## Ch-10: Wave Optics:

In geometric optics, the propagation of light is described by rays that travels in straight lines and obeys the laws of reflection and refraction. However, rules of geometric optics are only valid when the wavelength of light is much smaller than the dimensions of the apertures or barriers involved. When this is not the case, phenomena arising from the wave nature of light are observed.

Wavefront:

**A wavefront** is the locus of all points in space which receives the light waves from a source in phase.

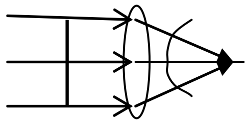
(i) A wavefront is always normal to the light rays.
(ii) A wavefront does not propagate in the backward direction.

**Types of wavefront**:-

1. **Spherical wavefront**: A point source of light gives spherical wavefront.
2. **Cylindrical wavefront**: When source of light is linear in shape, cylindrical wavefront is formed.
3. Plane wavefront: A small part of spherical or cylindrical wavefront originating from a distant source can be considered as a plane wavefront.

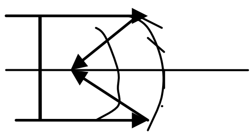
Shape of wave fronts in Convex, Concave (lens and mirror) and in prism:

1. Convex Lens:

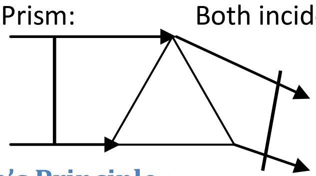


The refracted wavefront is spherical wave front converging , while incident Wave front is plane wave front.

2. Concave mirror: The reflected wavefront is spherical converging type while incident is PWF.



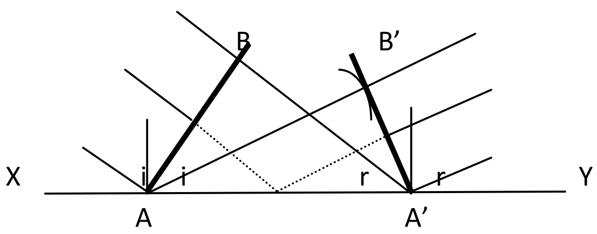
3. Prism: Both incident and refracted WFs are PWF.



### Huygen's Principle :

a) Each point on a given primary wavefront acts as a source of new disturbance called **secondary wavelets**, which travel in all directions with the velocity of light in the medium.
b) A surface touching these secondary wavelets, tangentially in the forward direction at any instant gives the new wavefront at that instant. This is called **secondary wavefront**.

**Reflection on the basis of wave theory**:




AB is a plane wavefront incident on a plane mirror XY at $\angle BAA' = i$ , 1, 2, 3 are corresponding incident rays perpendicular to AB.

According to Huygen's principle, every point on AB is a source of secondary wavelets. Let the secondary wavelet from B strikes XY at A' in time t sec. Therefore, BA' = ct

The secondary wavelets from A will travel the same distance c x t in the same time. Therefore, with A as a centre and c x t as radius, draw an arc B', so that AB' = ct

From A', draw a tangent plane A'B' touching the spherical arc tangentially at B'. Therefore, A'B' is the secondary wavefront after t sec. This would advance in the direction of rays 1', 2' and 3' which are the corresponding reflected rays perpendicular to A'B'.

In △s AA'B and AA'B', AA' is common, BA' = AB' = ct and ∠B = ∠B'

Therefore, triangles are congruent. Hence ∠BAA' = ∠B'A'A, i. e. ∠i = ∠r which is the first law of reflection. Further, the incident wavefront, reflecting surface and reflected wavefront all lie in the same plane of the paper. Therefore, incident ray, normal to the mirror XY and reflected rays all lie in the same plane of paper. This is second law of reflection.

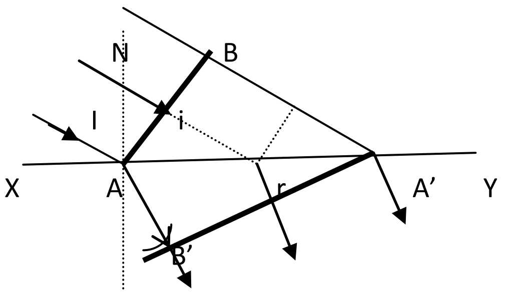
## Refraction on the basis of wave theory:

In the fig. Below, XY is a plane surface that separates a denser medium of refractive index μ from a rarer medium. If $c_1$ is the velocity of light in rarer medium and $c_2$ is the velocity of light in denser medium, then by definition, $\mu = \frac{C_1}{C_2}$.

Let AB is a plane wavefront incident on plane surface XY at ∠BAA' = i, Rays 1, 2, 3 are corresponding incident rays normal to AB.

According to Huygen's principle, every point on AB is a source of secondary wavelets. Let the secondary wavelets from B strikes XY at A' in t sec, so BA' = $c_1$ x t

During the time, the disturbance from B reaches A', the secondary wavelets from point A must have spread over a hemisphere of radius AB' = $c_2$ x t in the second medium. The tangent plane A'B' drawn from A' over this hemisphere will be the new refracted wavefront.




Angle of incidence is that incident wavefront makes with the refracting surface and angle of refraction is that refracted wavefront makes with the refracting surface.

In △ AA'B, $\sin i = \frac{BA'}{AA'} = \frac{c_1 t}{AA'}$

In △ AA'B', $\sin r = \frac{AB'}{AA'} = \frac{c_2 t}{AA'}$

∴ $\frac{\sin i}{\sin r} = \frac{c_1}{c_2} = \mu$, which proves the Snell's law of refraction.

It is also clear that incident rays, normal to the surface XY and refracted rays all lie in the same plane (i. e. in the plane of paper). This is the second law of refraction.

Hence, law of refraction is proved on the basis of wave theory.

Followings are merits and demerits of Huygens' principle:

## Merits of Huygens' principle:

On the basis of Huygens' Principle, many optical phenomena like interference, diffraction, law of refraction and reflection could be explained.

**Demerits of Huygens's principle**:

(1) Huygens' principle could not explain why wave front of secondary wavelets is formed in forward direction and not in backward direction?

(2) This theory assumes light waves as longitudinal waves, therefore could not explain polarisation of light.

(3) This principle could not explain electromagnetic nature of light.

**Note:** The angle of incident and reflected wave front from vertical as well as horizontal is same.

**Interference:**

The colour seen in oil films and soap bubbles area a result of interference between light reflected from the front and back surfaces of a thin film of oil or soap solution. The first successful optical interference was carried out by Thomas Young in 1801.

**Interference of light**: Young's double SLIT EXPERIMENT

It refers to any situation in which two or more waves overlap in space. When this occurs, the total wave at any point at any instant of time is governed by Principal of Superposition.

This principal states that: When two or more waves overlap, the resultant displacement at any instant may be found by adding the instantaneous displacements that would be produced at the point by individual wave if each were present alone.

" **When two light sources of same frequency and having zero or constant phase difference travelling in the same direction superpose each other, the intensity in the region of superposition gets redistributed, becoming maximum at some points and minimum at others. This phenomenon is called interference of light**."

**Type of interference:**

(1) ***Constructive interference*****:** The interference at the points, where the two waves meet in same phase, i. e. the intensity of light is maximum, is called the constructive interference.

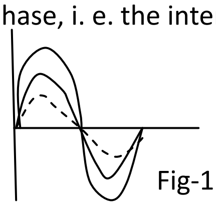


Fig-1

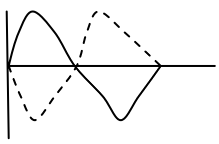


Fig. 2

(2) ***Destructive interference***: When two waves meet in opposite phase i. e. the intensity of light is minimum, is called the destructive interference. (as shown in fig. 2)

**Note**: During interference of light, redistribution of energy (intensity) takes place but total energy remains same.

**Conditions of interference**:

1. The sources must be coherent. (that is, they must maintain a constant phase difference with respect to each other).
2. Their frequencies (frequency of two waves) should be same and
3. Sources should be monochromatic.

**Expression for resultant intensity interference of two waves**:

Derive the expression for the intensity at any point on the observation screen in Young's double slit experiment and condition for constructive and destructive interference:

Let two waves have displacements as

$y_1 = A_1 \sin \omega t - - - - \qquad (1)$

$y_2 = A_2 \sin(\omega t + \varphi) - - - - (2)$

According to principal of superposition, the resultant wave would be given by

$y = A \sin(\omega t + \theta)$

So, $y = y_1 + y_2 = A_1 \sin \omega t + A_2 \sin(\omega t + \varphi) = A_1 \sin \omega t + A_2[\sin \omega t \cos \varphi + \cos \omega t \sin \varphi]$

$=(A_1 + A_2 \cos \varphi) \sin \omega t + A_2 \cos \omega t \sin \varphi$

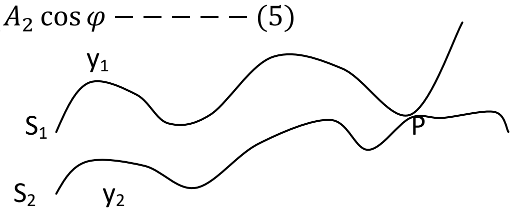
Let $A \cos \theta = A_1 + A_2 \cos \varphi ----(3)$

and $A \sin \theta = A_2 \sin \varphi ------(4)$, so

$y = A \cos \theta \sin \omega t + A \sin \theta \cos \omega t = A \sin(\omega t + \theta)$

From equation (3) and (4), $A^2 = {A_1}^2 + {A_2}^2 + 2A_1A_2 \cos \varphi ----- (5)$

,




Consider two coherent sources $S_1$ and $S_2$. Suppose two waves emanating from these two sources superpose at point P. The phase difference between them at P is ø (which is constant). If the amplitudes due to two individual sources at P is $A_1$ and $A_2$, then resultant amplitude is given by equation (5).

Since, intensity I is directly proportional to square of amplitude, that is, $I \propto A^2$

So, from equation (5), $I = I_1 + I_2 = 2\sqrt{I_1 I_2} \cos \varphi ----- (6)$ **Derived**.

[Here $I_1$ and $I_2$ are intensities due to independent sources. If the sources are incoherent sources, then the resultant intensity at P is given by $I = I_1 + I_2$]

**Constructive and destructive interference condition**:

From equation (5) and (6) above,

$A^2 = {A_1}^2 + {A_2}^2 + 2A_1A_2 \cos \varphi$ and $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \varphi$ where, ø is the constant phase angle between two waves, here,

1. **Constructive Interference**: If cos ø =1, $A = A_1 + A_2 = A_{max}$ and $I = I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$
   And this is the case of **constructive interference** i. e. **Intensity (I) should be maximum**.
   **Path difference** $\boldsymbol{\Delta x = n\lambda}$ where n = 0, $\pm1, \pm2\pm, \pm3, \dots$ .... and **phase difference** $\boldsymbol{\varphi = 2n\pi}$ where n = 0, $\pm1, \pm2\pm, \pm3, \dots$ ....
2. **Destructive Interference**: If cos ø = -1, $A = A_{min} = A_1 - A_2$ and accordingly, $I = I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$
   And this is the case of **destructive interference** i. e. intensity (I) should be minimum.
   **Path difference** $\boldsymbol{\Delta x = (2n - 1)\lambda/2}$ where n = 0, $\pm1, \pm2\pm, \pm3, \dots$ .... and **phase difference** $\boldsymbol{\varphi = (2n - 1)\pi}$ where n = 0, $\pm1, \pm2\pm, \pm3, \dots$ ....

**Comparison of intensities at maxima and minima**:

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ when cos ø = 1, that is, bright fringe corresponding to integer n is called $n^{th}$ order bright fringe or $n^{th}$ bright fringe. For n = 0, called central bright fringe (or zero order). For n = 1, first bright fringe or first order bright fringe and $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ when cos ø = -1, therefore,

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

**Q. 1.** Two sources with intensities $I_0$ and $4I_0$ respectively, interfere at a point in a medium. Find the ratio of (i) maximum and minimum possible intensities, (ii) ratio of amplitudes. [Ans. (i) 9:1, (ii) 2:1]

**Q. 2:** Two sources of intensities I and 4I are used in a experiment. Find the intensity at a point, where the waves from two sources superimpose with a phase difference of (i) 0, (ii) π/2 and (iii) π [Ans. 9I, 5I and I]

**Coherent and Incoherent sources**:
Two sources of light which continuously emit light waves of same frequency with zero or constant phase difference between them is called **coherent sources**.
Two sources of light which do not emit light waves with a constant phase difference are called **incoherent sources**.

**Coherent sources:**

a) Case I: **If cos $\varphi$ remains constant with time**, then total intensity at any point will be constant. The intensity will be maximum at point were cos $\varphi$ = 1 and maximum intensity will be $\left(\sqrt{I_1}+\sqrt{I_2}\right)^2$ and minimum intensity when cos $\varphi$ = -1 and minimum value of intensity is $\left(\sqrt{I_1}-\sqrt{I_2}\right)^2$. The sources in this case are coherent sources.

**Incoherent sources**:

b) Case-2: If **cos $\varphi$ varies continuously with time,** assuming both positive and negative value, then average value of cos $\varphi$ = 0 over time interval of measure. The interference term averages to zero. There will be same intensity I = $I_1 + I_2$. The two sources in this case are incoherent.

c) If both the slits are of equal width, $I_1 = I_2 = I_0$, we have $I_{max} = 4I_0$ and $I_{min} = 0$ and $I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\varphi = 2I_0(1+\cos\varphi) = 2I_0\left(1+2cos^2\frac{\varphi}{2}-1\right) = 4I_0 cos^2\frac{\varphi}{2}$

d) If slits are of unequal width, $I_1 \neq I_2$ and $I_{min} \neq 0$

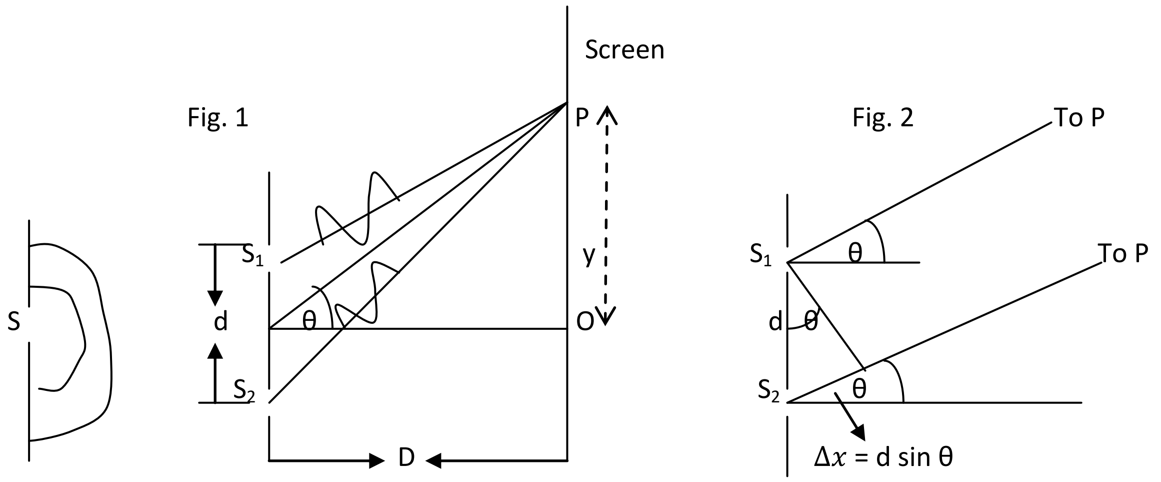
e) Intensity of light coming from a slit is proportional to slit width and slit width is proportional to square of amplitude. If $w_1$ and $w_2$ are widths of two slits, **then** $\frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2}$

**Q. 3:** Two incoherent sources of light emitting light of intensity $I_0$ and 3 $I_0$ interfere in a medium. Calculate the resultant intensity at any point. [Ans. 4 $I_0$]

Thomas Young was first to demonstrate the interference of light in 1801.

**YDSE (Young's double slit experiment)**: Principal: Monochromatic light (single wavelength) from a narrow slit (a narrow cut or opening) S falls on two other narrow slits $S_1$ and $S_2$ which are very close together and parallel to S. $S_1$ and $S_2$ act as two coherent sources. If S, $S_1$ and $S_2$ are very narrow, diffraction (bending of light at openings whose width is order of wavelength of light) causes the emerging beam to spread into the region beyond the slits. Superposition occurs in the shaded area where the diffraction beams overlap. Alternate bright and dark equally spaced vertical bands interference fringes) can be observed on a screen placed at some distance from the slits. If either of $S_1$ or $S_2$ is covered, the fringes disappear.

**Derivation of fringe width (separation of two bright or dark fringes):**



**Path difference**:

For bright fringe, $\Delta x$ = d sin θ = n $\lambda$ (integer multiple of $\lambda$ where n is the order of fringe)

For dark fringes, $\Delta x$ = d sin θ = odd multiple of $\lambda$ /2 or, (n + 1/2 ) $\lambda$

Since, d sin θ = n $\lambda$ for bright fringes so, $n = \frac{d}{\lambda}\sin\theta$, hence

Suppose a narrow slit S is illuminated by monochromatic light of wave length $\lambda$ . $S_1$ and $S_2$ are two narrow slits at equal distance at S. $S_1$ and $S_2$ act as two coherent sources separated by a small distance d. Interference fringes are obtained on a screen placed at distance D from the sources $S_1$ and $S_2$.

Fig 1 shows that the light waves from $S_1$ and $S_2$ meeting at an arbitrary point P on the screen. Since D >> d, the two light rays are approximately parallel with a path difference,

$\Delta\, x = S_2P - S_1P = d\sin\theta$, For two coherent sources, the resultant intensity is given by

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi - - - (1)$

**For maximum intensity at P:** cos ø = 1 ⇒ ø = 2nπ, where ø is the phase difference and given by

$\varphi = \frac{2\pi}{\lambda}(path\ difference) = \frac{2\pi}{\lambda}\Delta x$, so, $2n\pi = \frac{2\pi}{\lambda}d\sin\theta \Rightarrow d\sin\theta = n\lambda$

| Path difference = d sin θ = n $\lambda$ where $n = 0, \pm1, \pm2, \pm3, - - - -$ | ------(2) |
|---|---|

**For minimum intensity at P:** cos ø = -1⇒ ø = (2n-1) π ⇒ $\frac{\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}d\sin\theta \Rightarrow d\sin\theta = (2n-1)\frac{\lambda}{2}$,

sin $\theta$ = $\theta$ = $\tan\theta = \frac{y}{D}$

Hence,

| Path difference = $d\sin\theta = (2n-1)\frac{\lambda}{2}$ where $n = \pm1, \pm2, \pm3, - -$ | ------(3) |
|---|---|

**Note**: The first minima ( n = ±1) is adjacent to central maximum on either side.

**Position of bright fringes**:

Assumption: D > > d ( for two waves to be parallel) and d >> $\lambda$ (for very small angle), for visible light, so $\theta$ is very small. Hence,

sin $\theta$ = $\theta$ = $\tan\theta = \frac{y}{D}$ or $d\sin\theta = \frac{dy}{D}$ -------(4)

From equation (2), (3) and (4),

For bright fringe (max. Intensity), $\frac{dy_{bright}}{D} = n\lambda$ or,

So, $\boldsymbol{y_{bright}} = \frac{\boldsymbol{nD\lambda}}{\boldsymbol{d}}$ **------(5)**

Where, n = 0, ±1, ±2, ±3, --------- so,

For n = 0, $y_0 = 0$ (Central bright fringe)

For n = 1, $y_1 = \frac{D\lambda}{d}$ (First bright fringe), likewise

**Position of dark fringes**:

Assumption: D > > d ( for two waves to be parallel) and d >> $\lambda$ (for very small angle), for visible light, so $\theta$ is very small. Hence, sin $\theta$ = $\theta$ = $\tan\theta = \frac{y}{D}$ or $d\sin\theta = \frac{dy}{D} = (2n-1)\frac{\lambda}{2}$

$So, \boldsymbol{y_{dark}} = \frac{\boldsymbol{(2n-1)D\lambda}}{\boldsymbol{2d}}$ **-------(6)**

**To derive the fringe width:**

**Fringe width (β)** = distance between two adjacent bright (or dark fringe) (Fringe means a bright or dark band formed by interference or diffraction of light, that is visible bright/dark rows or bright/dark band or maxima/minima. The pattern of bright and dark fringes on the screen is called interference pattern)

**Width of a bright fringe**:-

$$\beta = y_n - y_{n-1} = \frac{Dn\lambda}{d} - \frac{D(n-1)\lambda}{d} = \frac{D\lambda}{d}$$

**Width of a dark fringe**:-

$$\beta = y_n - y_{n-1} = \frac{(2n-1)D\lambda}{2d} - \frac{(2n-3)D\lambda}{2d} = \frac{D\lambda}{d}$$

Clearly, both the bright and dark fringes are of equal width. Hence, expression of fringe width in YDSE can be written as $\beta = \frac{D\lambda}{d}$

**Angular fringe width**: $\alpha = \frac{\beta}{d} = \frac{\lambda}{d}$

**Note-1: Highest order of interference maxima**: will be when θ = $90^0$ and $n_{max} = \left[\frac{d}{\lambda}\right]$ where it represents the greatest integer function.

**Highest order of interference minima** : $= \left[\frac{d}{\lambda} + \frac{1}{2}\right]$

**Total number of maxima** : $2n_{max} + 1$

**Total number of minima** $= 2n_{min}$

**Condition that bright fringes due to both wavelengths $\lambda_1$ and $\lambda_2$ ($\lambda_2 > \lambda_1$) coincide is**

$$n\beta_2 = (n+1)\beta_1 \Rightarrow n\frac{D\lambda_2}{d} = (n+1)\frac{D\lambda_1}{d} \Rightarrow n\lambda_2 = (n+1)\lambda_1$$

Find the value of n and get least distance from central fringe as $y = n\frac{D\lambda_2}{d}$ or $(n+1)\frac{D\lambda_1}{d}$

**Note-2**: If Young's double slit apparatus is immersed in liquid of refractive index µ, the wave length of light decreases from to $\lambda_1$ to $\lambda_2$ and fringe width reduces from $\beta_1$ to $\beta_2$ as

$\beta_2 = \frac{\beta_1}{\mu}$ or $\mu_1\beta_1 = \mu_2\beta_2$ or $\mu\beta = constant$

[As per Snell's law, $\mu \sin\theta = constant$, $\mu\lambda = constant$ and $\mu v = constant$]

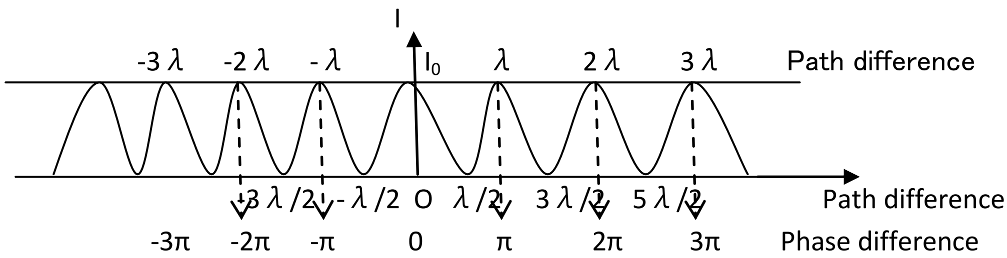
**Note:** If YDSE apparatus is immersed in liquid of refractive index µ the wavelength of light and hence fringe width decreases µ times.

**Trigonometry (Extra)**

1. If cos θ = 1, then its general value is θ = 2nπ { as cos θ = 1 = cos $0^0$ ⇒ θ = 2nπ ± $0^0$ = 2nπ}
2. If cos θ =-1⇒ cos θ = cos π ⇒ θ = 2nπ ± π = (2n + 1)π or (2n-1) π, so we take θ = (2 n +1) π

In all the above cases, n = 0, ±1, ±2, ±3, -------.

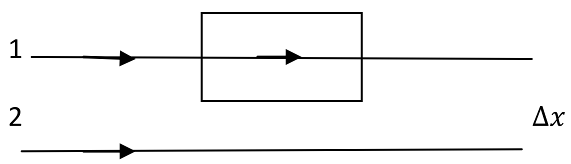
**Intensity distribution curve for interference**: A plot of intensity Vs. d sin θ (path difference)




**Interference pattern with white light**:

Different component colours of white light produce their own interference pattern. At the centre of the screen, the path difference is zero for all such components. So bright fringes of different colours overlap at the centre. Consequently, the central fringe is white. Since, fringe width $w = \frac{D\lambda}{d}$ or $w \propto \lambda$, so closest fringe on either side of central fringe is violet(since, violet colour has lowest wavelength), while the farthest fringe is red. After a few fringes, the interference pattern is lost due to large overlapping of fringes and uniform white illumination is seen on the screen.

**Path difference produced by a slab: (For NEET/IIT)**




Considering two light rays 1 and 2 moving in air parallel to each other. If a slab of refractive index μ and thickness t is inserted between the paths of one of the rays, then path difference $\Delta x = (\mu - 1)t$ is produced among them.

Proof: Time taken by ray to cross the slab 1 $T_1 = \frac{t}{v} = \frac{t}{\frac{c}{\mu}} = \frac{\mu t}{c}$ and time taken by ray 2 to cross the same thickness in air is $T_2 = \frac{t}{c}$ as $T_1 > T_2$ so difference in time is $T_1 - T_2 = \Delta T = (\mu - 1)\frac{t}{c}$ so

$\Delta x = (\mu - 1)t$

Note: Path difference produced when slab refractive index $\mu_2$ of thickness t is kept in a medium of refractive index $\mu_1$ ($<\mu_2$), then path difference $\Delta x = \left(\frac{\mu_2}{\mu_1} - 1\right)t$

Optical path length = μt

**Requirement to find coherent sources:**

**Two coherent sources can be obtained either by-**

(1) Two real images of same source (in YDSE)

(2) Two sources should give monochromatic light.

(3) The path difference between light waves from two sources should be small.

**Coherent sources are produced by –**

(1) Division of wave front (YDSE)

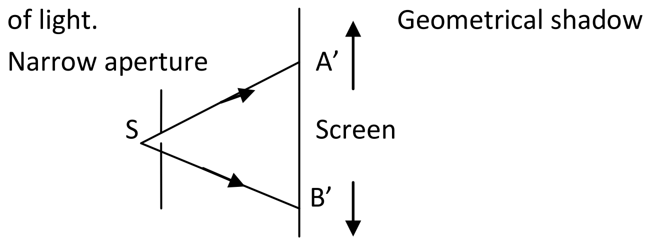
(2) Division of amplitude (partial reflection or refraction in thin films)

**Q.4:** If two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$ interfere at a point. What is the amplitude of resulting wave? [Ans. 6]

**Q.5:** In YDSE, the two slits are separated by 0.1 mm and they are 0.5 m from the screen. The wavelength of light used is 500 nm. Find the distance between $7^{th}$ maxima and $11^{th}$ minima. [Ans. 8.75 mm]

**Q. 6**: Two slits in YDSE are placed 2 mm from each other. Interference pattern is observed at a screen placed 2 m from the plane of slits. What is the fringe width for a light of wavelength 400 nm? [Ans. $2 \times 10^{-4}$ m]

**Diffraction of light**: The phenomenon of bending of light around the corners of small obstacles or apertures and its consequent spreading into the regions of geometrical shadow is called diffraction of light.

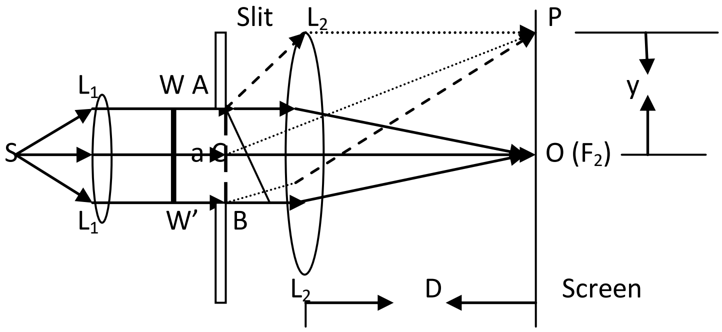



**"It is the phenomenon of bending of light around corners of an obstacle or aperture in the path of light".**

Diffraction is most pronounced when slit width is smaller than wavelength. (a < $\lambda$ ). Thus narrower the slit width, the greater is the spreading. For visible light, $\lambda$ is very small (order of $10^{-6}$ m), therefore, diffraction of visible light is not so common.

Diffraction occurs on account of mutual interference secondary wavelets which are not blocked by obstacle or from portions of wave fronts which are allowed to pass through the aperture.

Diffraction at single slit:




**Fraunhofer Diffraction**

A source S of monochromatic light is placed at the focus of a convex lens $L_1$. A parallel beam of light emerging from the lens $L_1$ with a plane wavefront WW' is made to fall on a single slit AB. As width of slit AB = a, is of the order of wavelength of light, therefore, diffraction occurs on passing through the slit. The diffraction pattern is focussed on to the screen XY with the help of a convex lens $L_2$.

The diffraction pattern is obtained on the screen consists of a central bright band, having alternate dark and weak bright bands of decreasing intensity on both sides of the central bright band.

The set of parallel rays falling on the slit form a plane wave front AB. According to Huygen's principle, each point on the unblocked portion (shown dotted) of plane wave-front AB sends out secondary wavelets in all the directions.

All parts of slits AB will become source of the secondary wavelets, which all starts in the same phase. These wavelets spread out in all directions, thus causing diffraction of light after it emerges through slit AB. The diffraction pattern is focussed by a convex lens $L_2$ on screen placed in its focal plane.

Path difference: Suppose the secondary wavelets diffracted at an angle θ are focussed at point P.

Path difference between the secondary waves reaching P from A and B = BN = AB sin θ = a sin θ

Dividing the slits in two equal parts, waves from upper half of the slit interfere destructively with waves from lower half when $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$ or when $a\sin\theta = \lambda$ or $\sin\theta = \frac{\lambda}{a}$

If we divide the slit into four equal parts, $\sin\theta = \frac{2\lambda}{a}$

If we divide the slit in six equal parts, darkness occurs on the screen when $\sin\theta = \frac{3\lambda}{a}$

1. **Position of nth secondary minima**:

   The general condition for destructive interference, $path\ difference = a\ \sin\theta = n\lambda$ or,

   $$\sin\theta = \frac{n\lambda}{a}, n = \ \pm1, \pm2, \pm3\_\_\_$$

   **Position of nth minima from the centre of the screen**:

   $$Since, \sin\theta = \theta = \tan\theta = \frac{y}{D} = \frac{n\lambda}{a}$$

   $$\therefore\ \boldsymbol{y_n} = \frac{\boldsymbol{nD\lambda}}{\boldsymbol{a}} - - - (1)$$

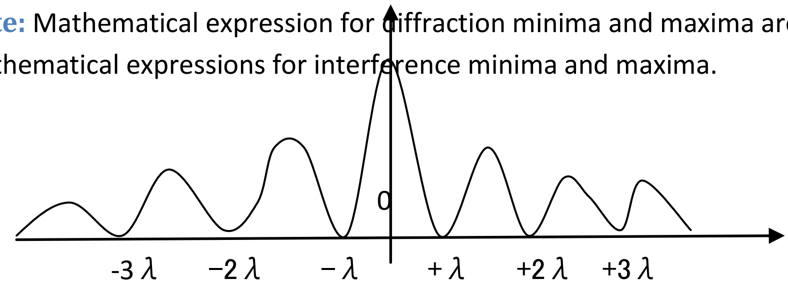
2. **Position of nth secondary maxima**: $Path\ difference\ BN = a\sin\theta = (2n+1)\frac{\lambda}{2}, n = \pm1, \pm2, \_\_$

### Position of nth maxima from the centre of screen:

$$Since, \sin\theta = \theta = \tan\theta = \frac{y}{D} = (2n+1)\frac{\lambda}{2a}$$

$$\therefore\ \boldsymbol{y_n} = \frac{\boldsymbol{(2n+1)D\lambda}}{\boldsymbol{2a}} --- (2)$$

**Note:** Mathematical expression for diffraction minima and maxima are exactly reverse of mathematical expressions for interference minima and maxima.




### Width of Central maximum:

The width of central maximum is the difference between first secondary minimum on either side of O. If P is the position of first secondary minimum and OP is x, then

$$\sin\theta = \theta = \tan\theta = \frac{y}{f} = \frac{y}{D} = \frac{\lambda}{a}$$

$$\therefore y = \frac{D\lambda}{a}, hence$$

$$Width\ of\ central\ maximum = \boldsymbol{2y} = \frac{\boldsymbol{2D\lambda}}{\boldsymbol{a}} = \frac{\boldsymbol{2f\lambda}}{\boldsymbol{a}}$$

Where, f = focal length of lens $L_2$ which is held very close to the slit.
So, we find that the width of central maximum decreases when the slit width increases.
Angular width of central maximum: sin θ = θ when θ in radian and small, therefore,

$$\sin\theta = \theta = \frac{\lambda}{a}, hence$$

$$\boldsymbol{Angular\ width\ of\ central\ maximum\ (2\theta)} = \frac{2\lambda}{a}, hence$$

$$\boldsymbol{Width\ of\ central\ maximum = D(angular\ width\ of\ central\ maximum)}$$

Where D = distance between slit and screen
Difference in single pattern of single slit due to monochromatic light and white light:
When source of light is monochromatic, the diffraction pattern consists of alternate bright and dark bands of unequal width. The central bright fringe has maximum intensity. The intensity of successive secondary maxima falls off rapidly.
When source is emitting white light, the diffraction pattern is coloured. The central maximum is white, but other bands are coloured. As $band\ width \propto \lambda$, the red band with higher wavelength is wider than the violet band with smaller wavelength.

**Q. 7:** In YDSE, separation between slits is 1 mm, distance of screen from slits is 2m. If wavelength of incident light is 500 nm, determine (i) fringe width, (ii) angular fringe with, (iii) distance between $4^{th}$ bright fringe and $3^{rd}$ dark fringe, (iv) if whole arrangement is immersed in water ($\mu_w = \frac{4}{3}$), new angular width. [Ans. 1 mm, 5 x $10^{-4}$ rad, 3/2 mm, 15/4 x $10^{-4}$ rad]

**Q. 8:** In YDSE set up using monochromatic light of wavelength $\lambda$, the intensity of light at a point, where path difference is 2 $\lambda$ is found to be $I_0$. What will be the intensity at a point when path difference is $\frac{\lambda}{3}$ ? [Ans. $I_0/4$]

**Q. 9:** The maximum intensity in YDSE is $I_0$. Find the intensity at a point on the screen, where (i) phase difference between the two interfering beam is $\frac{\pi}{3}$ . (ii) the path difference between them is $\frac{\lambda}{4}$ . [Ans. $3I_0/4$, $I_0/2$]

**Q. 10**: In a YDSE set up, the wavelength of light used is 546 mm. The distance of screen from slits is 1m and the slit separation is 0.3 mm. Compare the intensity at a point P distant 10 mm from the central fringe where the intensity is $I_0$. (ii) Find the number of bright fringes between P and the central fringe. [Ans. 3 x $10^{-4}$ $I_0$, 5.49 i. e. 5 (an integer)

Note: Regarding question No. 10, use path difference, $\Delta x = \frac{yd}{D}$ (here $I_0$ is the intensity at central fringe) and for number of bright fringes, use, $\frac{y}{\beta}$ (excluding central bright fringe), here y = 10 mm (given) and $= \frac{D\lambda}{d}$ ]

### Difference between interference pattern and diffraction pattern of light:

| | Interference Pattern | Diffraction Pattern |
|---|---|---|
| 1. | Interference is the result of superposition of waves from two different wave fronts. | Diffraction is the result due to superposition of wavelets from different points of the same wave front. |
| 2. | In the interference pattern, all the bright fringes (or bands) are of same intensity. | In diffraction pattern, all the bright bands are not of same intensity. |
| 3. | The dark fringes are almost black | Dark fringes are not perfectly black |
| 4. | The width of interference fringes are equal. | The width of diffraction bands is always unequal. |
| 5 | Bands are equally spaced. | Bands are not equally spaced. |

**Fresnel distance**: It is the minimum distance a beam of light can travel before its deviation from straight line path due to diffraction becomes significant.

Since, angle of diffraction (bending) for central maximum is $\theta = \sin\theta = \frac{\lambda}{a} = half\ angular\ width$

In travelling a distance Z, the half angular width become $\frac{Z\lambda}{a}$, due to diffraction of light. Now,

$\frac{Z\lambda}{a} \geq a\ only\ when\ Z \geq \frac{a^2}{\lambda} \therefore$ $\boldsymbol{Fresnel\ distance = Z_F = \frac{a^2}{\lambda}}$

Note: Deviation from straight line path becomes noticeable only when a beam after diffraction travels more than 1.67 m. [Given, average wavelength of visible light = 6000 Ǎ, diameter of pupil = 1 mm, so $Z_F = \frac{a^2}{\lambda} = 1.67\ m$ (after putting the values given)]

**Q. 11.** A screen is placed 50 cm from a single slit, which is illuminated with 6000 ʎ light. If distance between 1st and 3$^{rd}$ minima in the diffraction pattern is 3 mm, what is the width of the slit?[Ans. 0.2 mm]

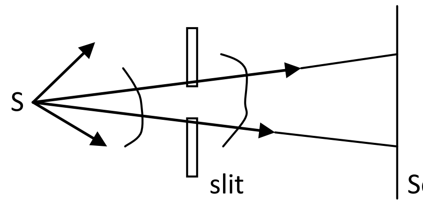
**Q.12**: A slit of width a is illuminated by red light of wavelength 6500 ʎ. For what value of a will the first maximum fall at angle of diffraction of $30^0$? [Ans. 1.95 x $10^{-6}$ m] [Use, $a\sin\theta = \frac{3}{2}\lambda$]

**Q. 13**: In a single slit diffraction experiment, first minimum for $\lambda_1$ = 600 nm coincides with the first maxima for wavelength $\lambda_2$. Calculate $\lambda_2$. [Ans. 440 nm] [Hint: The two will coincide if $\theta_1=\theta_2$] i. e. $a\sin\theta_1 = n\lambda_1$ and $a\sin\theta_2 = (2n+1)\frac{\lambda_2}{2}$]

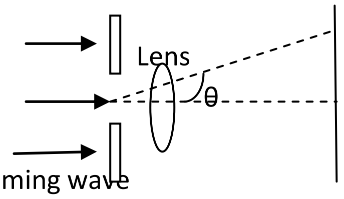
**Q. 14**: For what distance is ray optics a good approximation when a plane light wave is incident on a circular aperture of width 2 mm having wavelength 600 nm? [Ans. 6.7 m][Use Fresnel distance $Z_f = \frac{a^2}{\lambda}$

### Type of diffraction:

1. **Fresnel diffraction**: In this type, either source or screen or both are at finite distance from the diffracting device (obstacle or aperture). The diffracted wavefront is considered as spherical. No convex lens is used to focus the diffraction fringes on the screen.




2. **Fraunhofer diffraction**: In this type, both source and screen are effectively at infinite distance from the diffracting device. Fraunhofer diffraction is a particular limiting case of Fresnel diffraction.




### Some Extra Numerical:

1. In YDSE, using red and blue lights of wavelengths 600 nm and 480 nm respectively. Find the value of n for which nth red fringe coincides with (n + 1)th blue fringe.
   [Answer: n x 600 = (n +1) x 480 and n = 4]
2. In YDSE, how many maxima can be obtained on a screen (including central maxima) on both sides of central maxima?
   [Answer: No. of maxima, n = [d/λ] = [7000/2000] = [3.5] = 3 (highest integer.
   So, total maxima = 2 n + 1 = 2 x 3 +1 = 7]
3. Two beams of light having intensities I and 4 I interfere to produce fringe pattern on the screen. The phase difference between the beams is π/2 at point A and π at point B. Find the difference between the resultant intensities at point A and B.
   [Answer: At point A: $I_{r_1} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi_1 = I + 4I + 2\sqrt{(I)(4I)}\cos\frac{\pi}{2} = 5I$
   At point B: $I_{r_2} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi_2 = I + 4I + 2\sqrt{(I)(4I)}\cos\pi = I$
   Therefore, $I_{r_1}$-$I_{r_2}$ = 5 I – I = 4 I]
4. YDSE is made in a liquid. The tenth bright fringe in liquid lies on screen where $6^{th}$ dark fringe lies in vacuum. Find the refractive index of the liquid. [Ans. 1.8]

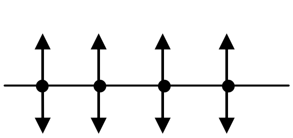
### Polarisation of wave:

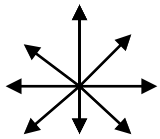
Waves are two types; transverse wave and longitudinal wave. Both types of these waves undergo reflection, refraction, interference and diffraction. Only transverse waves can be polarised. [Longitudinal waves cannot be polarised because these waves are symmetrical about the direction of propagation.

A transverse wave in which vibrations are present in all possible directions, in a plane perpendicular to the direction of propagation, is said to be unpolarised. If the vibrations of wave are present just in one direction, the wave is said to be polarised or plane polarised.

The phenomenon of restricting the oscillations of wave to just one direction in the transverse plane is called **polarisation of wave**.

Unpolarised light: A light which has vibrations in all directions in a plane perpendicular to the direction of propagation is said to be unpolarised light. Examples: The light from sun, sodium lamp, incandescent bulbs or candles.





Plane polarised light: If the electric field vector of light wave vibrates just in one direction, perpendicular to the direction of wave propagation, then it is said to be linearly polarised.

Fig. 1

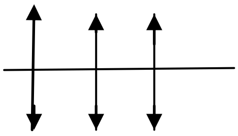


Fig. 2



In Fig. 1: Vibration parallel to the plane of paper, in fig. 2: Vibration perpendicular to the plane of paper

**The plane perpendicular to the plane of oscillation is called plane of polarisation**.

**Polarisers**: A device that plane polarises the unpolarised light, passed through it, is called polariser. Some commonly used polarisers are:

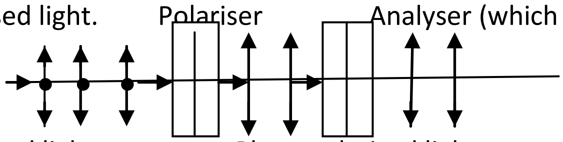
1. **Tourmaline crystal**: It is so cut that its plane contains its optic axis. When unpolarised light is incident on it normally, it allows only such electric field vibrations to pass through it which are parallel to its axis. Such a crystal can be used to polarise a beam of unpolarised light.
2. **Nicol prism**: It is an optical device used for producing an analysing plane polarised light. It consists of two pieces of calcite suitably cut and stuck together with Canada balsam.
3. **Polaroid:** It is a thin commercial sheet in the form of circular disc which makes use of the property of the selective absorption to produce an intense beam of plane polarised light.

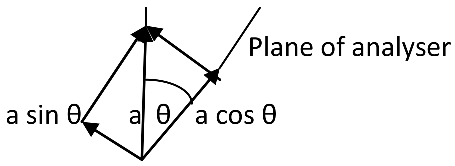
Polaroids allow the light oscillations parallel to the transmission axis, to pass through them.

**Law of Malus**: In 1809, ''E. N. Malus discovered that when a beam of completely plane polarised light is passed through analyser, the intensity of transmitted light varies directly as the square of the cosine of angle θ between the transmission direction (pass axis) of polariser and analyser.'' This statement is known as law of Malus. Mathematically,

$$I \propto cos^2\theta \text{ or } I = I_0 cos^2\theta$$

Where, $I_0$ is the maximum intensity of the transmitted light. $I_0$ is equal to half the intensity of unpolarised light.



Suppose plane of polariser and analyser are inclined to each other at an angle θ. Let $I_0$ be the intensity and a, the amplitude of plane polarised light transmitted by the polariser. The amplitude a of the light incident on the analyser has two rectangular components:

1. a cos θ is parallel to the plane of transmission (or pass axis) of the analyser and
2. a sin θ is perpendicular to the plane of transmission of the analyser.

So only the component a cos θ is transmitted by the analyser. The intensity of light transmitted by the analyser is

$I = k(a\cos\theta)^2 = ka^2 cos^2\theta = I_0 cos^2\theta ----(1)$

[Since $I \propto a^2$ and $I_0 = ka^2$]From equation (1), we see that

1. When θ = $0^0$ or $180^0$, i. e. pass axis of both polariser and analyser is parallel ($0^0$) or antiparallel, the maximum intensity of light will be transmitted by analyser and equal to the intensity emerging from the polariser.
2. When θ = $90^0$, I = 0 i. e. when transmission direction (pass axis) of polariser and analyser are perpendicular to each other (crossed), then intensity of light transmitted through analyser is zero.
3. When a beam of unpolarised light of intensity say $I_0$ is incident on the polariser, the intensity of polarised light (say $I_1$) after first polariser will be

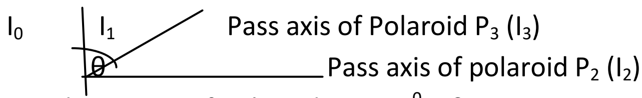
$I_1 = \frac{I_0}{2}$

$I_1 = I_0 < cos^2\theta >= \frac{I_0}{2}$

[Since, average value of $cos^2\theta$ over full cycle is ½.]

### Numerical concept based on law of Malus and Polaroid:

1. Suppose $I_0$ be the intensity of unpolarised light is incident on $P_1$. There are two polaroids $P_1$ and $P_2$ are placed in crossed positions and a third polaroid $P_3$ is kept in between $P_1$ and $P_2$ such that pass axis of $P_3$ makes an angle θ with the pass axis of $P_1$. The intensity of polarised light after $P_2$ is to be calculated as:     Pass axis of polaroid $P_1$




Here, the angle between Pass axis of polaroid $P_3$ and pass axis of polaroid $P_2$ is $90^0 - \theta$.

Hence, calculation will be as:

$I_1 = \frac{I_0}{2} --- (1)$

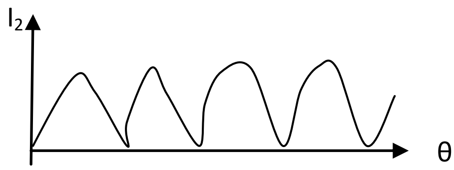
$I_3 = I_1 cos^2\theta --- (2)$

$I_2 = I_3 cos^2(90-\theta) --- (3)$

Solving equation (1), (2) and (3), we get

$I_2 = \frac{I_0}{8} sin^2 2\theta --- (4)$

The plot of equation (4) will be as: Plot of $I_2$ versus θ




### Difference between plane of vibration and plane of polarisation:

The plane containing the direction of vibration and the direction of wave propagation is called **plane of vibration.**

The plane passing through the direction of wave propagation and perpendicular the plane of vibration is called the **plane of polarisation**.

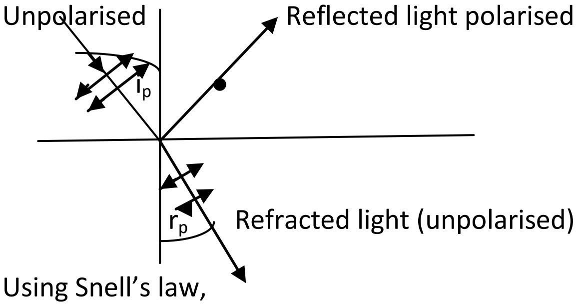
**Methods of producing the plane polarised light:**

(a) Reflection, (b) scattering, (c) double refraction and (d) selective absorption

**Polarisation by Reflection: Brewster's law** (British physicist David Brewster)

"The angle of incidence at which a beam of unpolarised light falling on a transparent surface is reflected as beam of completely plane polarised light is called Brewster angle ($i_p$)".

Brewster found that at polarising angle, the reflected ray and transmitted ray are perpendicular to each other. That is, $i_p + r_p = 90^0$.




Using Snell's law,

$(1)\sin i_p = \mu \sin r_p - - - - - (1)$

Since, $i_p + r_p = 90^0 \Rightarrow r_p = 90 - i_p - - - - - - (2)$

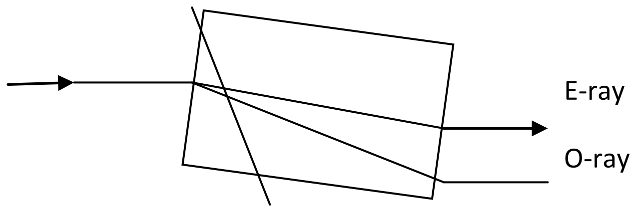
From equation (1) and (2),

$\sin i_p = \mu \sin(90 - i_p) = \mu \cos i_p \Rightarrow \mu = \tan i_p$

**Hence Brewster's law can be stated that "tangent of incidence polarising angle on a transparent surface is equal to the refractive index of the medium"**.

**Polarisation by double refraction**: Nicol Prism (An optical device based on the phenomenon of double refraction which is used for producing and analysing plane polarised light.)

When an unpolarised ray passes through certain crystals like quartz or calcite, it splits up into two rays. This phenomenon is called double refraction or birefringence.




**E-ray**: The ray which does not obey the law of refraction and has vibration parallel to the plane of incidence, is called E-ray or extra ordinary ray.

**O-ray**: The ray which obeys the ordinary law of refraction and has vibration perpendicular to the plane of incidence (dots), is called O-ray.

**Polarisation by selective absorption: Dichroism**

Certain doubly refracting crystals have the property of absorbing one of the doubly refracted beams to a greater extent than other. The crystals showing this property are said to be dichroic and the phenomenon is known as dichroism. Tourmaline is a naturally occurring crystal which shows the phenomenon of selective absorption.

**Polaroids:** Polaroids are thin commercial sheets which make use of the property of selective absorption to produce an intense beam of plane polarised light.

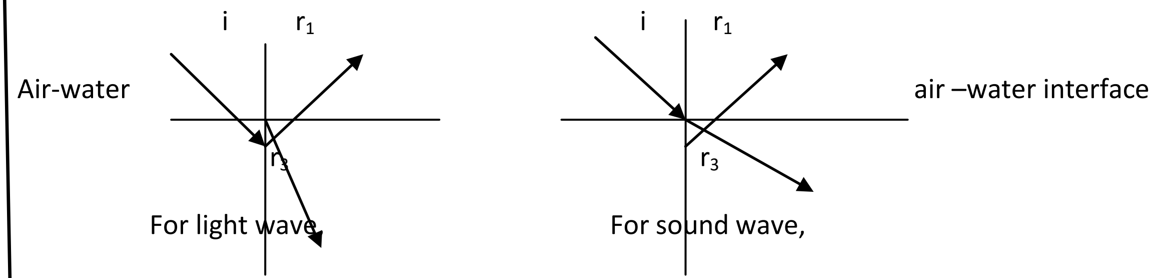
### Uses of Polaroids:

1. In sunglasses and camera filters
2. In window panes of aeroplanes
3. In 3-D movies
4. In LCDs (Liquid crystal display)

**Regarding reflection and transmission of wave: (For NEET/IIT)**

A medium is said to be denser (relative to other) if speed of wave in this medium is less than speed of wave in the other medium. Thus, it is the speed of wave which decides whether the medium is denser or rare for that particular wave. and $v_{denser} < v_{rare}$ .

If a medium is denser for one type of wave then at the same time the same medium can be rare for other type of wave. For example, water is denser for electromagnetic (light) waves compared to air because speed of light is less in water than in air. At the same time, for sound wave, water is a rarer medium because speed of sound wave in water is more. The laws of refraction 9from one medium to other) and reflection remain the same.




For light wave, $I = r_1$ and $r_2 < i$

For sound wave, $i = r_1$ and $r_2 > i$

Any type of wave is associated with following physical quantities:

(i) Speed of wave, (ii) frequency (f), time period (T) and angular frequency ($\omega$), (iii) wave length ($\lambda$) and wave number (k), (iv) amplitude (A) and intensity (I) and (v) Phase $\phi$

Speed of wave : It depends on medium and some of its characteristics ( tension and mass per unit length in case of stretched string in sound wave). So in reflection, speed of wave does not change. On the other hand, in refraction, medium and hence speed of wave do change.

Frequency, time period and angular frequency: frequency of wave depends on the source from where wave originates. In reflection and refraction, since, source does not change. Hence, none of three changes.

Wave length and wave number: $k = \frac{2\pi}{\lambda}$ and $\lambda = \frac{v}{f}$ here, v depends on medium and f on source. So, in reflection, medium and source both remain unchanged, so $\lambda$ does not change. In refraction, as the medium is changed, so $\lambda$ do change.

Amplitude and intensity: Intensity is the energy transmitted per unit area per unit time.

$$I = \frac{1}{2}\rho\omega^2 A^2 v \therefore I \propto A^2$$

When a wave is incident on a boundry, (separating two media) part of it is reflected and part is transmitted. Hence, intensity and amplitude both change in reflection as well as in refraction unless 100 % reflection or 100% transmission is there. Phase: In refraction (or transmission), no phase change takes place. While in reflection, phage change is zero if wave is refracted from a rare medium and it is π if it is reflected from a denser medium.

**Resolving power of an optical instrument**: It is the power or ability of instrument to produce distinctly separate images of two closely spaced objects i. e. **it is the ability of instruments to resolve or to see as separate, the images of two closely spaced objects**.

When two objects are very close to each other, every optical instrument has its own limit up to which it can produce distinctly separate images of these objects.

The minimum distance between two objects which can just be seen as separate by the optical instruments is called the limit of resolution of the instrument. (Obviously, smaller the limit of resolution, the greater is its resolving power and vice-versa).

Human eye is also an optical instrument. The limit of resolution of human eye is one minute or 1/60 degree.

**Resolving power of microscope:** It is the ability of microscope to show as separate images of two point objects lying close to each.

**The limit of resolution**: It is measured by the minimum distance (△d) between two point objects, whose images in the microscope are just seen as separate.

**The value of limit of resolution (△d) varies**

(i) Directly as wavelength ( $\lambda$ ) of the light used and

(ii) Inversely as cone angle of light rays from any one object entering the microscope

That is, $\Delta d \propto \frac{\lambda}{2\theta}$, where 2 θ is the cone angle of light rays entering the objective of the microscope.

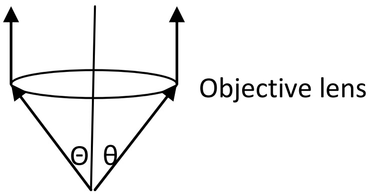
That is, limit of resolution of microscope is given as,

$$\Delta d = \frac{\lambda}{2\theta} = \frac{\lambda}{2\sin\theta}$$

(Replacing θ by sin θ)

If medium between object and objective lens of microscope is a transparent medium of refractive index μ instead of air, then,

$$Limit\ of\ resolution\ (\Delta d) = \frac{\lambda}{2\mu\sin\theta}, \quad \therefore \quad Resolving\ power\ of\ microscope\ (RP) = \frac{2\mu\sin\theta}{\lambda}$$



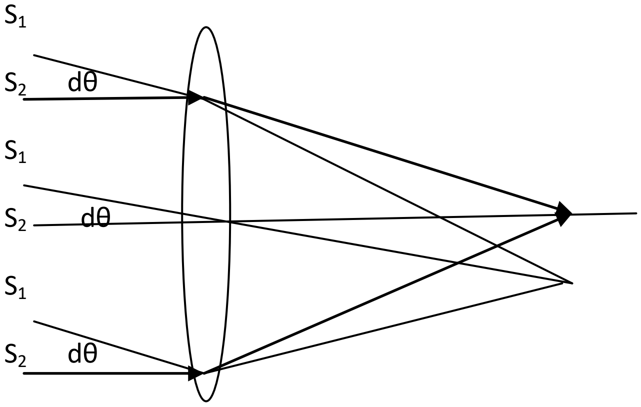

**Resolving power of telescope**: It is the ability to show distinctly the images of two distant objects, says star having small angular separation.

**Limit of resolution of telescope**: It is measured by angle dθ subtended at its objective by those two distant objects (say star) whose images through the telescope are just seen.

Limit of resolution dθ varies as(i) directly as wave length and (ii) Inversely as the aperture or diameter a of the telescope

That is, $d\theta \propto \frac{\lambda}{a}$, or $Limit\ of\ resolution\ (d\theta) = \frac{1.22\lambda}{a}$

$\therefore Resolving\ power\ of\ telescope\ (RP) = \frac{1}{d\theta} = \frac{a}{1.22\lambda}$




**Doppler's effect in light**: When source and observer approach each other, △f (apparent frequency) is positive, i. e. increases or apparent wavelength decreases. This is called blue shift. When source and observer recede away from each other, △f is negative, i.e. apparent frequency decreases or apparent wavelength increase. This is called red shift.

By measuring blue shift/red shift, we can calculate velocity of star from the given below equation:

$$f' = f\left(\frac{c+v}{c}\right)\ when\ source\ and\ observer\ approach\ each\ other ----(1)$$

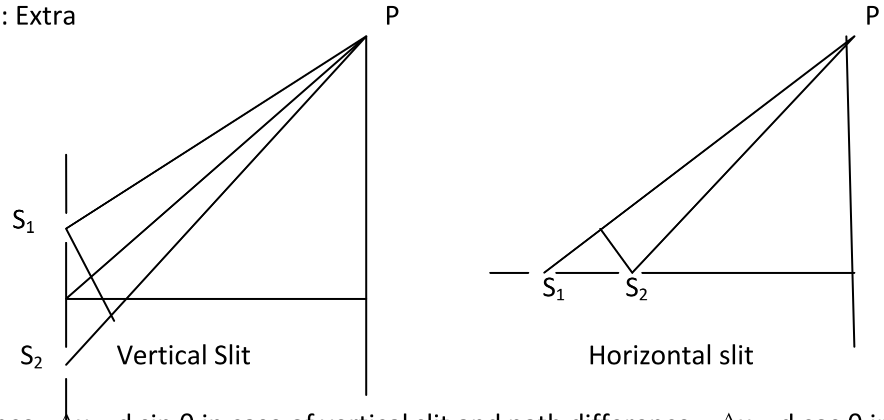
$$f' = f\left(\frac{c-v}{c}\right)\ when\ source\ and\ observer\ are\ moving\ away\ from\ each\ other ----(2)$$

$$\frac{f'}{f} = \frac{1 \pm \frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}\ Astronomical\ Doppler's effect ----(3)$$

$$f' - f = \Delta f = \pm\frac{v}{c}f ----(4)$$

$$\Delta\lambda = \mp\frac{v}{c}\lambda ------ (5)$$

Interference: Extra




Path difference =△x = d sin θ in case of vertical slit and path difference = △x = d cos θ in case of horizontal slit. For bright fringes, d sin θ = n $\lambda$ (vertical slit), for bright fringes, d cos $\theta$ = n $\lambda$ (horizontal slit). Sometimes, maximum number of maxima or minima are asked on the screen,: Then, sin $\theta$ or cos $\theta$ is maximum. Maximum value of sin $\theta$ = 1, hence, $\sin\theta = 1 = \frac{n\lambda}{d} \Rightarrow n = \frac{d}{\lambda}$

Suppose d/ $\lambda$ comes out 4.6, so total no. of maxima will be nine as n = 0, ±1, ±2, ±3, ±4 (9).

**Extra Points**:

1) Diffraction of light was discovered by Girmaldi.
2) The type of diffraction in YDSE is Fresnel.
3) The bending of light around corners of obstacles is called diffraction
4) To observe diffraction, the size of an obstacle should be as same order of the wavelength.
5) In YDSE, fringe width will be minimum for violet light.
6) The ultrasonic waves cannot be polarised because it is longitudinal wave.
7) An optically active compound rotates the plane polarised light.

## Numerical Type-1

**Q. 15** What is the angular resolution of a 10 cm diameter telescope at a wavelength of 0.6 μm? [Ans. $7.32 \times 10^{-6}$ radian]

**Q. 16:** When the light of particular wavelength falls on a plane surface at an angle of incidence $60^0$, then the reflected light becomes completely plane polarised. Find the refractive index of surface material and the angle of refraction through it. [Ans. $\sqrt{3}$, $30^0$]

**Q. 17**: Two polaroids are oriented with their plane perpendicular to incident light and transmission axis making an angle $30^0$ with each other. What fraction of incident unpolarised light is transmitted? [Ans. 37.5%]

**Q. 18:** A polarised light of intensity $I_0$ is passed through another polariser whose pass axis makes an angle of $60^0$ with the pass axis of the former. What is the intensity of emergent polarised light from second polariser? [Ans. ]

**Q. 19**: Two polaroids $P_1$ and $P_2$ are placed with their pass axis perpendicular to each other. Unpolarised light of intensity $I_0$ is incident on $P_1$. A third polaroid $P_3$ is kept in between $P_1$ and $P_2$ such that its pass axis makes an angle of $60^0$ with that of $P_1$. Determine the intensity of light transmitted through $P_2$. [Ans. $I_1 = \frac{I_0}{2}, I_2 = \frac{3I_0}{32}, I_3 = \frac{I_0}{8}$]

**Q. 20**: A narrow beam of unpolarised light of intensity $I_0$ is incident on a polaroid $P_1$. The light transmitted by it is then incident on second polaroid $P_2$ with it pass axis making an angle of $60^0$ relative to the pass axis of $P_1$. Find the intensity of the light transmitted by $P_2$. [CBSE 2017][Ans. $I_0/8$]

**Q. 21:** Find an expression for intensity of transmitted light when a polaroid is rotated between two crossed polaroids. In which position of the polaroid sheet will the transmitted intensity be maximum? [CBSE D 15, NCERT] [Ans. $\frac{I_0}{4} sin^2 2\theta$] [Ans. π/4]

## Numerical Type-2:

1. Define a wavefront and state Huygen's principle. Using this principle draw a diagram to show how a plane wave front incident at the interface of the two media gets refracted when it propagates from a rarer to a denser medium. Hence, verify Snell's law of refraction. [CBSE D 2013]
2. Use Huygen's geometrical construction to show the behaviour of a plane wavefront
    a) Passing through a biconvex lens
    b) Reflecting by a concave mirror
    c) Passing through a prism [F 2012]

3. Use Huygen's principle to verify law of reflection. [CBSE 2018]
4. Compare the interference pattern observed in Young's double slit experiment with single slit diffraction pattern, pointing out three distinguishing features. [2017, 2016]
5. In Young's double slit experiment, deduce the condition for (a) constructive and (b) destructive interference at a point on the screen. Draw a graph showing variation of intensity in the interference pattern against position 'X' on the screen. [2016]
6. Write the important characteristic features by which interference can be distinguished from the observed diffraction pattern. [AI 2015]
7. Light waves each of amplitudes 'a' and frequency '$\omega$', emanating from two coherent sources superpose at a point. If the displacements due to these waves is given by $y_1 = a\cos\omega t$ and $y_2 = a\cos(\omega t + \varphi)$, where ø is the phase difference between the two waves. Obtain the expression for the resultant intensity at the point. [D 14]
8. In YDSE, describe briefly, how bright and dark fringes are obtained on the screen kept in front of a double slit. Hence obtain the expression for the fringe width. [AI14]
9. Describe briefly how a diffraction pattern is obtained on the screen due to a single narrow slit illuminated by a monochromatic source of light. Hence, obtain the condition for angular width of secondary maxima and secondary minima. [F 14]
10. In single slit diffraction pattern, how does the angular separation between fringes change when distance of screen from slit is doubled? [CBSE 2012] [Ans. Angular separation = $\frac{\lambda}{a} = \frac{y}{D}$ Since, $\lambda$ and a remains unchanged, so angular separation between fringes remain the same.]
11. What is the essential condition for diffraction of light? [CBSE 2011] [Ans. Diffraction of light occurs only when size of obstacle/aperture is of the order of wavelength of light.]
12. Why do we not encounter the diffraction effect of light in everyday observations? [CBSE 2010][Ans. This is because objects around us are much bigger in size as compared to the wavelength of visible light (order of $10^{-6}$ m]
13. In single slit diffraction pattern, why is intensity of secondary maximum less than intensity of central maximum? [CBSE 2009] [Ans. Central maximum is due to constructive interference of secondary wavelets from all parts of the slits. With increase order of spectrum (n), lesser and lesser parts of slit produce constructive interference to form secondary maxima. That is why the intensity decreases.]
14. A monochromatic light of wavelength 500 nm is incident normally on a single slit of width 0.2 mm to produce a diffraction pattern. Find the angular width of the central maximum obtained on the screen. [Ans. 0.005 rad]
    Estimate the number of fringes obtained in Young's double slit experiment with fringe width 0.5 mm, which can be accommodated within the region of total angular spread of the central maximum due to single slit if distance of screen from slit is 1 m. [2017][Ans. n=10]
15. Two slits are made 1 mm apart and the screen is placed 1 m away. What should be the width of each slit to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern? [Guwahati, 15][Ans. 0.2 mm]
16. In Young's double slit experiment, the two slits are separated by a distance of 1.5 mm and the screen is placed 1 m away from the plane of the slits. A beam of light consisting of two wavelengths 650 nm and 520 nm is used to obtain interference fringes. Find (a) The distance of the third bright fringe for $\lambda$ =520 nm on the screen from the central maximum. [Ans. 1.04

mm](b) The least distance from the central maximum where the bright fringes due to both wavelengths coincide. [Ajmer, 15][1.73 mm]

17. In a double slit experiment using light of wavelength 600 nm, the angular width of the fringe formed on a distant screen is $0.1^0$. Find the spacing between the two slits.[D 15][Ans. 0.34 mm]
18. What would be the width of each slit to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern, for green light of wavelength 500 nm, if the separation between two slits is 1 mm? [Ans. 0.2 mm][AI 15]
19. In YDSE, using monochromatic light of wavelength $\lambda$, the intensity of light at a point on the screen where path difference is $\lambda/2$, is K/2 units. Find the intensity of light at a point where path difference is $\lambda$/3. [Ans. K]
20. The ratio of intensity at minima to the maxima in the YDSE is 9:25. Find the ratio of widths of the two slits. [AI 14] [Ans. 16:1]
21. A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that first minimum is at distance of 2.5 mm away from the centre. Find the width of the slit.[F14, AI 13][Ans. 0.2 mm]
22. A parallel beam of light of 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that $1^{st}$ minimum is at distance of 2.5 mm from the centre of the screen. Calculate the width of the slit. [AI 13][Ans. 0.2 mm]
23. Two wavelengths of sodium light 590 nm and 596 nm are used, in turn, to study the diffraction taking place at a single slit of aperture $2 \times 10^{-4}$ m. The distance between the slit and screen is 1.5 m. Calculate the separation between the positions of the first maxima of the diffraction pattern obtained in the two cases.[D13][Ans. $6.75 \times 10^{-5}$ m]
24. Light waves from two coherent sources superimpose at a point. The waves, at this point, can be expressed as $y_1 = a\sin[10^{15}\pi t]$ and $y_2 = 2a\sin[10^{15}\pi t + \varphi]$. Find the resultant amplitude if phase difference ø is (i) zero, (ii) π/3, (iii) π [Ans. (i) 3a, (ii) √7 a, (iii) a].Also find the frequency of resultant wave in each case. [Ans. $5 \times 10^{14}$ Hz]
25. In Young's experiment, the interfering waves have amplitudes in ratio 3:2. Find the ratio of (a) amplitudes and (ii) intensities between the bright and dark fringes. [Ans. $5, 2.5$]
26. Determine the angular separation between central maximum and first order maximum of the diffraction pattern due to a single slit of width 0.25 mm when light of wavelength is 589 nm is incident on it normally. [$3.534 \times 10^{-3}$ rad][CBSE 93]
27. In a YDSE, D = 1 m, d = 1 mm and $\lambda$ = 1/2 mm find (a) Distance between the first and central maxima on the screen (b) Find the number of maxima and minima obtained on the screen
28. In a single slit diffraction experiment, a slit of width d is illuminated by red light of wave length 650 nm. For what value of d will (a) The first minimum fall is at an angle of diffraction of $30^0$ and (b) The first maximum fall is at an angle of diffraction of $30^0$?
29. In YDSE, the two slits are separated by 0.1 mm and they are 0.5 m from the screen. The wavelength of light used is 5000 Ă. Find the distance between $7^{th}$ maxima and $11^{th}$ minima on the screen. [Ans. 8.75 mm]
30. Maximum intensity in YDSE is $I_0$. Find the intensity at a point on the screen where (a) The phase difference between the two interfering beams is π/3 [Ans. ¾ $I_0$](b) The path difference between them is $\lambda$/4 [Ans. $I_0/2$]

31. Two coherent sources are 0.3 mm apart. They are 0.9 m away from the screen. The second dark fringe is at a distance of o.3 cm from the centre. Find the distance of $4^{th}$ bright fringe from the centre. Also find the wavelength of light used. [Ans. 0.8 cm, $6.67 \times 10^{-7}$ m]
32. A screen is placed 50 cm from a single slit, which is illuminated with 6000 Ă light. If the distance between the first and third minima in the diffraction pattern is 3 mm, what is the width of the slit? [Ans. 0.2 mm]
33. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1.0 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. What is the distance between the first dark fringe on either side of the central bright fringe. [Ans. 2.4 mm]
34. In a single slit diffraction experiment first minima for wavelength $\lambda_1$ = 660 nm coincides with first maxima for wavelength $\lambda_2$. Calculate $\lambda_2$. [Ans. 440 nm]
35. State Brewster's law. The value of Brewster's angle for a transparent medium is different for lights of different colours. Give reasons. [2017] [Ans. Because different colours of light has different wavelengths and so different refractive indices. Hence, these lights have different Brewster's angle.]
36. Unpolarised light is incident on a plane glass. What should be the angle of incidence so that reflected rays are perpendicular to each other?[NCERT] [Ans. $56.3^0$]
37. For a given medium, the polarising angle is $60^0$. What will be the refractive index and the critical angle for this medium? [CBSE D 99] [Ans. $\sqrt{3}$, $35^0$ 16']
38. The velocity of light in air is $3 \times 10^8$ m/s and that in water is $2.2 \times 10^8$ m/s. Find the polarising angle of incidence. [Ans. $53.74^0$]
39. A beam of light travelling in water falls on a glass plate immersed in water. When the incident angle is $51^0$, the reflected beam of light is found to be completely plane polarised. Determine the refractive index of glass. Given refractive index of water = 4/3 [1.647]
40. Distinguish between unpolarised light and linearly polarised light. How does one get linearly polarised light with the help of a polaroid? [2017]
41. Find an expression for intensity of transmitted light when a polaroid sheet is rotated between two crossed polaroids. In which position of the polaroid sheet will the transmitted intensity be maximum? [2015] [Ans. $I_2 = I sin^2 2\theta$] [Ans. π/4]
42. In a two slits experiment with monochromatic light, fringes are obtained on a screen placed at some distance D from the slits. If the screen is moved 0.05 m towards the slits, the change in fringe width is 30 μm. If the distance between the slits is 1 mm, calculate the wavelength of light used. [Ans. 600 nm][CBSE 03, 06 C]
43. A narrow beam of unpolarised light of intensity $I_0$ is incident on a polaroid $P_1$. The light transmitted by it is then incident on second polaroid $P_2$ with it pass axis making an angle of $60^0$ relative to the pass axis of $P_1$. Find the intensity of the light transmitted by $P_2$. [CBSE 2017][Ans. $I_0/8$]
44. Two Polaroids $P_1$ and $P_2$ are placed in crossed positions. A third polaroid $P_3$ is kept between $P_1$ and $P_2$ such that pass axis of $P_3$ is parallel to that of $P_1$. How would the intensity of light transmitted through $P_2$ vary as $P_3$ is rotated? Draw a plot of final transmitted intensity versus angle θ between pass axis of $P_1$ and $P_3$. [CBSE D 15, OD 10, 15 C] [Ans. $I_2 = \frac{1}{4} I_1 sin^2 2\theta$]
45. Find an expression for intensity of transmitted light when a polaroid is rotated between two crossed polaroids. In which position of the polaroid sheet will the transmitted intensity be maximum? [CBSE D 15, NCERT] [Ans. $\frac{I_0}{4} sin^2 2\theta$] [Ans. π/4]

46. Two Polaroids $P_1$ and $P_2$ are placed with their pass axis perpendicular to each other. Unpolarised light of intensity $I_0$ is incident on $P_1$. A third polaroid $P_3$ is kept in between $P_1$ and $P_2$ such that its pass axis makes an angle of $60^0$ with that of $P_1$. Determine the intensity of light transmitted through $P_1$, $P_2$ and $P_3$. [CBSE OD 14] [Ans. $I_0/2$, $I_0/8$, $3I_0/32$]
47. Two polaroids $P_1$ and $P_2$ are placed with their pass axes perpendicular to each other. Unpolarised light of intensity $I_0$ is incident on $P_1$.A third polaroid $P_3$ is kept in between $P_1$ and $P_2$ such that its pass axis makes an angle of $45^0$ with that of $P_1$. Determine the intensity of light transmitted through $P_1$, $P_2$ and $P_3$. [CBSE OD 14] [Ans. $I_0/2$, $I_0/4$, $I_0/8$]
48. If the angle between pass axis of the polariser and the analyser is $45^0$, write the ratio of intensities of original light and the transmitted light after passing through the analyser. [CBSE D 09] [Ans. 4:1]
49. Unpolarised light of intensity $I_0$ passes through two polaroids $P_1$ and $P_2$ such that pass axis of $P_2$ makes an angle $\theta$ with the pass axis of $P_1$. A third polaroid $P_3$ making an angle $\beta$ with that of $P_1$. If $I_1$, $I_2$ and $I_3$ represent intensities of light transmitted by $P_1$, $P_2$ and $P_3$ respectively, determine the values of $\theta$ and $\beta$ for which $I_1 = I_2 = I_3$. [CBSE OD 14 C][Ans. $\theta = \beta = 0$ or $\pi$ rad]
50. Two Polaroids are used to study polarisation. One of them (polariser) is kept fixed and other (analyser) is initially kept with its axis parallel with the polariser axis. The analyser is then rotated through angles of $45^0$, $90^0$ and $180^0$ in turn. How would the intensity of light coming out of the analyser be affected for these angles of rotation as compared to the initial intensity and why? [CBSE D 03 C] [Ans. $I_0/2$, 0, $I_0$]
51. Two polarising sheets have their polarising directions so that the intensity of the transmitted light is maximum. Through what angles must the either sheet be turned if the intensity is to drop by one-half? [ Ans. $\pm45^0$, $\pm135^0$]
52. Two polaroids are crossed to each other. If one of them is rotated through $60^0$, then what percentage of the incident unpolarised light will be transmitted by the polaroids? [Ans. 37.5%]
53. Four polaroids are so placed that the transmission axis of each is inclined at an angle of $30^0$ from the axis of the previous polaroid in the same direction. If unpolarised light beam of intensity $I_0$ falls on the first polaroid, the what will be the intensity of light emerging from the last polaroid? [Ans. 0.21 $I_0$]
54. Assume that light of wavelength 6000 Ă is coming from a star. Find the limit of resolution of a telescope whose objective has a diameter 250 cm. [CBSE 2015 Guwahati][Ans. 2.928 x $10^{-7}$]