Chapter-11: Dual Nature of Radiation and Matter

The phenomena like interference, diffraction, polarisation etc. demonstrated by light wave by considering light as waves. But some other phenomena like photoelectric effect etc. could not be explained by wave theory of light. To explain photo electric effect, Einstein used Planck's quantum theory. According to which light consists of packet of energy. Each packet of energy is called photon or quantum of light. These packets of energy travel in a straight line with the speed of light. Thus particle nature of light was established.

Hence, it was concluded that light is of dual nature as some phenomena were explained by wave theory of light and some by particle nature of light.

Similar to light, matter also has dual nature, e. g. electron is a matter particle as it contains mass and charge but fast moving electrons, i. e. cathode rays show the properties of waves. Thus matter also exhibits dual nature.

Hertz's Observations: During the experiments of EM waves, Hertz noticed that sparking across the detector enhanced when the emitter plate was illuminated by ultraviolet light.

This was due to free charged particles (electrons) which escaped into the surrounding space from the metal surface on absorbing enough energy to overcome their force of attraction with positive ions in the materials.

Important terms and their definitions:

Electron emission

In metals, electrons in the outer shells are loosely bound to the atoms, hence they are free to move easily within the metal surface but cannot leave the metal surface. Such electrons are called free electrons.

These free electrons can be emitted from the metals if they have sufficient energy to overcome the attractive pull of metal surface.

The phenomenon of emission of electrons from the surface of a metal is called electron emission.

The energy required to liberate an electron from the metal surface may arise from various sources such as heat, light, electric field etc. Depending on the nature of source of energy, the following methods are possible:

1. **Thermionic emission**: The energy to the free electrons can be given by heating the metals. The electrons so emitted are known as **thermions**.
2. **Field emission**: When a conductor is put under strong electric field, the free electrons on it experiences an electric force in the opposite direction of field. Beyond a certain limit, electrons start coming out of the metal surface. Emission of electrons from a metal surface by this method is called the field emission.
   Field emission is also known as cold emission or cold cathode emission. One of the example of cold emission is spark plug.
3. **Secondary emission**: Emission of electrons from a metal surface by the bombardment of high speed electrons or other particles is known as secondary emission. The electrons so emitted are called secondary electrons.
4. **Photoelectric emission**: Emission of free electrons from a metal surface by falling light radiation of suitable frequency is called photoelectric emission. The electrons so emitted are called **photoelectrons**.

Work function (W) of a metal: It is the minimum energy required to emit (escape) the electrons which are just on the surface of the conductor.

It is measured in eV (electron volt). 1 eV is defined as the energy gained by an electron when it has been accelerated by a potential difference of 1 V. (1 eV = 1.6 x 10^{-19} J)

Cesium is regarded as the best photosensitive material as it has the lowest work function. (W or φ_0 = 2.14 eV) while platinum has the highest work function. (φ_0=5.65 eV).

Note: Work function depends on the (i) properties of metal and (ii) nature of its surface.

Work function decreases with increase in temperature.

Photon: A photon is a particle associated with electromagnetic radiations. According to quantum theory of radiations, photon is a packet of energy given by E = hf, where f is the frequency associated with photon and h is Planck's constant. The rest mass of photon is zero,

Photoelectric effect: The phenomenon of emission of electrons from the surface of metal, when radiation of suitable frequency falls on it, is known as photoelectric effect.

The emitted electrons are called photoelectrons and the current so produced is called photoelectric current. When certain metal surfaces are illuminated with UV radiation (and in some cases by visible light), the electrons are emitted from them.

Alkali metals like lithium, sodium, potassium, cesium etc. show photo electric effect with visible light, whereas the metals like zinc, cadmium, magnesium etc. are sensitive only to ultraviolet light.

The phenomenon of photoelectric effect was discovered by Heinrich Hertz in 1887 while detailed investigation on photoelectric emission was done by William Hallwachs and Philipp Lenard.

Points to remember:

For photo-emission to take place, energy of incident light photons should be greater than or equal to the work function of the metal. That is,

$E \geq W ----(1) \therefore \quad hf \geq W \ \ or\ f \geq \frac{W}{h}$

Here W/h is the minimum frequency required for the emission of electrons. This is known as threshold frequency f_0. Thus, $f_0 = \frac{W}{h} --- (2)$

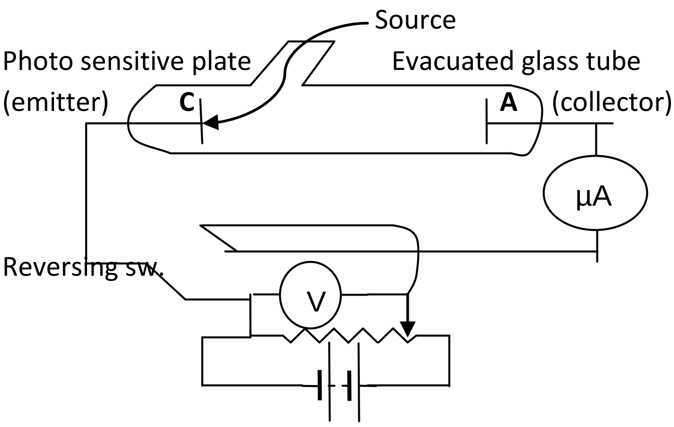
Further, equation (1) can be written as, $\frac{hc}{\lambda} \geq W \ \ or\ \lambda \leq \frac{hc}{W}$

Here hc/W is the largest wavelength beyond which photo-emission does not take place. This is called the threshold wavelength λ_0. Thus, $\lambda_0 = \frac{hc}{W} ----(3)$

Hence for photoemission to take place, either of the following conditions must be satisfied:

$\boldsymbol{E \geq W \ \ or\ \ f \geq f_0 \ \ \ or\ \ \lambda \leq \lambda_0 ----(4)}$

Experimental study of photoelectric effect: (Leonard observations)



It consists of an evacuated glass tube or quartz tube which encloses a photosensitive plate C (called emitter) and a metal plate A. A transparent quartz window is sealed onto the glass tube which permits ultraviolet radiation to pass through it and fall on photosensitive plate C. The electrons are emitted by the plate C and are collected by plate A. When the collector plate A is positive with respect to emitter plate C, the electrons are attracted to it and hence photoelectric current is constituted. The experimental arrangement is used to study the variation of photoelectric current with intensity of radiation, frequency of radiation and the potential difference between the plates A and C.

Effect of intensity of light on photoelectric current

For a fixed frequency of incident radiation and the accelerating potential the photoelectric current is increases linearly with the intensity of incident radiation as shown in the figure -1 below. Since the photoelectric current is directly proportional to the number of photoelectrons emitted per second which is also proportional to the intensity of incident radiation. Intensity ∝ photoelectric current ∝ number of photo electrons emitted per sec ∝ $\frac{1}{distance^2}$

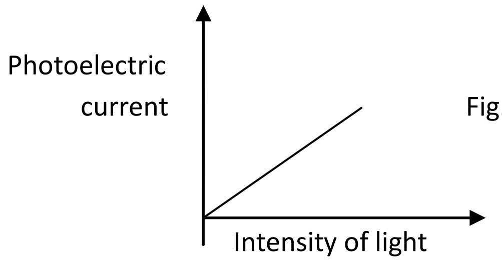


Fig.-2

Effect of potential on photoelectric current

For a fixed frequency and intensity of incident light, photoelectric current increases with increase in potential applied to the collector, as shown in the graph. (Fig. 3) From this graph, we observe that
(i)After certain value of accelerating potential, when all photoelectrons reach the plate A, the photocurrent does not increase. On increasing the value of accelerating potential, this maximum value of photoelectric current is called **saturation current**.
(ii)When the potential is decreased, the current decreases but does not becomes zero at at zero potential. This shows that even in absence of accelerating potential, few photoelectrons manage to reach the plate A on their own due to their kinetic energy.
(iii)For a particular frequency of incident radiation, the minimum negative potential V_0 (retarding potential) given to the plate A for which photoelectric current becomes zero is cut-off potential or stopping potential. [i. e. **cut-off potential** is the minimum negative potential given to plate A when photoelectric current becomes zero.]So no electron reaches to plate A even it is most energetic electron with maximum kinetic energy (K_{max}) as stopping potential is sufficient to repel it.
Hence, maximum kinetic energy of emitted electrons is given as $K_{max} = eV_0 = \frac{1}{2}mv_{max}{}^2$. Where m is the mass of photoelectron and V_{max} is the maximum velocity of photoelectrons.

Note: For radiation of given frequency and material of plate C, the value of stopping potential is independent of intensity of the incident radiation. It means the maximum K. E. of emitted photoelectron depends on the light source and the emitter plate material but it is independent of intensity of incident radiation.

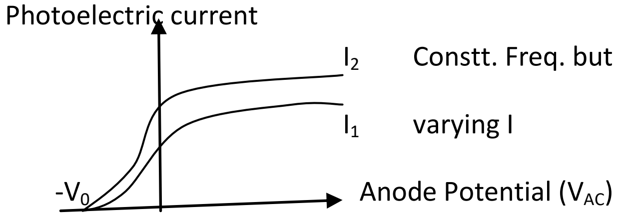


Figure-2

Numerical:

Q. 1: The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum K. E. of photoelectrons emitted? [Ans. 2.4×10^{-19} J]

Q. 2: In an experiment study of photoelectric effect, the stopping potential for a metallic surface is 5.68 V. What will be the maximum velocity of electrons? [Ans. 1.414×10^{6} m/s]

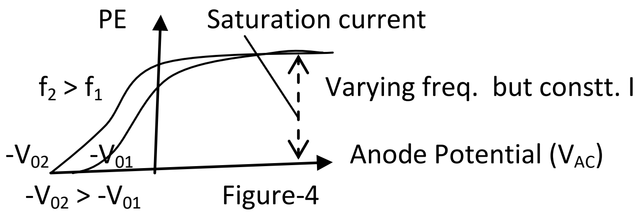


Figure-4

Effect of frequency of incident radiation on stopping potential

If we take radiation of different frequencies but of same intensity, for each radiation, we study the variation of photoelectric current against the potential difference between the plates as shown in the graph (Fig. 4). From the graph, we observe that

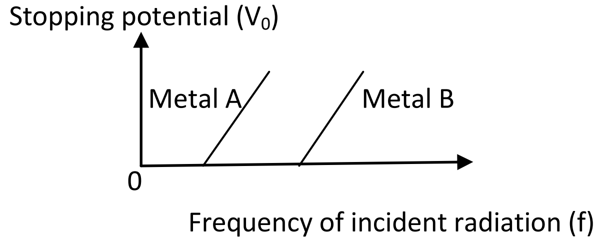
(i)The value of stopping potential is different for radiation of different frequencies but value of saturation current (for given intensity) remains same for all incident radiations.

(ii)The value of stopping potential is more negative for radiation of higher incident frequency. That means the energy of the emitted electrons depends on the frequency of incident radiations. Greater is the frequency of incident radiation, greater the K. E. of the photoelectrons, consequently, greater retarding potential is required to stop them completely.

(iii)The value of saturation current depends on the intensity of incident radiation but it is independent of frequency of incident radiation.

4. **Stopping potential versus frequency graph**:

If we plot a graph between stopping potential and the frequency of the incident radiation for two different metals A and B as shown in the fig. 5, we observe that



(i)Stopping potential V_0 varies linearly with the frequency of incident radiation for a given photosensitive material.

(ii)There exists a certain minimum cut-off frequency (f_0) for which the stopping potential is zero. This minimum frequency is called **threshold frequency**.

''It can be defined as minimum frequency of light which can emit photoelectrons from a material is called **threshold frequency or cut-off frequency** of the material.'' It is a characteristic property of material.

For a frequency lower that cut-off frequency, no photoelectric emission is possible even if the intensity is large.

Failure of wave theory in explaining photoelectric effect:

Three major features of photoelectric effect could not be explained by the wave theory of light which were later explained by Einstein's photon theory:

(That is failure of wave theory of light to explain the photoelectric effect)

1. Wave theory suggests that kinetic energy of photoelectrons should increase with the increase in intensity of light, however, $K_{max} = eV_0$ suggests that kinetic energy is independent of the intensity of light.
2. According to wave theory, the photoelectric effect should occur for any frequency of light, provided that the light is intense enough. However, equation (4) suggests that photo emission is possible only when frequency of incident light is either greater than or equal to the threshold frequency.
3. When the wave of light falls on a metal surface, the energy of the wave of light will not go to a particular electron but will be distributed to all the electrons present in the illuminated portion of metal surface. As a result of it, the electrons will take sometime in accumulating the energy required for their emission from metal surface. This is also against the experimental fact that the emission of photoelectrons takes place immediately after the light is incident on the metal surface.

Einstein's photoelectric equation

Einstein's photon's theory: In 1905, Albert Einstein succeeded in explaining the photoelectric effect by making a remarkable assumption that energy in a light beam travels through space in concentrated bundles, called photons. The energy E of a single photon is given by $E = hf$

When a photon of energy hf falls on a metal surface, the energy of the photon is absorbed by the electron and is used in the following two ways.

(i)A part of energy is utilised to overcome the surface barrier and come out of the metal surface. This part of energy is called work function. $W = hf_0$

(ii)The remaining part of energy is used in giving a velocity v to the emitted photoelectron. This is equal to maximum K. E. of the photoelectron. $K_{max} = ½ mv_{max}^2$.

According to the law of conservation of energy, $hf = W + \frac{1}{2}m{v_{max}}^2 = W + K_{max}$

So, **Einstein photoelectric equation** is, $K_{max} = hf - W$----(1)

Relation between stopping potential and frequency:

$K_{max} = h(f - f_0)$. If stopping potential is V_0 then $K_{max} = eV_0$

Therefore, $eV_0 = h(f - f_0)$, (here $f = \frac{c}{\lambda}$ and $f_0 = \frac{c}{\lambda_0}$)

Then, $V_0 = \frac{hf}{e} - \frac{hf_0}{e} = \frac{h}{e}(f - f_0)$ ------(2)

1. **Graph between K_{max} and f**:

   From the graph, K_{max} versus frequency, we can note the following points:

   a) $K_{max} = 0$ if $f = f_0$

   b) Slope of the straight line is h, a universal constant (=6.63 x 10^{-34} J-s). If graph is plotted for two different metals 1 and 2, slope of both the line is same.

c) The negative intercept of line is W (work function), so intercept of two different metals will be different. Further $W_2 > W_1$ therefore, $(f_0)_2 > (f_0)_1$, here f_0 is the threshold frequency,

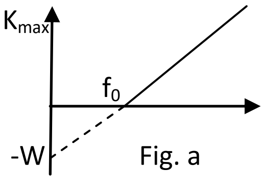


Fig. a

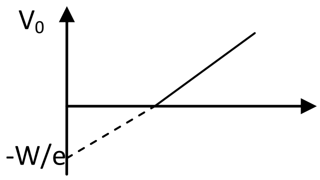


Fig. b

Graph between V₀ (stopping potential) and f (incident frequency)

Equation (2) can also be written as (since $K_{max} = eV_0$), $eV_0 = hf - W \quad or \quad V_0 = \left(\frac{h}{e}\right)f - \left(\frac{W}{e}\right) — (3)$

Stopping potential: It is the minimum retarding potential which should be applied across a photoelectric tube in order to make photoelectric current zero.

The photoelectrons of maximum kinetic energy 2 eV can thus be completely stopped by a potential difference of -2V. From the graph in Fig. , and the equation (2), we observe that-

Slope of the graph is h/e

(i) At the frequency $f = f_0$ (threshold frequency) , stopping potential is zero.

(ii) At stopping potential, the photoelectric current is zero.

(iii) From the equation (2), we see that stopping potential is directly proportional to the frequency of incident light.

Explanation of photoelectric effect from Einstein's equation: **OR,**

Three salient features observed in photoelectric effect from the equation (1) are;

(1) K_{max} depends linearly on frequency of incident light f only. It is independent of intensity of radiation. Einstein explained that single electron absorbs energy of a single photon.

(2) Photoelectric emission is possible only if $E > W \; or \; f > f_0$, if $f < f_0$, then K. E of electron comes out to be negative, which is not possible. So, no photoemission is possible below f_0, even if the incident radiation of high intensity and long duration falls on the surface.

(3) The intensity of radiation is proportional to the number of photon energy per unit area per unit time (n). The greater the number of photons, the greater is the number of photoelectrons emitted. Hence, the photoelectric current is proportional to the intensity.

Equation (2) can also be written as $K_{max} = hf - W = hf - hf_0 = h(f - f_0) - - - -(4)$

If K_{max} equals to zero, when $f = f_0$, then $W = hf_0$

(4) The absorption of photon by electron is instantaneous process, hence emission of electron is instantaneous.

Note: Practically, only one electron is ejected by 1 million photons.

Quantum efficiency = $\frac{No. of \; electrons \; ejected \; per \; sec}{No. of \; photons \; incident \; per \; sec}$

Stopping potent is independent of:

(a) Intensity of incident light

(b) Distance of source from metal surface and

(c) Illuminating power of source

Kinetic energy of photoelectron:

1. It depends on the frequency of incident light as $K_{max} = hf - W \; i.e. K_{max} \propto f \; for \; f > f_0$
2. It does not depend upon the intensity of incident radiation.

The number of

1. Photoelectrons emitted depend on the intensity of incident light.

2. Photoelectrons emitted Is independent of the frequency of incident light provided f > f_0

Q.3: If work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm? [1 nm = 10 Å] [Hint, $E = \frac{hc}{\lambda} = \frac{12375\ eV}{\lambda\ (in\ Å)}$][Ans. No]

Q.4: Threshold frequency for a certain metal is 3.3 x 10^{14} Hz. If the light of frequency 8.2 x 10^{14} Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission. [Hint: $V_0 = \frac{h}{e}(f - f_0)$] [Ans. 2.03 V]

Q. 5: In an experiment on photoelectric effect, the slope of the cut-off voltage to be 4.12 x 10^{-15} Vs. Calculate the value of Planck's constant.

Intensity of light: The intensity of light is defined as the number of photons falling per unit area per second. Or, energy crossing per unit area normally per second. If N be the number of photons, A be the surface area, t the time for which photons are incident on the surface, then $Intensity = \frac{N}{Axt}$

So, the intensity of light depends on the number of photons present in it. It does not depend on the frequency of incident light.

Total energy of photon = $Nhf = N\left(\frac{hc}{\lambda}\right)$

The, power of the source, $P = \frac{E}{t} = \frac{Nhf}{t} = nhf$ {where N = No. of photons, n = number of photons per second}

Or, $n = \frac{P}{E} = \frac{P}{hf} = \frac{P\lambda}{hc}$ and

intensity, $I = \frac{P}{A}$

Note: For a point source, Intensity is $I = \frac{P}{4\pi r^2}$, $i.e. I \propto \frac{1}{r^2}$

For a line source, $I = \frac{P}{2\pi rl}$ $or, I \propto \frac{1}{r}$

Dual nature of matter: De-Broglie Waves

De-Broglie put forward the bold hypothesis that material particles in motion should display wave-like properties. His reasoning was based on following two considerations.

1. The two physical quantities which govern all the forms of physical universe are mass and energy. The Einstein mass-energy relationship: $E = mc^2$ shows that there is a complete equivalence between matter (mass) and radiation (energy).
2. Radiation (light) has dual nature (that is, it sometimes behaves like a wave and sometimes as a particle), therefore, from symmetry considerations, de-Broglie predicted that matter must also possess dual nature. Thus particles like protons, electrons and neutrons should not only behave like mass points but they should also exhibits wave nature when in motion.

The waves associated with material particles in motion are called matter or de-Broglie waves and their wavelength is called de-Broglie wavelength.

De-Broglie wave equation:

Considering photons as an em wave of frequency f, its energy from Planck's quantum theory is given by $E = hf ----(1) where\ h\ is\ Planck'sconstant$

Considering photon as a particle of mass m, the energy associated with it is given by Einstein's mass energy relationship as $E = mc^2 ----(2)$

From equation (1) and (2), we get $hf = mc^2$ $\quad or, \frac{hc}{\lambda} = mc^2 \quad or, \lambda = \frac{h}{mc} = \frac{h}{p} ----(3)$

Where p (=mc) is the momentum of the photon. The above equation (3) has been derived for a photon of radiation.

According to de-Broglie hypothesis, it must be true for material particles like electrons, protons, neutrons etc. Hence a particle of mass m moving with velocity v must be associated with a matter wave of wavelength λ, given by $\lambda = \frac{h}{p} = \frac{h}{mv} ----(4)$

This equation (4) is the de-Broglie wave equation for material particles.

It explains the dual nature of matter as it connects the wave characteristic 'λ' with particle characteristic 'p'.

From de-Broglie's equation, we find that The wavelength of moving particle is inversely proportional to its momentum, that is, $\lambda \propto \frac{1}{p}$

1. If v = 0, the $\lambda = \infty$. This implies that waves are associated with material particles only when they are in motion.
2. To be associated with a de-Broglie wave, a particle need not have a charge. That is why, de-Broglie waves are also known as matter waves.
3. De-Broglie wave cannot be electromagnetic in nature because em waves are only associated with accelerated charged particles.

De-Broglie wavelength of an electron

Considering an electron of mass m and charge e, let v be the final velocity attained by the electron when it is accelerated from rest through a potential difference of V volt. By work-energy theorem, the kinetic energy gained by the electron equals the work done on the electron by the electric field.

Kinetic energy gained by the electron, $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Work done on the electron by electric field is $W = eV, \therefore \quad K = \frac{p^2}{2m} = eV \quad or, p = \sqrt{2mK} = \sqrt{2meV}$

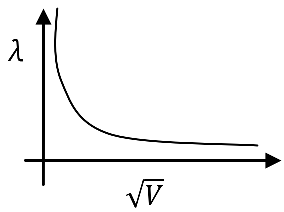
Hence, de-Broglie wavelength of electron is, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}}$ (in Å)----(1)

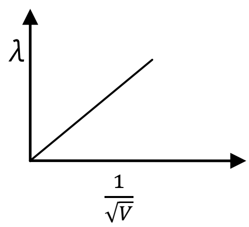
For any charge particle, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$

Similarly, de-Broglie wavelength for other charged particle can be given by

(1) For proton, $\lambda = \frac{0.286}{\sqrt{V}}$ (in Å) ,(2) For deuteron, $\lambda = \frac{0.2o2}{\sqrt{V}}$ (in Å), (3)For α-particle, $\lambda = \frac{0.101}{\sqrt{V}}$ (in Å)

Graph between the variation of de-Broglie wavelength (λ) associated with an electron with the square root of accelerating potential as mentioned in equation (1) is-





Characteristics of matter waves:

(1) Matter waves are not electromagnetic waves in nature.
(2) Matter waves are non-mechanical waves. That is, they can travel in vacuum.
(3) Matter waves are independent of charge i. e. they are associated with every moving particle (whether charged or uncharged).
(4) Observation of matter waves is possible only when the de-Broglie wavelength is of the order of size of the particle.
(5) The phase velocity of matter waves can be greater than the speed of light.
(6) The number of de-Broglie waves associated with nth orbital electron is n.
(7) It is applicable to both charged particle and neutral particle.

$\lambda_{thermal\ neutron} = \frac{h}{\sqrt{2mkT}}$ i. e. $\lambda_{thermal\ neutron} \propto \frac{1}{\sqrt{T}}$, k = Boltzmann constant (k = 1.38 X 10^{-23}), T is absolute temperature

Numerical and theoretical questions:

1. An electron and proton have same kinetic energy. Which one has smaller de-Broglie wavelength and why? [AI 2012] [Ans. Proton]
2. Name an experiment which shows wave nature of electrons. Which phenomenon was observed in this experiment using an electron beam? [F 2010] [Hint: Davison and Garner, diffraction]
3. The de-Broglie wavelength of a particle of kinetic energy K is λ. What would be the wavelength of the particle if its kinetic energy were K/4. [Ans. 2 λ]
4. An electron and photon each have a wavelength 1.00 nm. Find (a) their momenta, (b) the energy of the photon and (c) the kinetic energy of the electron [Momentum of electron = momentum of photon, $p = \frac{h}{\lambda}$ = 6.63 x 10^{-25} kg m/s, energy of photon =$\frac{hc}{\lambda}$ = 2 x 10^{-16} J, $K = \frac{p^2}{2m_e}$ = 2.42 x 10^{-19} J] [D 2011] [h = 6.63 x 10^{-34} Js, m_e = 9.1 x 10^{-31} kg]
5. The maximum kinetic energy of a photoelectron is 3eV. What is its stopping potential? [Ans. -3V] [Hint $K_{max} = eV_0$]
6. Determine the d-Broglie wavelength of a proton whose kinetic energy is equal to the rest mass energy of an electron. Mass of proton is 1836 times mass of electron. In which region of the electromagnetic spectrum does this wavelength lie? [Ans. 4 x 10^{-14} m, ϒ-ray region]
7. In a plot of photoelectric current versus anode potential, how does

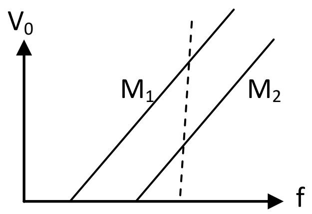
(i) The saturation current varies with anode potential for incident radiations of different frequencies but same intensity?

(ii) The stopping potential varies for different intensities but same frequency?

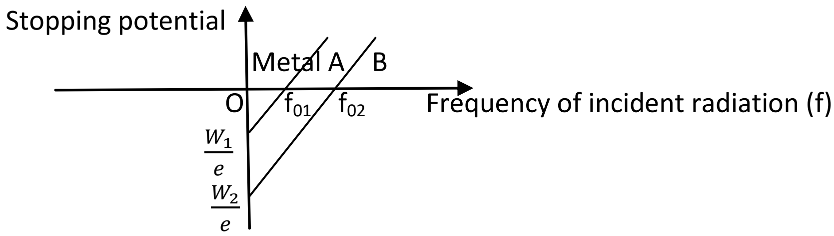
(iii) Photoelectric current vary for different intensities but for same frequency of incident radiations? Justify your answer in each case.

8. The figure shows variation of stopping potential (V_0) with the frequency f for two photosensitive materials M_1 and M_2. Why is the slope same for both lines?

(i) For which material will the emitted electrons have greater kinetic energy for the incident radiations of the same frequency? Justify your answer.



9. Light of wavelength 2500 Ǻ falls on a metal surface of work function 3.5 eV. What is the kinetic energy in eV of (i) the fastest and (ii) the slowest electrons emitted from the surface? If the same light falls on another surface of work function 5.5 eV, what will be the energy of emitted electrons? [F 2011] [Ans. 1.4 eV, 0 eV, no photoelectric emission will occur.]
10. The graph shows variation of stopping potential with frequency of incident radiation for two photosensitive metals A and B. Which one of two has higher work function? Justify your answer. [Ans. Metal B has higher value of work function because the slope of both materials is constant and the intercept of line depends on work function.]



11. A particle of mass 1 mg has the same wavelength as on electron with a velocity of 6 x 10^6 m/s. What is the velocity of the particle? [Ans. 2. 7 x 10^{-18} m/s]
12. What is the (a) momentum, (b) speed and (c) de-Broglie wavelength of an electron with K. E. of 120 eV? [Ans. 5.91 x 10^{-24} kg m/s, 6.5 x 10^6 m/s, 1.12 x 10^{-10} m]