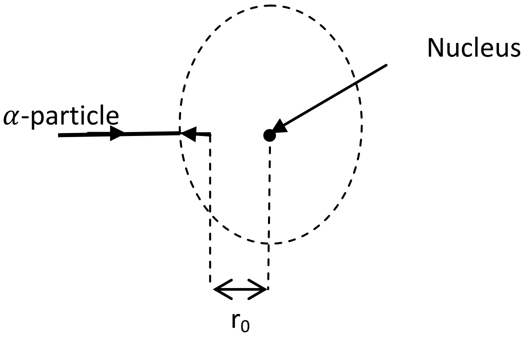
Chapter-12: Atom

Distance of closest approach-Estimation of nuclear size



The minimum distance from the nucleus up to which, α particle approaches and retraces its path, is called the distance of closest approach.

At this distance (when particle deflect by 180^0) whole of the K. E. of α-particle is converted into electrostatic potential energy.

Suppose an α-particle of mass m and initial velocity v moves directly towards the centre of the nucleus of an atom. As it approaches the positive nucleus, it experiences repulsive force and its kinetic energy gets progressively converted into electrostatic potential energy. When the kinetic energy equals to potential energy, at a certain distance r_0 from the nucleus, it begins to retrace its path. That is, it is scattered through an angle of 180^0. At this distance r_0, the entire initial K. E. of the α-particle gets converted into electrostatic potential energy.

The charge on the α-particle is $q_1 = + Z e$ and charge on nucleus, $q_2 = + Ze$ where Z is the atomic number of foil atoms. So electrostatic potential energy of α-particle and nucleus at distance r_0 is

$$U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{2Ze^2}{r_0}\right)$$

This potential energy is equal to initial kinetic energy, so, initial K. E. of α-particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0}\left(\frac{2Ze^2}{r_0}\right) \Rightarrow r_0 = \frac{Ze^2}{m\pi\epsilon_0 v^2}$$

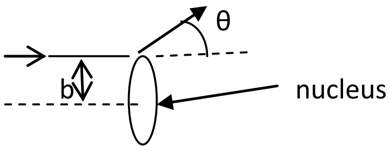
Q. 1: An α-particle of energy 5 MeV is scattered through 180^0 by a fixed uranium nucleus. Determine the distance of the closest approach. Given atomic number of Uranium, Z = 92, e = 1.6×10^{-19} C
[Hint: Charge of α-particle q_1 =2e, Charge of 92 U, q_2 = 92e, K. E. =5 Mev, Ans. 5.3×10^{-14} m]

Impact parameter:

The scattering of α particle from a nucleus depends upon the closest approach to the nucleus.

''It is defined as the perpendicular distance of velocity vector of the α-particle from the central line of the nucleus when it is far away from the atom''.

The shape of the trajectory of scattered α-particle depends on the (i) impact parameter b and (ii) the nature of the potential field.



Rutherford deduced the following relationship between the impact parameter b and the scattering angle θ as

$$b = \frac{1}{4\pi\epsilon_0}\left(\frac{Ze^2 cot\frac{\theta}{2}}{K}\right), where\ K = \frac{1}{2}mv^2, \qquad Or, b \propto cot\left(\frac{\theta}{2}\right)$$

Smaller the impact parameter (b), the greater is the angle(θ) and grater the K. E.

Impact parameter b and scattering angle θ

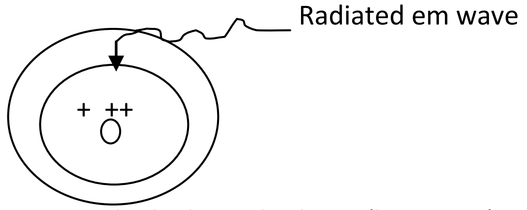
From the experiments, it can be noticed the following points:

1. For large impact parameters, the repulsive force experienced by the α-particle is weak because of its inverse square law character.
2. For small impact parameter , the repulsive force is large and so α-particle is scattered through large angle.
3. For a head-on collision, the impact parameter b= 0, scattering angle θ = 180^0. That ie, α-particle is reversed back along its original path.

Q. 2: In Rutherford scattering experiment, what will be the correct angle for α-scattering of for an impact parameter b =0? [Ans. 180^0]

Bohr's atomic model: Postulates of Bohr's theory of hydrogen atom: Bohr's model based on the following **three postulates**.

1. **Quantum condition**: The electrons can revolve only in those orbits in which the angular momentum is the integral multiple of h/2 π. That is,
   $L_n = mvr = \frac{nh}{2\pi}, where\ n = 1, 2, 3, - - - - (1)$ are principal quantum number.
2. **Stationary orbit**: An electron in atom can move around nucleus in certain circular stable orbits without emitting radiations, is called stationary orbit.
3. **Frequency condition**: When the atom receives the energy from outside, then one (or more) of its outer electrons leaves its orbit and goes to some higher orbit. This state of atom is called **excited state**. The electron in the higher orbit stays only for 10^{-8} s and returns back to lower orbit. While returning back, the electron radiates energy in the form of electromagnetic waves



If the energy of the electron in the higher orbit be E_2 (here is E_i) and that in the lower orbit be E_1 (here is E_f) So, frequency of the emitted radiation is given by

$$f = \frac{E_2 - E_1}{h}, - - - - (2) where, h = Planck's\ constant$$

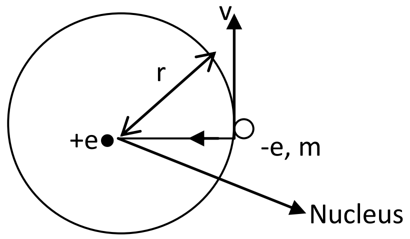
This equation is called Bohr's frequency condition.

Q.3: Compute the angular momentum in the fourth orbit, if L is the angular momentum of the electron in the 2^{nd} orbit of hydrogen atom. [Ans. 2L]

Solve NCERT 12.4 based on frequency condition.

Mathematical analysis Bohr's theory hydrogen atom and hydrogen like atom:

To derive an expression of radius of nth orbit of an electron of hydrogen atom and speed of nth orbit:



According to Bohr, a hydrogen atom consists of a nucleus with a positive charge e (positive proton) and an electron of charge –e, which revolves around it in a circular orbit of radius r. The magnitude of electrostatic force of attraction between nucleus and electron is given by

$$F = \frac{1}{4\pi\epsilon_0}\left[\frac{(e)(e)}{r^2}\right] - - - (1)$$

To keep the electron in its orbit, the radial inward centripetal force must be equal to the electrostatic force of attraction. Therefore, $\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\left(\frac{e^2}{r^2}\right) - - - (2)$

Where m is the mass of an electron and v is its speed in an orbit of radius r.

Since, Bohr's quantization condition is, $mvr = \frac{nh}{2\pi} - - - (3)$

From equation (2) and (3), since, $v = \frac{nh}{2\pi mr}$, so, $\frac{m}{r}\left(\frac{nh}{2\pi mr}\right)^2 = \frac{1}{4\pi\epsilon_0}\left(\frac{e^2}{r^2}\right)$, hence we get

nth orbit radius of hydrogen atom is, $r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2} - - - - (4)$ ■

So, the smallest radius corresponds to n = 1 {putting in equation (4)} denoting this minimum radius, called Bohr's radius as $r_1 = \frac{\epsilon_0 h^2}{\pi m e^2}$

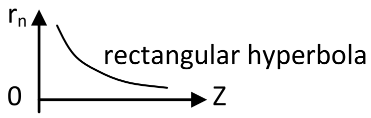
Substituting the values of ϵ_0=8.86 x 10^{-12} F/m, h = 6.63 x 10^{-34} Js, m = 9.11 x 10^{-31} kg, e = 1.6 x 10^{-19} C, we get r_1 = 0.53 Ă (1 Ă= 10^{-10} m)

Equation (4) in terms of r_1 can be written as $r_n = n^2 r_1 \; or \; r_n \propto n^2$

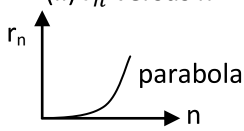
For hydrogen like atoms (He^+, Li^{++}), $r_n = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}$ or $r_n = \frac{n^2 r_1}{Z}$

Variation of r_n :

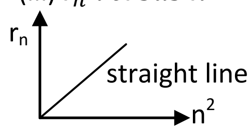
(i)r_n versus Z



(ii) r_n versus n



(iii) r_n $versus$ n^2



Speed of nth orbit in Bohr's model of hydrogen atom: $v_n = \frac{e^2}{2\epsilon_0 nh} - - - - (5)$■

Similarly, substituting values of e, ϵ_0 and h with n = 1 in equation (5), we get,

v_1 = 2.19 x 10^6 m/s = c/137-------(6), here c is the speed of light = 3 x 10^8 m/s

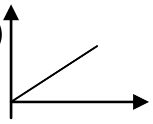
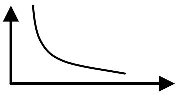
This is the greatest possible speed of electron in the hydrogen atom.

So, for hydrogen atom, $v_n = \frac{v_1}{n}$ and For hydrogen like atoms, $v_n = Z\frac{v_1}{n}$

Fine structure constant $\alpha = \frac{e^2}{2\epsilon_0 ch}, so \; v = \alpha\frac{c}{n}$ ------(7)

Hence, speed of electron in the innermost orbit is 1/137 of the speed of light in vacuum, while the speed of outer orbit varies inversely with n.

Variation of v_n: $v_n = Z\frac{v_1}{n}$ (i)v_n versus Z (ii) v_n versus n

(i)  (ii) 

Q.4: Using known values for hydrogen atom, calculate radius of 3rd orbit for Li^{2+} (Z =3)[Ans. 1.587 Ʌ]

Q.5: Using known values for hydrogen atom, calculate speed of electron in 4th orbit of He^{+}[Ans. 1.1 x 10^6 m/s]

Energy of the electron: The total energy in the nth orbit is the sum of kinetic energy and potential energy. So, E = K + U or, $E_n = K_n + U_n$

Here, $K_n = \frac{1}{2}m{v_n}^2 = \frac{me^4}{8{\epsilon_0}^2n^2h^2}$ and, $U_n = \left(-\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n}\right) = -\frac{me^4}{4{\epsilon_0}^2n^2h^2}$ =- 2 K_n

$$E_n = K_n + U_n = \frac{1}{2}m{v_n}^2 + \left(-\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n}\right) = \frac{me^4}{8{\epsilon_0}^2n^2h^2} - \frac{me^4}{4{\epsilon_0}^2n^2h^2} = -\frac{me^4}{8{\epsilon_0}^2n^2h^2} - - - (1)$$

$$E_n = K_n + U_n = K_n - 2K_n = -K_n$$

In terms of Rydberg's constant, $K_n = \frac{RhcZ^2}{n^2}$ and $U_n = -2Rhc\frac{Z^2}{n^2}$

$$Kinetic\ energy\ of\ electron = -Total\ energy$$

$$Potential\ energy\ of\ electron = +2\ (total\ energy)$$

Substituting the value of m, e, ϵ_0 and h with n = 1 in equation (1), we get the least energy of the atom in 1st orbit is E_1= - 13.6 eV. Hence, $E_1 = \frac{-me^4}{8{\epsilon_0}^2h^2}$ $and\ E_n = -\frac{me^4}{8{\epsilon_0}^2n^2h^2}, therefore$

$$\boxed{E_n = \frac{E_1}{n^2} = \frac{-13.6}{n^2} = \frac{-Rhc}{n^2} - - - - (2) \text{ where n = 2, 3, 4-------}}$$

Substituting n = 2, 3, 4, -------∞

E_2 = -3.40 eV, E_3 = -1.51 eV, ------- E_∞ =0, in ground state, (n = 1), energy of atom is 13.6 eV and energy corresponding to n = ∞ is zero. Hence energy required to remove the electron is 13.6 eV.

Bohr's model of Hydrogen like atoms:

The Bohr model of hydrogen can be extended to hydrogen like atoms, i. e. one electron atoms, such singly ionized helium (H^+), doubly ionised lithium (Li^{++}) and so on. In such atom, nuclear charge is +Ze where Z is atomic number equal to atomic number of proton in the nucleus.

Replacing e^2 by Ze^2, we have r_n, v_n and E_n are $r_n = \frac{\epsilon_0 n^2h^2}{\pi mZe^2} = r_1\frac{n^2}{Z}$ $or\ r_n \propto \frac{n^2}{Z} - - - - !)$

Where r_1 = 0.529 Ă (radius of first orbit of H)= Bohr's radius.

Speed of electron in nth orbit, $v_n = \frac{Ze^2}{2\epsilon_0 nh} = v_1\frac{Z}{n}$ $or\ v_n \propto \frac{Z}{n} - - - - (2)$

Where v_1 = 2.19 x 10^6 m/s (speed of electron in first orbit of H)

Energy of electron in nth orbit, $E_n = -\frac{mZ^2e^4}{8{\epsilon_0}^2n^2h^2} = E_1\frac{Z^2}{n^2}$ $or\ E_n \propto \frac{Z^2}{n^2} - - - - (3)$

Where E_1 = - 13.6 eV (energy of atom in first orbit of H)

For, He^+ , Z = 2. H and He^+ have many spectrum lines that have almost the same wavelengths.

Some general points related to energy of electron:

(i)Rhc = 1 Rydberg = 13.6 eV, (ii)For hydrogen atom, Z = 1, $E_n = \frac{-13.6}{n^2}\ eV$

(iii) Negative energy of electron shows that electron is bound to the nucleus and is not free to leave .

(iv) For hydrogen like atom as for He^+, Z =2, For Li^{++} Z = 3

Q. 6: The ground state energy of hydrogen atom is -13.6 eV. What is the kinetic energy and potential energy of electron in this orbit? [Ans. 13.6 eV, -27.2 eV]

Q.7: The total energy of electron in the ground state of hydrogen atom is -13.6 eV. Find the kinetic energy of electron in the 1st excited state. [Ans. 3.4 eV]

Q. 8: Find the kinetic energy, potential energy and total energy of electron in 1st and 2nd orbit of hydrogen atom, if potential energy in 1st orbit is taken to be zero. [Ans. For n = 1, $E_1 = -13.6\ eV$, so K = 13.6 eV, $U_1 = -27.2eV$, but $U_1 = 0$ as given, hence potential energy has been increased by 27.2 eV, so, E and U will be increased in all energy states except kinetic energy. Hence, final answer is-

Orbit	K (in eV)	U (in eV)	E (in eV)
First	13.6	0	13.6
Second	3.4	20.4	23.8

Energy of nth orbit of the hydrogen like atom is $E_n = \frac{-13.6}{n^2}\ eV$, Where, Z is atomic number of atom.

Some common terms used:

Absorption of energy: When the electron in the atom can jump from lower orbit to higher orbit, it absorbs energy.

Emission of energy: When an electron jumps from higher to lower orbit, it emits energy.

Energy of photon is equal to the difference of two orbits. $[E_i - E_f = hf]$

Ground state and ground state energy: The lowest state of an atom (n =1) is called ground state and energy associated with ground state is called ground state energy.

Ionisation energy: Energy required to ionise an atom is called ionisation energy. It is the energy required to make the electron jump from present orbit to the infinite orbit.

$$E_{ionisation} = E_\infty - E_n = 0 - (E_n) = -\left(\frac{-13.6Z^2}{n^2}\right) = +\frac{13.6Z^2}{n^2}$$

Energy needed to remove the electron from any energy level means Ionisation Energy.

(Minimum energy required to free the electron from ground state of the hydrogen atom is 13.6 eV, it is called ionisation energy.)

Ionisation Potential : The potential difference through which an electron should be accelerated to get ionisation energy is called ionisation potential.

Excitation energy: The energy required to take an electron from lower state to higher state is known as excitation energy. If lower state energy is E_{n_1} and higher state energy is E_{n_2}, then excitation energy is $\Delta E_{n_1 n_2} = E_{n_2} - E_{n_2}$

Note: Excitation energy ΔE_{12} i. e. 1st excitation energy (n = 2) is $\Delta E_{12} = E_2 - E_1$

First excitation energy of the hydrogen, $\Delta E_{12} = E_2 - E_1 = -3.4 - (-13.6) = 10.2\ eV$

Second excitation energy of the hydrogen, $\Delta E_{13} = E_3 - E_1 = -1.51 — 13.6 = 12.09\ eV$

Excitation potential: It is that accelerating potential which gives rise to a bombarding electron, sufficient energy to excite the target atom by raising one of its electrons from an inner to an outer orbit. First excitation potential of the hydrogen $\Delta V_{12} = V_2 - V_1 = -3.4 - (-13.6) = 10.2\ V$

Second excitation potential of the hydrogen, $\Delta V_{13} = V_3 - V_1 = -1.51 — 13.6 = 12.09\ V$

Wavelength of photon emitted in De-excitation:

According to Bohr, when an atom makes a transition from one energy level to a lower level, it emits a photon with energy equals to the energy difference between the initial and final level. If E_i is the initial energy of the atom before such a transition, E_f is its final energy after the transition. And the photon energy is $hf = \frac{hc}{\lambda}$, then conservation of energy gives

$$E = hf = \frac{hc}{\lambda} = E_i - E_f - - - - - - (1) energy\ of\ emitted\ photon$$

$$E_n = -\frac{Rhc}{n^2} = -\frac{13.6}{n^2}, where\ Rhc = 13.6\ eV$$

Limitation of Bohr's theory:

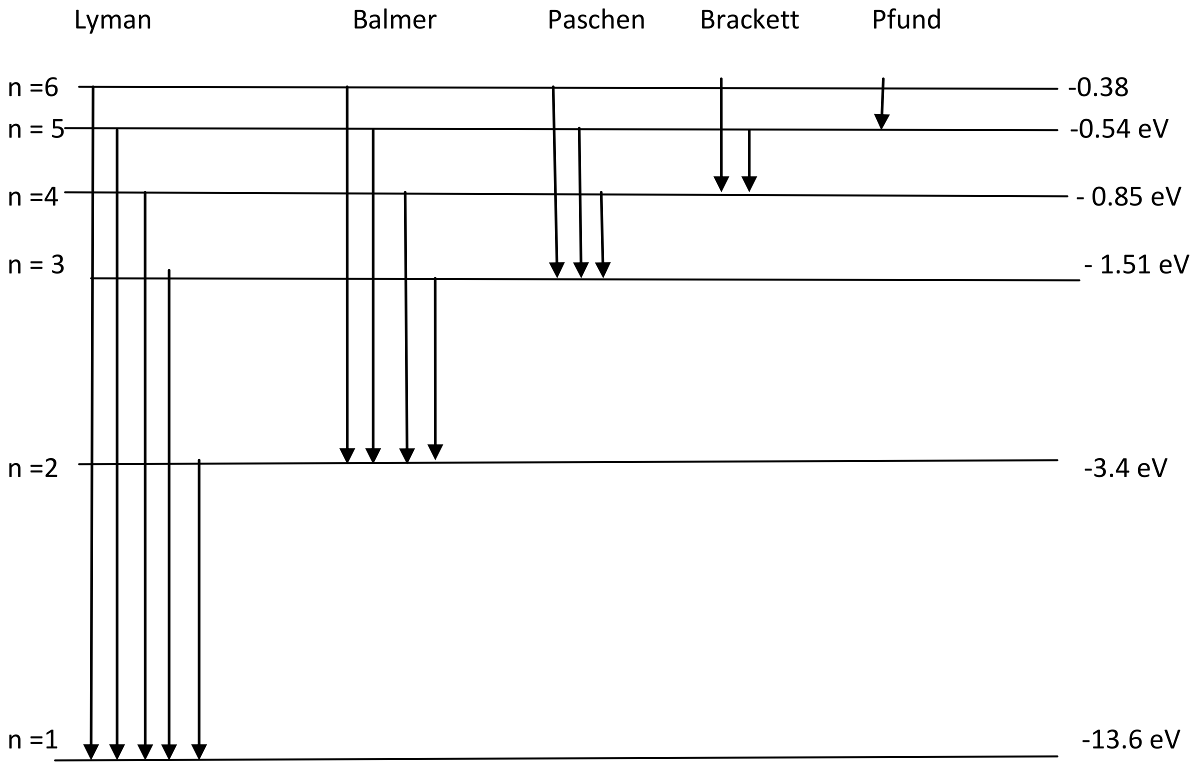
1. The theory is applicable only to hydrogen like single electron atoms and fails in the case of atoms of two or more electrons.
2. It does not explain why only circular orbits should be chosen when elliptical orbits are also possible.
3. It does not explain the further splitting of spectral lines in a magnetic field (Zeeman effect) or in the electric field (Stark effect)

Neil Bohr was awarded the 1922 Noble prize in Physics for this work.

Emission spectrum of hydrogen atom: Under normal condition, the single electron in hydrogen atom stays in ground state (n =1).It is excited to some higher energy state when it acquires some energy from some external source. But it hardly stays there for more than 10^{-8} second.

A photon corresponding to a particular spectrum line is emitted when an atom makes a transition from a state in an excited level to a state in a lower excited level.

Let n_i be the initial energy state and n_f the final energy state then depending on final energy state, the following series are observed in the emission spectrum of hydrogen atom.



For the Lyman series: $n_f = 1$, for Balmer series: $n_f = 2$, for Paschen series:$n_f = 3$, for Brackett series: $n_f = 4$, for Pfund series: $n_f = 5$

Energy of atom:

Absorption of energy: When electron jumps from lower to higher orbit, it absorbs photon.

Emission of energy: When electron jumps from higher to lower orbit, it emits photon.

Energy of photon is equal to the difference of energies of two orbits.

Let an electron of atom jumps from n_1 to n_2 ($n_2 > n_1$)

$$\therefore\ E_{n_2} - E_{n_1} = -13.6\frac{Z^2}{{n_2}^2} - (-13.6\frac{Z^2}{{n_1}^2}) = 13.6Z^2\left[\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right]$$

Energy of absorbed photon, $\Delta E = hf = \frac{hc}{\lambda}$ or, $\frac{1}{\lambda} = Z^2R\left[\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right]$

Where, R is the Rydberg constant and R = 1.097 X 10^7/m

Energy level for hydrogen atom:

Energy of nth state for hydrogen atom is, $E_n = -13.6\frac{Z^2}{n^2}eV$---(1)

For hydrogen, Z = 1, so, $E_n = \frac{-13.6}{n^2}eV$

The energy of an atom is least when n = 1 and $E_1 = -13.6\ eV$ (this is the lowest state or ground state of an atom.)

When n = 2, $E_2 = -3.4\ eV$, this is the 1st excited state or energy in the second orbit. It means energy required to excite an electron of hydrogen atom in 1st excited state is $E_2 - E_1 = -3.4 - (-13.6)$=10.2 eV and so on so forth.

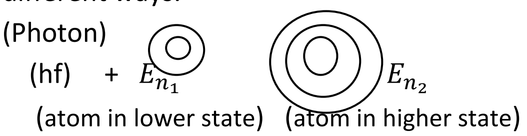
Q. 9: A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when an atom makes a transition from the upper level to the lower level?[Ans. 5.6 X 10^{14} Hz]

Q.10: Determine the wavelength of the radiation required to excite the electron in Li^{++} from 1st to the 3rd Bohr orbit. [Ans. 11.4 nm]

Q.11: A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n =4 level. Determine the wavelength and frequency of photon. [974 Å, 3.8 X 10^{15} Hz]

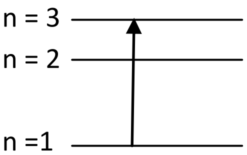
Absorption and emission spectrum:

An atom can be excited by incident photon or by collision. If an atom absorbs a photon, it goes to higher orbit/state and remains there for a time of 10^{-8} sec and returns back to ground state by different ways.



From above diagram, it is clear that $E_{n_1} + hf = E_{n_2}$

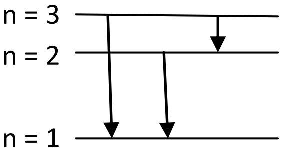
Example: A photon of energy 12.09 eV is incident on hydrogen atom in ground state, so, energy of of the atom after absorption $= 12.09 + (-13.6) = -1.51\ eV = E_3$ so, after absorbing, electron goes to the state n = 3



Absorption spectrum

In this state, atom remains for the time 10^{-8} s and returns to ground state by emitting photons(s). This atom can come directly to n = 1 by emitting one photon, or by emitting two photons from n = 2 to n = 1.

Therefore, possible transitions are 3 → 2, 2 → 1, 3 → 1 as shown in the figure below.



Emission spectrum

In the above figure, number of emission spectrum equals to 2.

Out of three transitions, frequency of emitted photons will be maximum corresponds to 3 → 1 (since $\Delta E = hf$), therefore, wavelength will be minimum. In the transition, 3 → 2, frequency is minimum and hence, wavelength will be maximum.

No. of lines in emission spectrum = $\frac{n(n-1)}{2}$

Note: *Emission/absorption of photons only takes place when the energy of the photon is exactly equal to the energy difference*.

Q. 12: In which of the following transitions, will the wavelength be the minimum?

(a) n = 5 to n = 4 (b) n = 4 to n = 3 (c) n = 3 to n = 2 (d) n = 2 to n =1

[Ans. (d) as $\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$

Q. 13: Excitation energy of a hydrogen like ion in its 1st excitation state is 40.8 eV. Energy needed to remove the electron from the ion in the ground state is __________. [Ans. 54.4 eV]

Q. 14: Monochromatic radiation of wavelength λ is incident on a hydrogen sample in the ground state. Hydrogen atom absorbs a fraction of light and subsequently emits radiation of 10 different wavelengths. Find the value of λ . [Ans. 947.5 Å]

Q. 15: Ionisation potential for hydrogen atom is 13.6 V. Hydrogen atom in the ground state are excited by monochromatic radiation of photon energy 12.1 eV. Find the possible spectral lines emitted by hydrogen. [Ans. n = 3 second excited state and no. of spectral lines = 3]

Spectral series of hydrogen atom:

From Bohr's theory, energy of electron in the nth orbit of hydrogen atom is given by $E_n = -\frac{me^4}{8\epsilon_0^2 n^2 h^2}$

According to Bohr's frequency condition, whenever an electron makes a transition from a higher energy level n_2 to a lower energy level n_1, the difference of energy appears in the form of photon. The frequency v of the emitted photon is given by

$$hv = E_{n_2} - E_{n_1} = \frac{me^4}{8\epsilon_0^2 h^2}\left[\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right] = h\frac{c}{\lambda} \Rightarrow \frac{1}{\lambda} = \frac{me^4}{8\epsilon_0^2 h^3 c}\left[\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right] ----(1)$$

Where 1/λ = Wave number (number of wave per unit length)

Since, $\frac{me^4}{8\epsilon_0^2 h^3 c} = R$ (Rydberg's constant) and the value of R found to be equal to 1.0973 x 10^7/m. So, the origin of various series in the hydrogen spectrum can be explained as follows:

Rydberg formula: $\frac{1}{\lambda} = Z^2 R\left[\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right]$

Energy of the emitted radiation, $E = Rhc\left[\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right]$, here Rhc = -13.6 eV

Note: Reciprocal of wavelength is called wave number. Here, $n_2 > n_1$

Hydrogen Spectral series:

1)*Lyman series*: If an electron jumps from any higher energy levels n_2 =n = 2, 3, 4---to a lower energy level n_1 = 1, we get a set of spectral lines called Lyman series which belongs to the **ultraviolet region** of the electromagnetic spectrum. This series is given by $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$.

Where n = 2, 3, 4------ [wavelength corresponding to n =2 is the longest wavelength of first line of Lyman series and wavelength corresponding to n = ∞ is the shortest wavelength of Lyman series]

2)Balmer series (a Swiss teacher): Spectral series corresponding to the transition n_2 = n = 3, 4, 5----- and n_1 = 2 lies in ***the visible region*** and is called Balmer series. This series is given by $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$.

Where n = 3, 4, 5------------- [wavelength corresponding to n =3 is the longest wavelength of first line of Balmer series and wavelength corresponding to n = ∞ is the shortest wavelength of Balmer series]

3)Paschen series: If n_2 = n = 4, 5, 6---- and n_1 = 3, we get the **spectral series in the infrared region** which is called Paschen series. This series is given by $\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n^2}\right]$.

Where n = 4, 5, 6------------- [wavelength corresponding to n =4 is the longest wavelength of first line of Paschen series and wavelength corresponding to n = ∞ is the shortest wavelength of Paschen series]

4)Brackett series: Brackett series: If n_2 = n = 5, 6, 7---- and n_1 = 4, we get the **spectral series in the *infrared region*** which is called Paschen series. This series is given by $\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n^2}\right]$.

Where n = 5, 6, 7------------- [wavelength corresponding to n =5 is the longest wavelength of first line of Paschen series and wavelength corresponding to n = ∞ is the shortest wavelength of Brackett series]

5)Pfund series: Pfund series: If n_2 = n = 6, 7, 8---- and n_1 = 5, we get the **spectral series in the *infrared region*** which is called Paschen series. This series is given by $\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n^2}\right]$.

Where n = 6, 7, 8------------- [wavelength corresponding to n =6 is the longest wavelength of first line of Paschen series and wavelength corresponding to n = ∞ is the shortest wavelength of Brackett series]

Series limit: The transition corresponding to maximum energy or smallest wavelength of a given series of hydrogen spectrum is called series limit. We use $n_2 = \infty$ this also corresponds to shortest wavelength.

For Lyman, it is $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{\infty}\right]$ or $= \frac{1}{R}$, For Balmer, $\lambda = \frac{4}{R}$

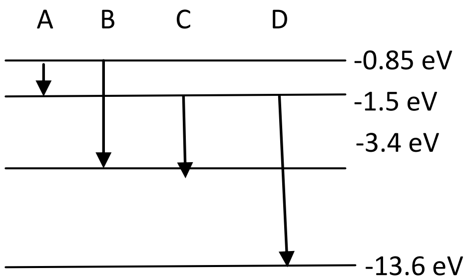
In general, series limit, $\frac{1}{\lambda} = \frac{R}{{n_1}^2}$, or, $\lambda = \frac{{n_1}^2}{R}$, where n_1 = 1 for Lyman, n_1 = 2 for Balmer, n_1 =3 for Paschen, n_1= 4 for Bracket and n_1= 5 for Pfund series.

Note: In Balmer series, First line is also called H_α line and second line as H_β line and so on.

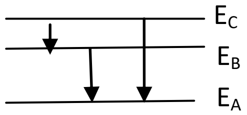
Numerical on Atom

[Constants: Rydberg's constant R = 1.097 x 10^{7} /m, Planck's constant h = 6.63 x 10^{-34} Js, c = 3 x 10^{8} m/s, 1eV = 1.6 x 10^{-19} J]

1. The energy level diagram of an element is given here. Identify by doing necessary calculations, which transition corresponds to the emission of a spectral line of wavelength 102.7 nm? [Ans. D]



2. Energy level A, B and C of a certain atom corresponding to increasing value of energy $E_A < E_B < E_C$. If λ_1, λ_2 and λ_3 are wave lengths of radiations corresponding to transition C to B, B to A and C to A. Obtain the relation between these three wavelengths.



Since, $E_C - E_B = \lambda_1, E_B - E_A = \lambda_2, E_C - E_A = \lambda_3$ so, get the answer $\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_3}$

3. The energy of an electron in the nth orbit is given by $E_n = -\frac{13.6}{n^2}$ eV. Calculate the energy required to excite an electron from ground state to the second excited state. [Ans. 12.09 eV]
4. Calculate the shortest wavelength in the Balmer series of Hydrogen atom. In which region of hydrogen spectrum does this (short) wavelength lies? [Ans. 3637 Ǎ]
5. Find the longest and shortest wavelengths in the Lyman series for hydrogen. In what region of electromagnetic spectrum does each series lie? [Ans. 1215 Ǎ, 911 Ǎ, both lie in ultraviolet region of electromagnetic spectrum]
6. Using the known values for hydrogen atom, calculate (a) radius of third orbit for Li^{++} (b) speed of electron in fourth orbit for He^+ [Hint For lithium, Z = 3, for He, Z =2, a_0 = 0.529 Ǎ] [Ans. 1.587 Ǎ, 1.095 x 10^6 m/s]
7. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelength will be emitted? [Ans. 993 Å]
8. Calculate (i) the wavelength, and (ii) the frequency of H_β line of Balmer series for hydrogen. [Ans. 4.9 X 10^{-7} m, 6.12 X 10^{14} Hz]

Note: The minimum energy required to ionise an atom is the energy equal to remove one outermost electron from the gaseous state of atom.

9. The shortest wavelength of the Brackett series of hydrogen like atom of atomic number Z is same as the shortest wavelength of the Balmer series of hydrogen atom. Find the value of Z. [Ans. 2]
10. If an electron in hydrogen atom jumps from 3rd orbit to 2nd orbit, it emits a photon of wavelength λ. When it jumps from 4th orbit to 3rd orbit, find the corresponding wavelength of photon. [Ans. $\frac{20\lambda}{7}$]
11. Find excitation potential of hydrogen atom in the first excited state.[Ans. 10.2 V]

Period of revolution: (Time period):

Time period of revolution of electron in nth orbit is $T = \frac{2\pi r}{v}$

$$\therefore T = 2\pi \frac{\frac{n^2h^2}{\pi m Z e^2}}{\frac{e^2 Z}{2\epsilon_0 nh}} = \frac{4\epsilon_0 n^3 h^3}{mZ^2e^4}, that\ is\ T \propto \frac{n^3}{Z^2} ----(1)$$

We know that nth radius of hydrogen like ion or atom is $r \propto \frac{n^2}{Z}$ and if Z = 1 for hydrogen atom, then

$$\sqrt{r} \propto n, \quad hence, \qquad T \propto r^{\frac{3}{2}} ----(2)$$

12. Find the ratio of period of revolution of an electron in n = 2 and n =1 orbit in Bohr model of hydrogen atom. [Ans. 8:1]
13. In an atom, the two electrons move round the nucleus in circular orbits of radii R and 4R. Find the ratio of time taken to complete one revolution. [Ans. 1:8]

Change in angular momentum:

$$\Delta L = L_2 - L_1 = \frac{n_2 h}{2\pi} - \frac{n_1 h}{2\pi}$$

14. When an electron in hydrogen atom is excited from its 4^{th} to 5^{th} stationary orbit what is the change in angular momentum of electron? [Ans. 1.05 X 10^{-34} Js]

Binding Energy: The energy required to separate the constituent of a nucleus to infinity is called binding energy. The binding energy of hydrogen atom in ground state is, $BE = -(E_1)$ = 13.6 eV

15. Hydrogen atoms emits blue light when it changes from n = 4 energy level to n = 2 energy level. Which colour of light would the atom emit when it changes from n = 5 level to n =2 level. [Ans. Violet light]