Chapter-13: Nuclei Key Points

Nucleus:

As per the Rutherford's experiment, it was concluded that the entire positive charge and mass of an atom was present at its centre known as nucleus.

Composition of nucleus:

Nucleus is composed of two types of particles-(i) proton (positively charged particles) and (ii) neutrons, (neutral particles. Mass of neutron is slightly greater than the mass of proton.

Mass of proton, $m_p \approx 1840 m_e$ where m_e = mass of electron = 9.1 X 10^{-31} kg $\approx 0.0009 \times 10^{-27}$kg

Mass of proton $m_n = 1.6726 \times 10^{-27}$kg, (mass of neutron $m_n = 1.6749 \times 10^{-27}$kg)

Atomic number

Atomic number of an element is the number of proton inside the nucleus of a neutral atom. It is also equal to number of electrons revolving in various orbits around the nucleus of the atom. It is represented by Z. So, **Z = no. of proton = no. of electron**

Mass number (A):

The total number of neutrons and protons is called mass number. A chemical symbol or element is represented as ${}_{Z}^{A}X$ where X is any element. Total charge of nucleus is Ze, where e the charge on proton.

Discovery of Neutron

In 1935, James Chadwick discovered neutron. He bombarded beryllium with α-particles from polonium of about 5 MeV energy and suggested that a particle having highly penetrating power is produced. He found that the particle was uncharged and of approximately same mass as that of proton. ${}_{4}^{9}Be + {}_{2}^{4}He \rightarrow {}_{6}^{12}C + {}_{0}^{1}n + Q$

Properties of Neutron

a)Very high penetrating power, (b) Very low ionising power, (c) It is stable within the nucleus. (d) outside the nucleus, it decays as ${}_{0}^{1}n \rightarrow {}_{1}^{1}H + {}_{-1}^{0}e + \bar{\nu}$ (e) Its mean life is approx. 1000 seconds. The slow neutrons are called thermal neutrons. These can be obtained by allowing fast moving neutrons to pass through some hydrogen containing substances like paraffin wax, heavy water or graphite.

Mass Number (A): The total number of nucleons (protons and neutrons) in a nucleus is called its mass number. So, mass number A = Z + N, where N is the number of neutrons. In a nuclear physics, nucleus of an element is represented as ${}_{Z}^{A}X$.

Diameter of atomic nucleus ranges from 10^{-15} m to 10^{-14} m.

m_p = 1.6726 x 10^{-27} kg = 1.007825 amu; m_n = 1.6749 x 10^{-27} kg = 1.008665 amu [1 amu = 1.6596 X 10^{-27} kg]

Nuclear size: The volume of a nucleus is proportional to it mass number.

$$\frac{4}{3}\pi R^3 \propto A \quad (R \text{ is the radius of nucleus}) \therefore \quad R \propto A^{\frac{1}{3}} \text{ or } R = R_0 A^{\frac{1}{3}}$$

R_0 is a constant= range of nuclear size. It is known as nuclear unit radius and its value is 1.2 x 10^{-15} m (1.2 fm).Hence, different nuclei of different elements have different size.

Nuclear Density:

It is the ratio of mass of nucleus to the volume of nucleus.

Let m be the average mass of nucleons (1.67 X 10^{-27} kg) and R is the radius of nucleus. If 'A' is the mass number of element, then $\rho = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3m}{4\pi {R_0}^3} \quad (R^3 = {R_0}^3 A)$

Thus, the density of a nucleus is constant and is independent of mass number A for all nuclei and ρ is found to be 2.98 x 10^{17} kg/m^3. Hence, matter in the nucleus is very densely packed.

Nuclide: It is a specific nucleus of an atom which is characterised by its atomic number Z and mass number A. example, ${}^{32}_{16}S$ together taken as one nuclide.

Nuclear spin: The total angular momentum of nucleus is is known as nuclear spin of that nucleus.

Isotopes, Isobars and Isotones

Isotopes: Atoms having same atomic number (Z) but different mass number (A).

For example, ${}^{235}_{92}U, {}^{238}_{92}U$. ${}^{1}_{1}H$, ${}^{2}_{1}H$, ${}^{3}_{1}H$, ${}^{16}_{8}O, {}^{17}_{8}O$, ${}^{18}_{8}O$; ${}^{3}_{2}He$, ${}^{4}_{2}He$

Isobars: Atoms having same mass number (A) but different atomic number (Z)

For example,

Isotones: Nuclei containing same number of neutrons are called isotones. ${}^{37}_{17}Cl, {}^{39}_{19}K$.

Mirror nuclei: The nuclei having same mass number A but with proton number and neutron number interchange are called mirror nuclei. e. g. ${}^{3}_{1}H$ and ${}^{3}_{2}H$ and ${}^{7}_{3}Li$, ${}^{7}_{4}Be$

Atomic mass unit and energy equivalent to it

A unit in which atomic and nuclear masses are measured is called atomic mass unit (u).

1 u = $\frac{1}{12}th$ of mass of an atom of ${}^{12}_{6}C$ isotope of carbon. 1 u (or 1 amu) = 1.66 x 10^{-27} kg.

1 u (or amu) represents the average mass of a nucleon.

1 eV = 1.6 x 10^{-19} J, 1u mass has energy 931.5 MeV.

Mass-energy relationship

According to Einstein, $E = mc^2$, here c is speed of light in vacuum and its value is 3 x 10^8 m/s.

Mass defect and Nuclear Binding Energy

Mass defect (Δm) = (sum of masses of neutrons and protons, or mass of nucleon) – actual mass of the nucleus

Certain mass disappears in the formation of a nucleus in the form of mass defect. That is,
$Zm_p + (A - Z)m_n - m_N$= mass defect. This mass loss appears in the form of energy (Einstein theory of mass-energy equivalence) called

Binding energy. Thus, binding energy is the minimum energy required to separate its nucleon and place them at rest at infinite distance apart. Or, it is defined as the energy equivalent to the mass defect of the nucleus.

$Binding\ energy = \Delta mc^2 = \left[Zm_p + (A - Z)m_n - m_N\right]c^2$. This binding energy is expressed in joule when Δm is in kg.

If Δm is measured in amu, then binding energy, $E_b = \Delta m\ amu = \{Zm_p + (A - Z)m_n - m_N\}$ = $\Delta m \times$ 931MeV$where, Mass\ of\ nucleus\ or\ mass\ of\ atom, m_N = [m({}^{A}_{Z}X)]$

Here, m_p is the mass of proton and m_n is the mass of neutron.

$$Binding\ energy\ per\ nucleon, E_{be} = \frac{Total\ binding\ energy, E_b}{Mass\ number, A}$$

Note: *If binding energy per nucleon is greater for any nuclei, that nuclei is more stable.*

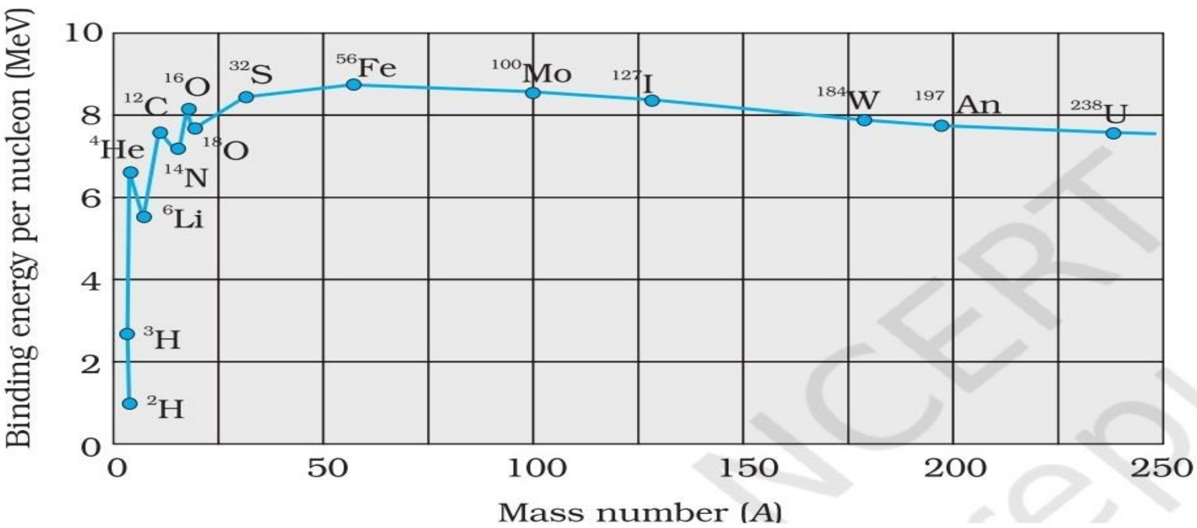
Binding energy curve

(Variation in binding energy per nucleon with mass number)

The curve reveals that

a) Light nuclei like ${}^{1}_{1}H, {}^{2}_{1}H\ and\ {}^{3}_{1}H$ have small value of binding energy per A

b) For mass numbers ranging from 2 to 20, there are sharp peak corresponding to nuclei having mass number in multiple of 4 (containing equal number of protons and neutrons)
Peak indicates that these nuclei are more stable than the nuclei in the neighbourhood.



Some features of this curve:

(i) Nuclei having mass number between 50 and 80 are most stable. Fe having A = 56 and BE/nucleon 8.8 MeV has maximum stability.
(ii) For nuclei having mass number more than 80, the average binding energy per nucleon decreases slowly and drop to about 7.6 MeV for uranium 238. The lower value of binding energy per nucleon fails to overcome the repulsion among protons in nuclei having large number of protons. That is nuclei of heavier atoms beyond $^{209}_{83}Bi$ are radioactive.
(iii) For nuclei having mass number below 20, it decreases sharply. So, nuclei having mass number below 20 are comparatively less stable.
(iv) Below A =50, the curve does not fall continuously, but has subsidiary peaks at $^{16}_{8}O$, $^{12}_{6}C$ and $^{4}_{2}He$. This shows that these even-even nuclei are more stable than their immediate neighbours.

Conclusions from the graph:

(i)The curve shows that very heavy and very light nuclei have a lower average BE/nucleon than the nuclei of intermediate masses. So, if a very heavy nucleus such as U^{238} be split up into two lighter nuclei near the flat maximum of the curve, the BE/nucleon will increase. Hence energy will be released in this process. This method of releasing nuclear energy by breaking up a heavy nucleus into two lighter nuclei of comparable masses is called nuclear fission. It is the basis of nuclear bombs (atom bombs) and nuclear reaction.
(ii) Alternatively, if two or more very light nuclei (such as H^2) can be combined to form a heavier nucleus (such as He^4), the BE /nucleon will again increase and now much a greater amount than fission process. This will result in much larger release of energy. This method of releasing nuclear energy is called nuclear fusion. It occurs inside sun and other stars and is the source of their energy. (hydrogen bomb)

Nuclear Stability of a nucleus depends upon

(i)High value of binding energy per mass number
(ii) Even-even nucleus (even Z, even A): It is found to be more stable.
(iii) An even- odd nucleus or odd-even nucleus is less stable and
(iv) An odd-odd nucleus is least stable.
(v) Nuclei having higher or lower value of neutron to proton ratio than the stable nuclei are also unstable.
(iv) $\frac{N}{Z}$ ratio (Neutron-proton ratio): For lighter nuclei, N/Z is 1, it is most stable. Heavy nuclei are stable when they have more neutrons than protons. It is unstable, if no. of protons is greater than neutrons. The nuclei $^{209}_{81}Bi$ is the heaviest stable nucleus.
Note: Experimental study shows that the stable nuclei contains even number of protons or neutrons or both.

Nuclear forces

Nucleus consists of protons (positively charged particles) and neutrons (neutral particles). The protons due to their positive charge repel each other, so there must be strong attractive forces inside the nucleus which overcome the repulsive force of protons and bind them together inside the nucleus. This force is called nuclear force.

Therefore, forces that hold the nucleons together inside the nucleus of an atom are called nuclear forces.

[The relative strength of gravitational, Coulomb's and nuclear are $F_g : F_e : F_n :: 1: 10^{36} : 10^{38}$]

Some important properties of nuclear forces are given below:

(i)Nuclear forces act between a pair of neutrons, a pair of protons and also between a neutron and proton pair, with the same strength. This shows that **nuclear forces are charge independent**.

(ii)**Nuclear forces are short range forces**. They do not exist at large distance greater than 10^{-15} m.

(iii) These are attractive force and cause stability of the nucleus.

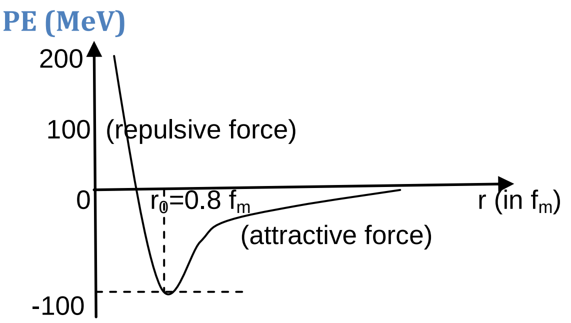
(iv) Nuclear forces are non-central force.

(v)Electrical and gravitational forces follow inverse square law but nuclear forces are complicated in nature and do not follow it.

(vi) The nuclear forces are the strongest forces in nature.

(vi) Nuclear forces depend upon the direction of spins (or angular momentum of nuclei) of the nucleons. The force is stronger if the spin of the nucleons is parallel and weaker if it is antiparallel.

To plot the graph between potential energy versus separation between the nucleons:-



(a) The potential energy is minimum at distance r_0 = 0.8 fm. At this distance, force between nucleons is zero.

(a) For $r < r_0$, the negative potential energy decreases to zero and then becomes positive. The nuclear forces are repulsive..

(c) For $r > r_0$, the negative P. E. goes on decreasing. The nuclear forces are attractive in nature.

Q-value of a nuclear reaction

Consider a nuclear reaction in which a target nucleus X is bombarded by a particle a resulting in a daughter nucleus Y and a particle b. Then, $a + X \rightarrow Y + b$

Sometimes this reaction is written as X(a,b)Y.

The reaction energy Q associated with a nuclear reaction is defined as the total energy released as a result of the reaction.

A reaction for which Q is positive is called **exothermic**. A reaction for which Q is negative is called **endothermic**.

In an exothermic reaction, the total mass of incoming particles is greater than that of the outgoing particles and the Q value is positive. If the total mass of the incoming particles is less than that of outgoing particles, energy is required for reaction to take place and the reaction is said to be endothermic.

Thus, an endothermic reaction does not occur unless bombarding particles has a kinetic energy greater than $|Q|$. The minimum energy necessary for which such a reaction to occur is called

Nuclear Fission (Divide and Conquer)

The process, in which a heavy nucleus splits up into two lighter nuclei, is called nuclear fission. Fission is initiated when a heavy nucleus captures a thermal neutron (slow neutrons) Example: The fission by thermal neutron can be represented by the equation,

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n + Energy$$

As mass of parent nucleus is more than the total mass of daughter nuclei, a large amount of energy is released in the reaction. Mass defect appears as energy according to the relation $E = (\Delta m)c^2$. The energy released in this process is 200 MeV.

$^{236}_{92}U$, is an intermediate excited state that lasts only for 10^{-12} s before breaking into nuclei Ba and Kr. The fission energy appears mostly as kinetic energy of the fission.

The disintegration energy in fission events first appears as kinetic energy of the fragments and neutrons, Eventually, it is transferred to the surrounding matter appearing as heat. The source of energy in nuclear reactors, which produce electricity, is nuclear fission. The enormous energy released in an atom bomb comes from uncontrolled nuclear fission.

Chain reaction

A nuclear reaction which produces particles that started the reaction originally, is called a chain reaction. The chain reaction once started, continues automatically.

In the **atomic bomb**, an increasing uncontrolled chain reaction occurs in a very short time when two pieces of U^{235} are rapidly brought together to form a mass greater than the critical size.

Nuclear reactor: In it, the chain reaction is steady and controlled so that on average, only one neutron from each fission produces fission. It is known experimentally that slow neutrons are much more likely to cause fission in $^{235}_{92}U$ than fast neutrons. Also, fast neutrons liberated in fission would escape instead of causing another fission reaction. The reaction rate is adjusted by inserting neutron - absorbing **rods of boron** steel into the U-235. The graphite core is used as a **moderator** to slow down the neutrons. In spite of graphite, water and heavy water (D_2O) are also commonly used as moderators, Natural uranium contains over 99% of Uranium 238 and 1% of Uranium 235. Deuterium in heavy water and carbon as graphite are both suitable as moderator. To control the power level, **control rods** are used. These rods are made of materials such as cadmium, that are very efficient in absorbing neutrons. The first nuclear reactor was built by Enrico Fermi and his team at the university of Chicago in 1942.

Nuclear Fusion

Binding energy of light nuclei (A< 20) is much smaller than the binding energy of heavier nuclei. This suggests a process that is reverse of fission. In this process, when two nuclei fuse (combine to form) together to form a single heavy nucleus. The mass of the single heavy nucleus formed is less than the total initial mass of the parent nuclei. Such reaction has been achieved in **hydrogen bomb** and it is believed to be the principle source of Sun's energy. The difference in mass appears as the energy according to $E = mc^2$. Examples of energy liberating nuclear fusion reactions are:

$$^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + e^{-} + \boldsymbol{\nu} + \mathbf{0.42\ MeV}$$

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + n + \mathbf{3.27\ MeV}$$

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{1}He + ^{1}_{1}H + \mathbf{4.03\ MeV}$$

When fusion is achieved by raising the temperature of the system so that particles have enough kinetic energy to overcome the coulomb repulsive behaviour, it is called thermonuclear fusion. Fusion can be carried out only at extremely high temperature of the order of $10^{8}\,{}^{0}$C.

Numerical:

1. What is the nuclear radius of 125_{Fe}, if that of 27_{Al} is 6.4 Fermi? [Ans. 10 fm]
2. assuming that proton and neutron have equal masses. Calculate how many times nuclear matter is denser than water. Take mass of nucleon = 1.67 X 10^{-27} kg and R_0 = 1.2 X 10^{-15} m. [Ans. 2.3 X 10^{14}]
3. Select the pairs of isotones from the following nuclei.

$^{24}_{12}Mg$, $^{3}_{1}H$, $^{23}_{11}Na$, $^{4}_{2}He$

4. Calculate the mass defect of He ($^{4}_{2}He$); mass of proton = 1.007276 u, mass of neutron = 1.008665 u, mass of $^{4}_{2}He$ = 4.001506 u [Ans. 0.030376 u]
5. A neutron breaks into a proton and electron. Calculate the energy produced in this reaction. Given m_e =9 X 10^{-31} kg, m_p = 1.6725 X 10^{-27} kg, m_n= 1.6747 X 10^{-27} kg, c = 3 X 10^8 m/s [Ans. 0.73 MeV] [$^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e$, here, $\Delta m = m_n - (m_p + m_e)$= 1.3 X 10^{-31} kg and energy produced = Δmc^2 in J]
6. Calculate the energy required to remove the least tightly bound neutron in $^{40}_{20}Ca$. Given that mass of $^{40}_{20}Ca$ = 39.962589 amu, mass of $^{39}_{20}Ca$ =38.970691 amu, mass of neutron = 1.008665 [Ans. 15.6 MeV][1 amu = 931 MeV][$^{40}_{20}Ca \rightarrow {}^{39}_{20}Ca - {}^{1}_{0}n$][$\Delta m$ =0.016767 amu]
7. Binding energy of $^{35}_{17}Cl$ and $^{31}_{15}P$ are 287.67 MeV262.48 MeV respectively. Which of the two nuclei is more stable? [Ans. $^{31}_{15}P$ is more stable as its E_{be} is greater than that of $^{35}_{17}Cl$]
8. Calculate the binding of an α-particle from the following data. Mass of helium nucleus m($^{4}_{2}He$) =4.001265 u, mass of proton m_p = 1.007277u, mass of neutron m_n = 1.00866 u, take 1 u = 931.5 MeV [Ans. 28.52 MeV]
9. Binding energy of $^{35}_{17}Cl$ =287.67 MeV and of $^{31}_{15}P$ = 262.48 MeV. Which of the two nuclei is more stable? [Ans. $^{31}_{15}P$ as BE per nucleon is more]
10. If the BE/nucleon of deuteron is 1.115 MeV, find its mass defect in atomic mass unit. [Ans. 0.0024u]
11. Calculate the energy equivalent of 1 g of substance. [Ans. 9 X 10^{13} J]
12. Obtain the binding energy of the nuclei $^{56}_{26}Fe$ *and* $^{209}_{83}Bi$ in unit of MeV from the following data: m($^{56}_{26}Fe$) =55.934939 u and m($^{209}_{83}Bi$) = 208.980388 u, Given that mass of proton = 1.007825 u, mass of neutron = 1.008665 [Ans. 8.790 MeV, 7.848 MeV]

    [Hint: Binding energy per nucleon = $\frac{mass\ defect\ x\ 931.5}{56}$, mass defect = 56.46340-55.934939=0.528461]
13. Find the minimum K. E. of an $\alpha - particle$ to cause the reaction $^{14}_{7}N(\alpha, p)^{17}_{8}O$. The masses of $N^{14}, He^4, H^1, O^{17}$ are respectively 14.00307 u, 4.00260 u, 1.00783 u and 16.99913 u.

    [Hint: Q = (14.00307 u + 4.00260 u - 1.00783 u - 16.99913 u) x 931.5 MeV/u]
14. Calculate the energy release in MeV in deuterium-tritium fusion reaction,$^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n$ Given that, m ($^{2}_{1}H$)=2.014102 u, m($^{3}_{1}H$) = 3.016049, m($^{4}_{2}He$) = 4.002603 u, 1 u = 931.5 MeV/c^2. [Ans. 17.59 MeV] [D 2015]
15. Obtain the binding energy (in MeV) of a nitrogen nucleus $^{14}_{7}N$, given $m(^{14}_{7}N)$ 14.00307 u. Given that 1 u = 931.5 MeV /c^2.
16. Draw a plot of potential energy of a pair of nucleus as a function of their separation. Write two important conclusions that can be drawn from the graph.
17. In the nucleus of helium if F_1 is the net force between two protons, F_2 is the net force between two neutrons and F_3 is the net force between a proton and a neutrons, then

    a)$F_1 = F_2 = F_3$ b) $F_1 > F_2 > F_3$ c) $F_2 > F_3 > F_1$ d) $F_2 = F_3 > F_1$
18. What is the ratio of nuclear radii of $^{1}_{1}H$ and $^{27}_{13}Al$?

    a)1:2 b) 2:1 c) 1:3 d) 3:1
19. **Read the text carefully and answer the questions:** Neutrons and protons are identical particles in the sense that their masses are nearly the same and the force, called nuclear force, does into distinguishing them. The nuclear force is the strongest force. The stability of the nucleus is determined by the neutron **-** proton ratio or mass defect or packing fraction. The shape of the

nucleus is calculated by quadrupole moment and the spin of the nucleus depends on even or odd mass numbers. The volume of the nucleus depends on the mass number. The whole mass of the atom (nearly 99%) is centered at the nucleus.

1) The correct statements about the nuclear force is/are
   a) all of these b) change independent c) Short - range force d) Non - conservative force
2) The range of nuclear force is the order of
   a) 2×10^{-10} m b) 1.5×10^{-20} m c) 1.4×10^{-15} m d) 1.2×10^{-4} m
3) A force between two protons is same as the force between proton and neutron. The nature of the force is
   a) gravitational force b) electrical force c) weak nuclear force d) strong nuclear force.
4) Two protons are kept at a separation of 40A^{o} . Fn is the nuclear force and F $_{e}$ is the electrostatic force between them. Then
   a) F$_{n}$<< F $_{e}$ b) F$_{n}$ = F $_{e}$ c) F$_{n}$>> F $_{e}$ d) F$_{n}$ $\approx$ F $_{e}$

   OR

   All the nucleons in an atom are held by
   a) nuclear forces b) van der Waal's forces c) tensor forces d) coulomb force

20. **Read the text carefully and answer the questions:** The density of nuclear matter is the ratio of the mass of a nucleus to its volume. As the volume of a nucleus is directly proportional to its mass number A, so the density of nuclear matter is independent of the size of the nucleus. Thus, the nuclear matter behaves like a liquid of constant density. Different nuclei are like drops of this liquid, of different sizes but of same density. Let A be the mass number and R be the radius of a nucleus. If m is the average mass of a nucleon, then Mass of nucleus = mA Volume of nucleus = $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(R_0 A^{1/3}\right)^3 = \frac{4}{3}\pi R_0^3 A$ ∴ Nuclear density, $\rho_{\text{nu}} = \frac{Mass of nucleus}{Volume of nucleus}$ or $\rho_{\text{nu}} = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$ Clearly, nuclear density is independent of mass number A or the size of the nucleus. The nuclear mass density is of the order 10 ^{17} kg m $^{-3}$. This density is very large as compared to the density of ordinary matter, say water, for which = 1.0 × 10 ^{3} kg m $^{-3}$.

1) The nuclear radius of $^{16}_{8}$O is 3 × 10 $^{-15}$ m. The density of nuclear matter is
   a) 2.9× 10 ^{34} kgm $^{-3}$ b) 2.4×10 ^{17} kgm $^{-3}$
   c) 16× 10 ^{27} kgm $^{-3}$ d) 1.2× 10 ^{17} kgm $^{-3}$
2) What is the density of hydrogen nucleus in SI units? Given R$_0$ =1.1 fermi and m $_{p}$ = 1.007825 amu.
   a) 2.98× 10 ^{17} kg m $^{-3}$ b) 7.85× 10 ^{17} kg m $^{-3}$
   c) 1.99× 10 ^{11} kg m $^{-3}$ d) 3.0× 10 ^{34}kg m $^{-3}$
3) Density of a nucleus is
   a) more for heavier elements and less for lighter elements
   b) a constant
   c) very less compared to ordinary matter
   d) more for lighter elements and less for heavier elements
4) The nuclear mass of $^{56}_{26}$Fe is 55.85 amu. The its nuclear density is
   a) 9.2× 10 ^{26} kgm $^{-3}$ b) 5.0× 10 ^{19} kgm $^{-3}$
   c) 1.5× 10 ^{19} kgm $^{-3}$ d) 2.9× 10 ^{17} kgm $^{-3}$

   OR

   If the nucleus of $^{27}_{13}$Al has a nuclear radius of about 3.6 fm, then $^{125}_{52}$Te would have its radius approximately as
   a) 12 fm b) 6 fm c) 9.6 fm d) 4.8 fm