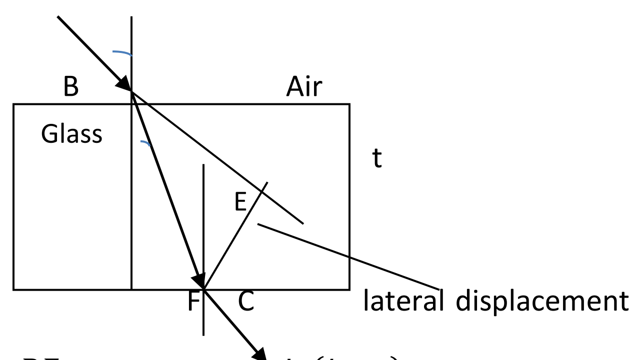
i=incidence angle ,r = refracted angle

t =thickness of glass

Lateral displacement (d):

d=CE=BC sin (i-r)

Hence,



$$d = BC\sin(i-r) = \frac{BF}{\cos r}\sin(i-r) = t\frac{\sin(i-r)}{\cos r}$$

$$d = t\sin i\left[1 - \frac{\cos i}{(\mu^2 - sin^2 i)^{\frac{1}{2}}}\right]$$

25. Relation between actual (real) depth, apparent depth and refractive index

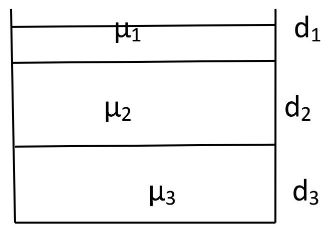
$$Apparent\ depth = \frac{Real\ depth}{refractive\ index}$$

26. Normal shift(t): d=real depth, μ is refractive index

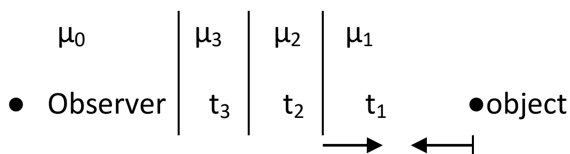
$$d = t\left(1 - \frac{1}{\mu}\right)$$

27. Refraction through composite slabs or through a number of parallel media as seen from μ_0

Three immiscible liquids of refractive indices μ_1, μ_2 and μ_3 ($\mu_3 > \mu_2 > \mu_1$) are filled in a vessel. Their depths are d_1, d_2 and d_3 respectively. The apparent depth (for normal incidence) when seen from top of first liquid will be



$d_{app} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3}$ when top of first liquid is air.



$$Apparent\ depth\ D_{app} = \sum \frac{d}{\mu_{rel}} = \frac{t_1}{\frac{\mu_1}{\mu_0}} + \frac{t_2}{\frac{\mu_2}{\mu_0}} + \frac{t_3}{\frac{\mu_3}{\mu_0}} + - - -$$

$$Apparent\ shift\ s = t_1\left(1 - \frac{1}{\mu_{1rel}}\right) + t_2\left(1 - \frac{1}{\mu_{2rel}}\right) + t_3\left(1 - \frac{1}{\mu_{3rel}}\right) + - - - -$$

Total Internal Reflection:

28. Necessary condition for total internal reflection:

1. Light must travel from an optically denser to rarer medium.
2. The angle of incidence in the denser medium must be greater than the critical angle for the two media.

Refraction of light through curved convex surface

29. When the light passes from rarer to denser medium, object lies in rarer medium, whether the surface is convex or concave and the image is real or virtual, the formula is given as

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

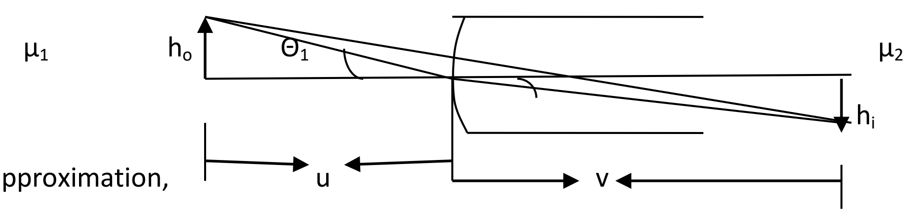
30. When the light passes from dense to rarer medium, object lies in denser medium, whether the surface is convex or concave and the image is real or virtual, the formula is given as

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

(Note: All formulae must be derived with sign convention)

31. **Lateral magnification through a curved surface:**

We take the curved surface as convex. The figure is shown below-



With the paraxial approximation,

$$\sin\theta_1 = \theta_1 = \tan\theta_1 = \frac{h_o}{-u} \text{ and } \sin\theta_2 = \theta_2 = \tan\theta_2 = \frac{-h_i}{v}$$

Applying Snell's law, $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

So, Magnification, $m = \frac{h_i}{h_o} = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right)$

32. **Lens Maker Formula,** If lens has refractive index μ_2 and the medium outside lens,both sides is μ_1, then focal length of lens is

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = Power\ (P)$$

33. If media is different on two sides a bi-convex lens,then lens maker formula becomes as:

$$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$$

$For\ u = \infty, v = f, then\ lens\ make\ formula\ becomes\ as$ $\frac{\mu_3}{f} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$

34. When lenses placed at distance from each other, then two cases,

$Case - I$: Lenses placed apart (distance d from each other) but object is at infinity, then

$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$Case - II$: Lenses placed apart and object at finite distance, then apply lens formula for each lens separately. The image formed by first lens acts as an object for the second lens and so on so forth. For lens L_1,apply $\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$; find v_1 for u_1 and f_1 given. Then for second lens L_2, take u_2 from optical centre of lens L_2 by subtracting or adding v_1. If image from lens L_1 is formed right side from optical centre of L_1 lens ,then subtract it from u_2 as $u_2 = d_1 - v_1$ but if image distance v_1 is left of optical centre of L_1,then $u_2 = d_1 + v_1$

35. **Two thin lenses in contact**: Suppose a lenses of focal length f_1 is placed in contact with another lens of focal length f_2, then combined focal length is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

36. Similarly, if more than two lenses in contact, the equivalent focal length is

$$\frac{1}{f}=\sum_{i=1}^{i=n}\frac{1}{f_i}$$

Here f_1, f_2 etc. are substituted with sign.

37. For n thin lenses in contact, the equivalent power is given by

$$P=P_1+P_2+P_3\pm----$$

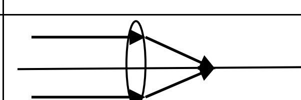
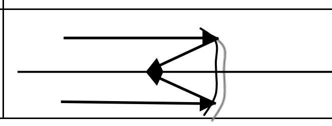
38. Construct the ray diagram to show the nature of the images as for convex lens:

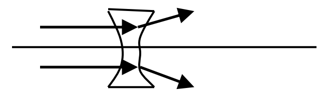
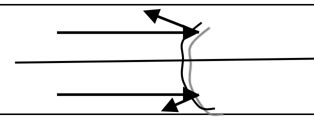
Object Position	Image Position	Size of image.	Nature of image
At infinity	At focus	Very small (Diminished)	Real & inverted
Between 2F and ∞	Between 2F & F	small	Real & inverted
At 2F	At 2F	Equal as object	Real & inverted
Between 2 F & F	Between 2F & ∞	Large	Real & inverted
At F	At infinity	Very large	Real & inverted
Between F and Optical centre	Beyond first focus	Large	Virtual & erect

Construct the ray diagram to show the nature of the images as for concave lens:

Object position	Image Position	Size of image	Nature of image
At any position on principal axis	Between F and Optical centre	Smaller	Virtual and Erect

39. The shorter the focal length of a lens or mirror, the more it converges or diverges, i. e. the bending of light is more for shorter focal length.

Nature of lens/Mirror	Focal length (f)	Power	Converging /diverging	Ray diagram
Convex lens	Positive	$P=\frac{1}{f}$ (Positive)	Converging	
Concave mirror	Negative	$P=-\frac{1}{f}$ (Positive)	Converging	

Concave lens	Negative	$P = \frac{1}{f}$ (Negative)	Diverging	
Convex mirror	Positive	$P = -\frac{1}{f}$ (Negative)	Diverging	

40. **Refraction of light through a prism**: If the minimum deviation is δ_{min},

δ_{min} = 2i – A or $i = \frac{A+\delta_m}{2}$ and A = r_1 + r_2 = 2r or $r = \frac{A}{2}$ So, refractive index

$$\mu_g = \frac{\sin i}{\sin r} = \frac{\sin \frac{A+\delta_m}{2}}{\sin \frac{A}{2}}$$

41. Minimum deviation occurs when refracted ray becomes parallel to the base of the prism.

42. Simple Microscope

(i) When final image is formed at least distance of distinct vision(D = 25 cm):Magnifying power or angular magnification M_D is:

$$M_D = \left(\frac{1}{D} + \frac{1}{f}\right) D = 1 + \frac{D}{f}$$

(ii) When final image is formed at infinity, the magnifying power is:

$$M_\infty = \frac{D}{f}$$

43. Compound Microscope:

(i) When final image is formed at least distance of distinct vision(D = 25 cm):Magnifying power or angular magnification M_D is

$$M_D = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

(ii) When final image is formed at infinity, the magnifying power is:

$$M_\infty = -\frac{v_0}{u_0}\left(\frac{D}{f_e}\right)$$

(iii) Tube length or distance between a convex lens of small aperture and focal length (objective) (f_o) and another convex lens of bigger aperture and large focal length (eye-piece) (f_e) when final image is formed at least distance of distinct vision is,

$$L_D = v_0 + \frac{Df_e}{D + f_e} \quad and \quad L = v_0 + u_e$$

(iv) Tube length or distance between a convex lens of small aperture and focal length (objective) (f_o) and another convex lens of bigger aperture and large focal length (eye-piece) (f_e) when final image is formed at least distance of distinct vision is,

$$L_\infty = v_0 + f_e$$

44. Astronomical Telescope: Refraction type (Objective is large and Eye-piece is small)

(i) Final image formed at least distance of distinct vision,

$$M_D = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

(ii) When final image is formed at infinity,

$$M_\infty = -\frac{f_0}{f_e}$$

(iii) Tube length when final image formed at D:

$$L_D = f_o + |u_e|$$

(iv) Tube length when final image is formed at infinity,

$$L_\infty = f_o + f_e$$