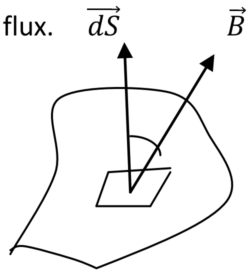
Chapter-6: Electromagnetic Induction

Whenever magnetic lines of force are cut by a closed circuit, a current is produced which is known as induced current and lasts only so long as the changes last. The induced current giving rise to such changes in the circuit is called the electromotive force (emf) and the phenomenon is called the **electromagnetic induction (EMI).**

That is, ''the phenomenon of production of induced emf and hence induced current due to a change of magnetic flux linked with a closed circuit is called electromagnetic induction.''

If the magnetic flux through a circuit changes, an emf and a current are induced in the circuit. EMI was discovered in 1830.The central principle of EMI is Faraday's law which relates induced emf to change in magnetic flux in any loop, including a closed circuit.

Magnetic flux: The flux associated with magnetic field is called magnetic flux.



$d\varphi = \vec{B}.\overrightarrow{dS}$

In magnitude form,

$$d\varphi = B\ dS\ \cos\theta$$

Here, $\overrightarrow{dS}$ is a vector perpendicular to the surface and has magnitude equal to area dS and θ is the angle between $\vec{B}\ and\ \overrightarrow{dS}$ at that element. The total magnetic flux through the surface is the sum of contribution from the individual area element.

$\varphi = \int \vec{B}.\overrightarrow{dS} = \int B\ dS\ \cos\theta$

The following points regarding magnetic flux:

1. The magnetic flux is a scalar quantity.
2. The SI unit of magnetic flux is T-m^2 or weber (Wb).
   1 Wb = 1 T-m^2 = 1 N-m/A
3. In the special case in which $\vec{B}$ is uniform over a plane surface with total area S,
   $$\varphi = BS\cos\theta$$
   If $\vec{B}$ is perpendicular to the surface, θ = 0^0 and $\varphi = BS$

Gauss's Law for Magnetism:

The surface integral of magnetic field over a closed surface is zero. Or, total magnetic flux through a closed surface is zero. That is,

$$\oint \vec{B}.\overrightarrow{dS} = 0$$

Faraday's law of electromagnetic induction:

Faraday gave two laws of EMI as:

1. When a flux of the magnetic induction through a circuit changes, an emf is induced in the circuit.

2. The magnitude of induced emf produced in a coil is equal to the negative rate of change of flux due the lines of force threading the coil. That is,

$$e = -\frac{d\varphi}{dt} ----(1)$$

This equation (1) is known as Faraday's law of induction and also known as Neumann's law. Neumann's law could be stated as '' when the magnetic flux linked with a coil (or circuit) is changed in any manner, then an emf is set up in the circuit such that the produced emf is proportional to the rate of change of the flux linkage with the circuit.''

The induced emf around the closed path is given by the line integral of the induced electric field E around the closed path.

$$e = \oint \vec{E}.\vec{dl}$$

So, $-\frac{d\varphi}{dt} = \oint \vec{E}.\vec{dl} ----(2)$

Hence, we can say that the electric fields associated with changing magnetic fields are non-conservative while the fields associated with stationary charges are conservative.

Also, $e = \oint \vec{E}.\vec{dl} = -\frac{d}{dt}\oint \vec{B}.\vec{dS} ----(3)$

Thus the Faraday's law can be stated as ,''The line integral of electric field intensity around any closed path is equal to the time rate in decrease of magnetic flux induction through the surface enclosed by that curve''.

If a circuit in coil contains N loops or turns, in the same area and if φ is the flux through one loop, an emf is induced in every loop, thus the total induced emf in the coil is given by

$$e = -N\frac{d\varphi}{dt}$$

Following points regarding Faraday's law:

1. Induced emf is produced only when there is a change in a magnetic flux passing through a loop. The flux passing through the loop is given by

$$\varphi = BS\cos\theta$$

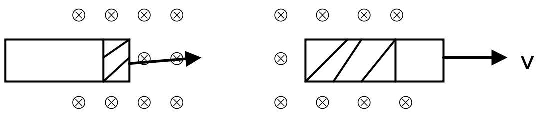
Thus, ***flux can be changed in several ways***:

a) The magnitude of $\vec{B}$ can change with time. If magnetic field is given a function of time, it implies that magnetic field is changing with time. Thus,

$$B = B(t)$$

b) The current producing in magnetic field can change with time. I = i(t)

c) The area enclosed by the loop can change with time. This can be done by pulling a loop inside (or outside) a magnetic field. By doing so, the area enclosed by loop (hatched) can be increased.



d) The angle θ between $\vec{B}$ and normal to the loop can change with time. This can be done by rotating a loop in a magnetic field.

2. When the magnetic flux passing through a loop is changed, an induced emf and hence, an induced current is produced in the circuit. If R is the resistance of the circuit, then induced current is given by,

$$i = \frac{e}{R} = \frac{1}{R}\left(\frac{-d\varphi}{dt}\right)$$

Current starts flowing in the circuit means flow of charge takes place. Charge flows in the circuit is given by

$$dq = idt = -\frac{d\varphi}{R}$$

So, for a time interval Δt,

$$e = -\frac{\Delta\varphi}{\Delta t}, i = \frac{1}{R}\left(-\frac{\Delta\varphi}{\Delta t}\right) \text{ and } \Delta q = \frac{1}{R}(\Delta\varphi)$$

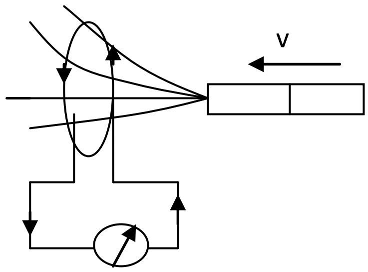
We can see that e and i are inversely proportional to Δt while Δq is independent of Δt.

Faraday's experiment:

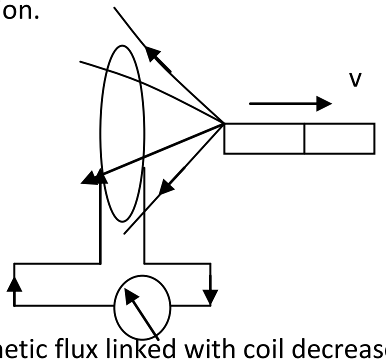
Experiment 1: Induced emf with a stationary coil and moving magnet

Taking a circular coil of thick insulated copper wire connected to a sensitive galvanometer.

i. When the north pole of a strong bar magnet is moved towards the coil, the galvanometer shows a deflection, say to the right of zero mark
ii. When the north pole of the bar magnet is moved away from the coil, the galvanometer shows a deflection in the opposite direction
iii. When south pole of the magnet is brought or away from the fixed coil, the direction of current is opposite to that in I and ii. Respectively.
iv. When the magnet is held stationary anywhere near or inside the coil, the galvanometer does not show any deflection.



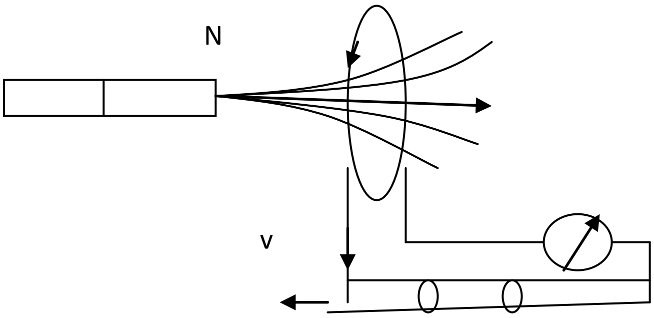
(Magnetic flux linked with coil increases)



(Magnetic flux linked with coil decreases)

Experiment 2: Induced emf with a stationary magnet and moving coil:

When the relative motion between the coil and the magnet is fast, the deflection in the galvanometer is large and when the relative motion is slow, the galvanometer deflection is small.



Lenz's law: A German physicist Heinrich Lenz gave a general law for determining the direction of induced emf and hence that of induced current in a circuit.

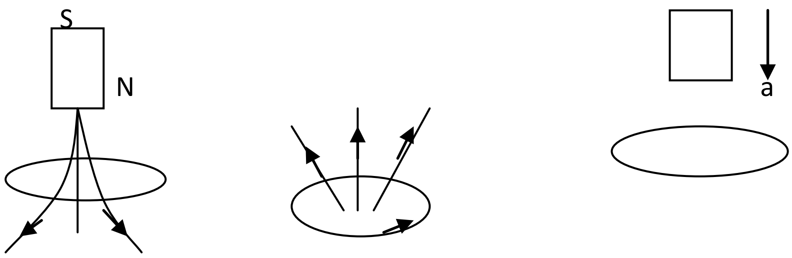
‘’It states that the direction of induced current in a circuit is such that it opposes the cause or the change which produces it.’’

It show that Lenz’s law is a consequence of the law of conservation of energy.

Whether a magnet is moved towards or away from a closed coil, the induced current always opposes the motion of the magnet. When a north pole of the magnet is brought closer to the coil, its face towards the magnet develops north polarity and thus repels north pole of the magnet. Work has to be done in moving the magnet closer to the coil against this force of repulsion. Similarly, when north pole the magnet is moved away from the coil, its face towards magnet develops south polarity and thus attracts north pole of the magnet. Here work has to be done in moving the magnet away from the coil against this force of attraction. It is this work done against the force of repulsion or attraction that appears as electric energy in the form of induced current.

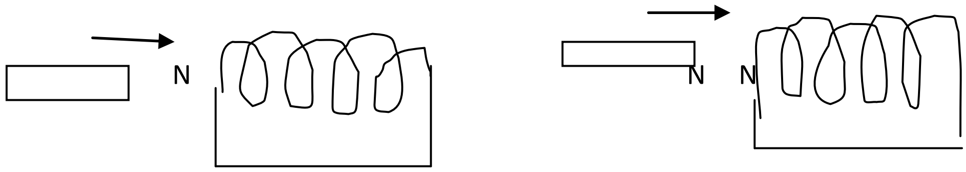
Suppose that Lenz’s law is not valid. Then induced current flows through the coil in a direction 6pposite to one dictated by Lenz’s law. The resulting force on the magnet makes it move faster and faster, i.e. magnet gains its speed and hence kinetic energy without expanding an equivalent amount of energy. This sets up a perpetual motion machine, violating law of conservation of energy. Thus, Lenz’s law is valid and is a consequence of the law of conservation of energy.

Q.1. A bar magnet is freely falling along the axis of a circular loop. State whether its acceleration a is equal to, greater than or less than the acceleration due to gravity.



Like poles repel each other and hence, a<g.

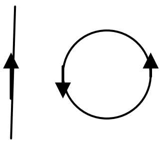
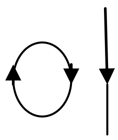
Q.2. A bar magnet is brought near a solenoid as shown. Will the solenoid attract or repel the magnet?



Repel each other

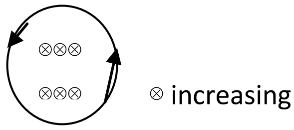
To apply Lenz’s law, it can be remembered RIN or ⊗ IN

RIN: R stands for right, I stands for increasing and N for north pole (anticlockwise). It means if a loop is placed on the right side of a straight current carrying conductor and the current I in the conductor is increasing, then induced current in the loop is anticlockwise.

I increasing   I decreasing

(Current in the loop is anticlockwise) (Current in the loop is clockwise)

⊗ IN: Suppose the magnetic field in the loop is perpendicular to paper inwards (⊗) and this field is increasing, then induced current in the loop is anticlockwise.

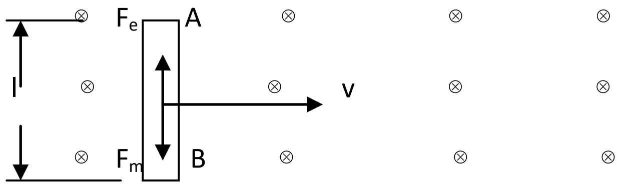


Motional Electromotive Force:

From Lorentz force:

We have studied an emf is induced in a stationary circuit placed in a magnetic field when the field changes with time.

A motional emf is the emf induced due to motion of a conductor in a constant magnetic field.



We assume that conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent. According to Fleming Left hand Rule, the magnetic force in the conductor acts in the direction AB {q is negative (e)}. The electrons in the conductor experiences a force

$$\vec{F}_m = -e(\vec{v}x\vec{B})$$

Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end. As a result of this charge separation, an electric field is produced inside the conductor (from negative to positive i. e. from lower to higher potential from B to A). The charge accumulates at both ends until the downward magnetic force is balanced by upward electric force. At this point, electrons stop moving. At the equilibrium condition, eE = evB ⇒E= vB

The potential difference across the ends of the conductor,

$$\Delta V = El = vBl$$

Where, upper end is at higher electric potential than the lower end.

A potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform electric field.

Suppose moving rod slides along a stationary U-shaped conductor, forming a complete circuit. No magnetic force acts on the charges in the stationary U-shaped conductor but there is an electric field caused by the charge accumulation at a and b.

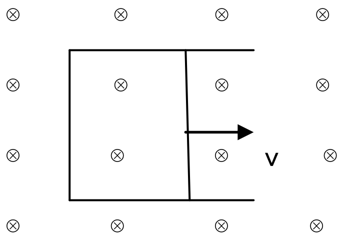


Fig. 1 (a)

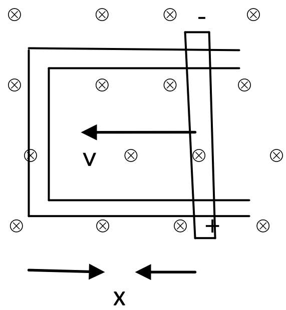


Fig. 1 (b)

Fleming Right hand Rule:

The direction of induced current can be known by Fleming Right Hand Rule, stated as: ''If we stretch thumb and two fingers of our right hand in mutually perpendicular direction and if the forefinger points in the direction of magnetic field, thumb in the direction of motion of the conductor; then the central fingers points in the direction of current induced in the conductor.''

Positive charge moves from lower to higher potential in the rod and in the remaining part, charge moves from higher to lower potential.

This $\Delta V = e = motional\ emf = vBl$

Second method: Motional EMF by Faraday's law of induction

$$e = -\frac{d\varphi}{dt} = -\frac{d}{dt}(BA) = -Bl\frac{dx}{dt} = -Blv - - - (1)$$

The negative sign shows that velocity v is in the decreasing direction of x as shown in Fig. 1 (b).

Current induced in the loop:

If R is the resistance of the circuit then current in the circuit is

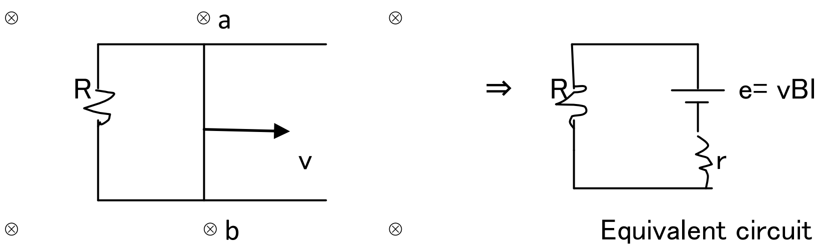
$$i = \frac{e}{R} = \frac{vBl}{R}$$

Force on the movable arm:

As shown in the Fig. 1 b, Force experienced in the movable conductor of length l carrying current I, is given by

$$F = Ilb\sin 90^0 = IlB = \frac{vBl}{R}(lB) = \frac{B^2l^2v}{R}$$

This force (due to induced current) acts in the outward direction (By Fleming Left hand Rule) opposite to the velocity of the arm in accordance with Lenz's law. Hence to move the arm with a constant velocity v, it should be pulled with a constant force F.



(From magnetic circuit to electric circuit)

The moving rod ab is replaced by a battery of emf vBl with the positive terminal at a and the negative terminal at b. The resistance r of the rod ab may be treated as the internal resistance of the battery. Hence, the current in the circuit is,

$$i = \frac{e}{R+r} = \frac{vBl}{R+r}$$

Power delivered by the external force:

$$P_{del} = Fv = \frac{B^2l^2v}{R}v = \frac{B^2l^2v^2}{R}$$

Power dissipated as Joule loss:

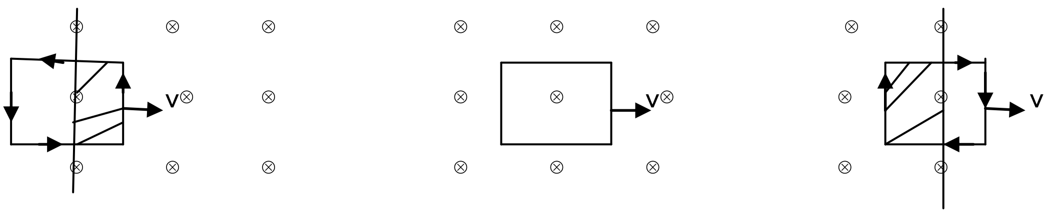
$$P_{dissipated} = i^2R = \frac{B^2l^2v^2}{R^2}R = \frac{B^2l^2v^2}{R}$$

This is just equal to the rate at which work is done by the applied force.

Note 1:

$e = -\frac{d\varphi}{dt}$ is best applied to the problem in which there is a changing flux through a closed loop while e= Blv is applied to problem in which conductor moves through a magnetic field.

Note 2: If conductor is moving in a magnetic field but circuit is not closed, then only P.D. will be asked between two points of the conductor. If the circuit is closed, then current will be asked in the circuit.



(Loop enters in increasing field) Loop enters into field) (Loop is coming out of decreasing field)

In the above figures, in first case, applying ⊗ IN rule, induced current in the loop will be anticlockwise (North). In the second case, induced current is zero (by Right hand rule) and the third case, induced current in the loop is clockwise (decreasing field south).

⊗ IN: means field increasing so induced current will be as north pole (anticlockwise).

⊙ IN: If field increasing, induced current in the loop will be clockwise

While in magnetic field, ⊗ (cross) means South Pole so magnetic field will be in clockwise and ⊙ (dot) means North Pole so magnetic field will be in anticlockwise direction as in case of solenoid.

Self Inductance and Inductors:

Self induced emf:

The induced emf in a circuit due to flux changes in its own circuit which in turn due to current changes in its own circuit is called self induced emf.

Self Inductance:

First definition- Total flux linked with the coil is directly proportional to current in the coil. That is,

$$N\varphi \propto i \text{ or } N\varphi = Li \Rightarrow L = \frac{N\varphi}{i}$$

So, it is total flux linkage per unit current. The SI unit of self- inductance is Henry (H).

Second definition-If a current i is passed in a circuit and it is changed with a rate di/dt, the linked emf produced in the circuit is directly proportional to rate of change of current. That is,

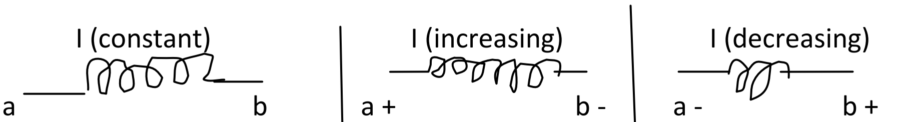
$$e \propto -\frac{di}{dt} \ or \ e = -L\frac{di}{dt} \ or \ L = \left|\frac{-e}{\frac{di}{dt}}\right|$$

Negative sign shows that self -inductance opposes any change in the current in the circuit. And it is defined as magnitude of self -induced emf per unit rate of change of current.

A circuit or a part of circuit that is designed to have a particular inductance is called an inductor.

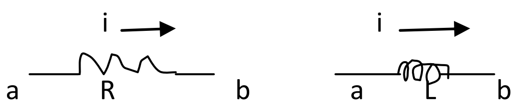
An inductor is a circuit element which opposes the change in current through it. It may be a circular coil, solenoid etc.

Significance of self- inductance and inductor: Like capacitors and resistors, inductors are circuit elements of modern electronics. Its purpose is to oppose any variation in the current through the circuit. In a DC circuit, an inductor helps to maintain a steady state current despite fluctuations in the applied emf.
In an AC circuit, an inductor tends to suppress variations of current that are more rapid than desired. It becomes active when current changes in the circuit. Every inductor has some self inductance which depends upon (i) size, (ii) shape (iii) number of turns and (iv) magnetic properties of material enclosed by the circuit. For N turns close together, it is always proportional to N^2.
Potential difference across an inductor:



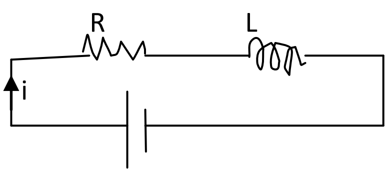
$\frac{di}{dt} = 0$, e =0, V_{ab} =0 | $\frac{di}{dt} > 0$, $e_{ab} < 0$, $V_{ab} > 0$ | $\frac{di}{dt} < 0$, $e_{ab} > 0$, $V_{ab} < 0$

Assume that inductor has negligible resistance, so that PD $V_{ab} = V_a - V_b$. Here, V_{ab} = -e = $L\frac{di}{dt}$
Difference between Resistor and inductor: A resistor opposes the current i while inductor opposes the change in current (di/dt).



$V_{ab} = iR,$ $\qquad$ $V_{ab} = L\frac{di}{dt}$

Kirchhoff's Second law with a inductor:



$E - iR - L\frac{di}{dt} = 0$ E

Methods of finding self inductance of a circuit:

1. Assume that current i flowing through the circuit
2. Determine the magnetic field B produced by the current
3. Obtain the magnetic flux
4. With the flux known, self inductance can be found from $L = \frac{N\varphi}{i}$

Inductance of a Solenoid:

Assume that a solenoid is uniformly wound and having N turns and length l. Also assume that l is much longer than radius of the windings and that the core of the solenoid is air.

We know that interior magnetic field due to a current I is uniform and given by equation

$$B = \mu_0 nI$$

Where n = no. of turns per unit length, let A be cross section area of the solenoid, so flux through turn is $\varphi = BA$

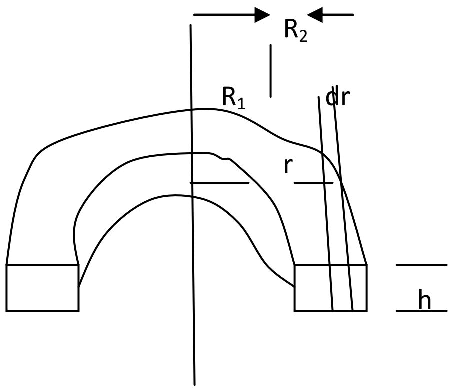
Now inductance of the solenoid is,

$$L = \frac{N\varphi}{I} = \frac{NBA}{I} = \frac{\mu_0 nINA}{l.I} = \frac{\mu_0 NINA}{I.l} = \frac{\mu_0 N^2 A}{l}$$

This shows that L depends upon as: $L \propto N^2, L \propto A \text{ and } L \propto \frac{1}{l}$

We can also express $L = \mu_0 N^2 V$, where V =Al is the volume of the solenoid.

Inductance of a rectangular toroid:



Let inner and outer radii are R_1 and R_2.

And h be the height of toroid.

The magnetic field inside a rectangular

Toroid is given by-

$$B = \frac{\mu_0 NI}{2\pi r}$$

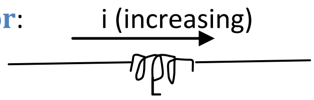
Using infinitesimal cross-sectional area element dS = h dr

$$\varphi = \int_{R_1}^{R_2} B\,dS = \int_{R_1}^{R_2} \left(\frac{\mu_0 NI}{2\pi r}\right)(hdr) = \frac{\mu_0 NIh}{2\pi} ln\frac{R_2}{R_1}$$

Therefore, self -inductance of a rectangular solenoid is given by

$$L = \frac{N\varphi}{I} = \frac{\mu_0 N^2 h}{2\pi} ln \frac{R_2}{R_1}$$

Energy stored in an inductor:



The energy of a capacitor is stored in electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field. An increasing current in an inductor causes an emf between its terminals. The work done per unit time is power.

$$P = \frac{dW}{dt} \Rightarrow dW = Pdt = eidt$$

Since, $e = -L\frac{di}{dt}$, So,

$$dW = -Li\frac{di}{dt}(dt) = -Lidi$$

Since, dW=-dU, so

$$dU = Lidi$$

The total energy U is supplied while the current increases from zero to a final value i is,

$$U = \int_0^i Lidi = \frac{1}{2}Li^2$$

Relation between energy density and magnetic field:

Assuming the case of an ideal long cylindrical solenoid, the energy stored in a solenoid when current i flows,

$$U = \frac{1}{2}Li^2$$

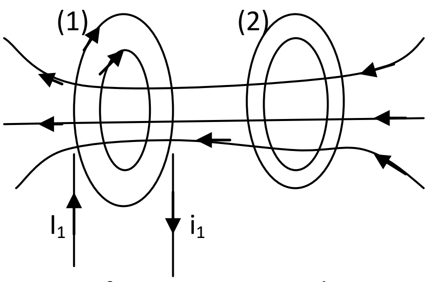
Since, L = $\mu_0 n^2 V$ and energy density

$$u = \frac{U}{V} = \frac{1}{2V}(\mu_0 n^2 V)i^2 = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2\mu_0}(\mu_0 ni)^2 = \frac{1}{2}\frac{B^2}{\mu_0}$$

This expression is similar to energy density in electric field as $u = \frac{1}{2}\epsilon_0 E^2$.

Mutual Inductance (M):

A change in current in one circuit can induce a current in a circuit, is called mutual induction.



First Definition: Suppose the circuit 1 has a current i_1 flowing in it. Then the total flux $N_2\, \phi_2$ linked with circuit 2 is proportional to the current in 1. Thus,

$$N_2\varphi_2 \propto I_1 \; or \; N_2\varphi_2 = MI_1$$

So, mutual inductance between two circuits is

$$M = \frac{N_2\varphi_2}{I_1}$$

Second definition: If we change the current in circuit 1 at the rate of di_1/dt, an induced emf e_2 is developed in circuit 2 which is proportional to the rate di_1/dt. Thus,

$$e_2 \propto \frac{di_1}{dt} \; or, e_2 = -M\frac{di_1}{dt}$$

So, magnitude of mutual inductance will be as-induced emf in circuit 2 per unit rate of change in current in circuit 1.

$$M = \left|\frac{-e_2}{\frac{di_1}{dt}}\right|$$

Note:

1. The SI unit of mutual inductance is henry (H)
2. M depends upon the closeness of the two circuits, their orientation and sizes and the number of turns etc.

Reciprocity theorem: $M_{21} = M_{12} = M$

$$e_2 = -M\frac{di_1}{dt} \; and \; e_1 = -M\frac{di_2}{dt}$$

$$M_{12} = \frac{N_2\varphi_2}{i_1} \; and \; M_{21} = \frac{N_1\varphi_1}{i_2}$$

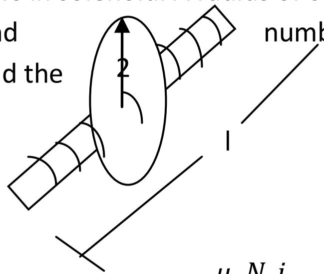
The following steps worth noting for calculating the mutual inductance of two circuits:

1. Assume any one of the circuits as primary (first) and the other as secondary (second).
2. Suppose a current i_1 flows through the primary circuit
3. Determine the magnetic field $\vec{B}$ produced by current i_1
4. Obtain the magnetic flux ϕ_2
5. With the flux known, mutual inductance can be found from,

$$M = \frac{N_2\varphi_2}{i_1}$$

<u>Mutual inductance of a solenoid surrounded by a coil:</u>

Assuming current i_1 flows in solenoid. A radius of coil R_2 and number of turns N_2 surrounds a solenoid of radius R_1 and number of turns N_1. There is no magnetic field outside the solenoid and the field inside has magnitude,



$$B = \frac{\mu_0 N_1 i_1}{l}$$

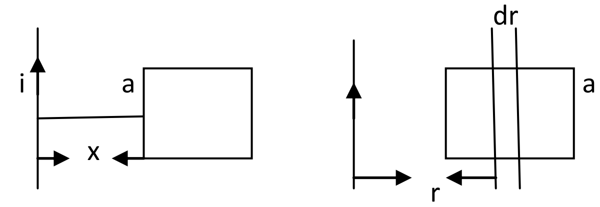
This field is directed parallel to the solenoid axis. The magnetic flux through the surrounding coil is,

$$\varphi_2 = BS_1 = B(\pi {R_1}^2) = \frac{\mu_0 N_1 i_1 \pi {R_1}^2}{l}$$

$$M = \frac{N_2 \varphi_2}{i_1} = \frac{N_2}{i_1}\left(\frac{\mu_0 N_1 i_1 \pi {R_1}^2}{l}\right) = \frac{\mu_0 N_1 N_2 \pi {R_1}^2}{l}$$

Note that M is independent upon radius R_2. This is because solenoid's magnetic field is confined to its interior.

N. B. : Obtain an expression for mutual inductance between a long straight wire and a square loop of side a as shown in the figure below.



The magnetic field at distance r due to a long conductor in which current I flows, is given as,

$$B = \frac{\mu_0 I}{2\pi r}$$

The flux linked in square coil due to element dr is

$$d\varphi = B(S) = B(adr)$$

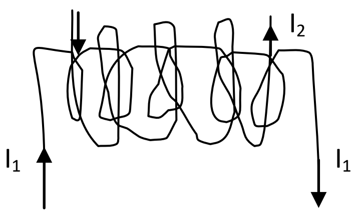
So, total flux linked with square coil is,

$$\varphi_2 = \int d\varphi = \int_{r=x}^{r=x+a} Badr = \int_{x}^{x+a} \frac{\mu_0 I a}{2\pi r} dr = \left(\frac{\mu_0 I a}{2\pi}\right) ln \frac{x+a}{x}$$

So, mutual inductance M is given by,

$$M = \frac{\varphi_2}{I} = \frac{\mu_0 a}{2\pi} ln\left(1 + \frac{a}{x}\right)$$

Mutual inductance of two long solenoids:



Let flux linked in solenoid 1 is$B_1 = \mu_0 n I_1$

Suppose two solenoids have same length l and their radii are << l.

Now, flux linkage in coil 2 is

$$N_2 \varphi_2 = N_2 B_1 A = N_2\, A(\mu_0 n_1 I_1) = N_2 A\left(\frac{\mu_0 N_1 I_1}{l}\right)$$

So, mutual inductance between the two is,

$$M = \frac{N_2\varphi_2}{I_1} = \frac{\mu_0 N_1 N_2 A}{l}$$

Factors on which mutual inductance depend upon:

1. Number of turns, $M \propto N_1 N_2$
2. Common cross sectional area
3. Relative separation, larger the distance between two solenoids, smaller will be magnetic flux linked and hence smaller will be mutual inductance.
4. Relative orientation of the two coils, M will be minimum when two coils are perpendicular to each other, M will be maximum when entire flux of primary is linked with the secondary.
5. Permeability of the core material: If two coils are wound over iron core, M increases μ_r times.

Coefficient of coupling:

We know that

$$M_{12} = \frac{N_2\varphi_2}{I_1}\ and\ M_{21} = \frac{N_1\varphi_1}{I_2}$$

Similarly,

$$L_1 = \frac{N_1\varphi_1}{I_1}\ and\ L_2 = \frac{N_2\varphi_2}{I_2}$$

If all the flux of coil 2 links with coil 1 and vice-versa then,

$\phi_2 = \phi_1$. Since $M_{12} = M_{21} = M$, hence we have,

$$M_{12}M_{21} = M^2 = \frac{N_2\varphi_2 N_1\varphi_1}{I_1 I_2} = \left(\frac{N_1\varphi_1}{I_1}\right)\left(\frac{N_2\varphi_2}{I_2}\right) = L_1 L_2$$

So, $M_{max} = \sqrt{L_1 L_2}$

This is the maximum possible value of M as total flux associated with coil 1 links with the coil 2. In general, there is a fraction (K_1) of ϕ_1 passes through coil 2 and a fraction (K_2) of ϕ_2 passes through coil 1. Hence,

$$\varphi_2 = K_1\varphi_1\ and\ \varphi_1 = K_2\varphi_2$$

So,

$$M_{12}M_{21} = M^2 = \left(\frac{N_2\varphi_2}{I_1}\right)\left(\frac{N_1\varphi_1}{I_2}\right) = \left(\frac{N_2 K_1\varphi_1}{I_1}\right)\left(\frac{N_1 K_2\varphi_2}{I_2}\right) = \left(\frac{N_1\varphi_1}{I_1}\right)\left(\frac{N_2\varphi_2}{I_2}\right)(K_1K_2)$$

Or,

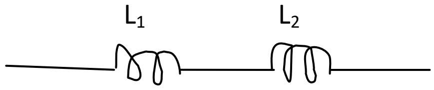
$$M^2 = (L_1L_2)(K_1K_2) = K^2 L_1 L_2 \Rightarrow M = K\sqrt{L_1L_2}$$

Where, $K = \sqrt{K_1K_2}$ = Coefficient of coupling and is a number depending on the geometry of the coils and their relative closeness, having value between 0 and 1 ($K \leq 1$).

Combination of Inductances:

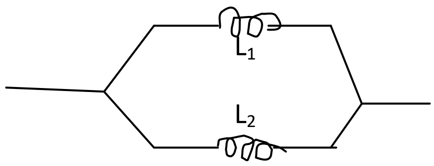
(i) If flux of one inductance does not links other (separation of coils are large, so that mutual inductances are negligible.

Series combination of inductances:



$$e = e_1 + e_2 \Rightarrow -L\frac{di}{dt} = -L_1\frac{di}{dt} - L_2\frac{di}{dt} \Rightarrow L = L_1 + L_2$$

Parallel combination of inductances:



$$e = -L_1\frac{di_1}{dt} \; or \; \frac{e}{L_1} = -\frac{di_1}{dt}$$

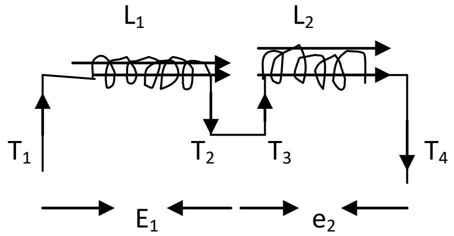
And, $\frac{e}{L_2} = -\frac{di_2}{dt}$ and $-\frac{di}{dt} = \frac{E}{L}$ hence,

$$-\frac{d}{dt}(i) = -\frac{d}{dt}(i_1 + i_2) = \frac{e}{L_1} + \frac{e}{L_2} = \frac{e}{L} \Rightarrow \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

(ii) When coils are placed very near to each other (flux of one coils links with other)

Obtain a formula for the equivalent inductance in the circuit-NCERT]

Fig. 1 a



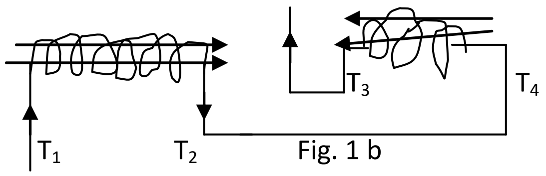


Fig. 1 b

(Inductances in series when fluxes get added) (Inductances in series when fluxes get subtracted)

a) Let the series connection be such that the current flows in the same sense in the two coils.

$$e_1 = -L_1\frac{di}{dt} - M\frac{di}{dt} \quad \text{and } e_2 = -L_2\frac{di}{dt} - M\frac{di}{dt} \text{ and } e_{eq} = -L_{eq}\frac{di}{dt} = e_1 + e_2$$

[EMF in two coils opposes the flux]

$$-L_{eq}\frac{di}{dt} = -L_1\frac{di}{dt} - L_2\frac{di}{dt} - 2M\frac{di}{dt} \Rightarrow L_{eq} = L_1 + L_2 + 2M$$

b) Let the series combination be such that flows in opposite senses in the two coils as shown in figure 1. B

$$e_1 = -L_1 \frac{di}{dt} + M \frac{di}{dt} \; and \; e_2 = -L_2 \frac{di}{dt} + M \frac{di}{dt}$$

$$e = e_1 + e_2 = -L_{eq} \frac{di}{dt} \Rightarrow L_{eq} = L_1 + L_2 - 2M$$

Describe some phenomena associated with self -induction:

1. ***Sparking***: The break of a circuit is very sudden. When the circuit is switched off, a large self induced emf is set up in the circuit in the same direction as the original emf. This causes a big spark across the switch.
2. Eddy current:

Currents can be induced not only in conducting coil but also in conducting sheets or blocks. Whenever the magnetic flux links with a metal sheets or blocks changes, an emf is induced in it. These currents look like eddies or whirl pools in water and so they are known as eddy currents. These currents were first discovered by Focault in 1895 so eddy currents are also known as Focault currents.

''Eddy currents are current induced in solid metallic masses when the magnetic flux threading through them changes.''

Eddy currents are produced inside the iron cores of the rotating armatures of electric motors and dynamos and also in cores of transformers, which experience flux changes, when they are in use.

Undesirable effects of eddy current:

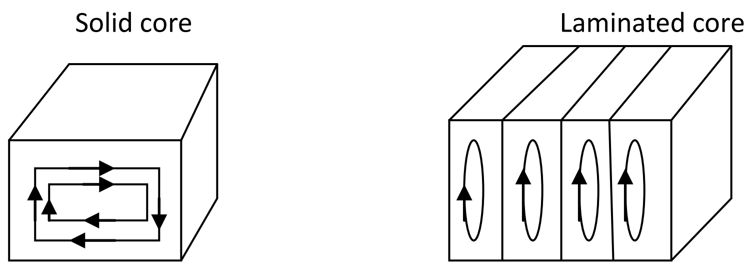
(i) Eddy currents cause unnecessary heating and wastage of power
(ii) Heat produced by eddy currents may even damage the insulation of the coils.

How eddy currents are reduced or minimised?

Answer: Eddy currents can be reduced by using laminated core which instead of a single solid mass consists of thin sheet of metal, insulated from each other by a thin layer of varnish. The planes of sheet are placed perpendicular to the direction of the current that would be set up by the emf induced in the material. The insulation between the sheets then offers high resistance to the induced emf and eddy currents are substantially reduced.

$$W_e = K f^2 t^2 B^2$$

Where, t = thickness, f= frequency, B=magnetic field induction

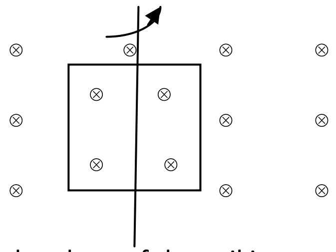


Applications of eddy currents: Although eddy currents are undesirable, still they find applications in the following devices.

1. **Induction furnace**: If a metal specimen is placed in a rapidly changing magnetic field (produced by high frequency AC), very large eddy currents are set up. The heat produced is sufficient to even melt the metal. This process is used in the extraction of some metals from the ores.
2. **Electromagnetic damping**: When a current is passed through a galvanometer, its coil suffers little oscillation before coming to rest in the final position. As the coil moves in the magnetic field, induced current is set up in the coil which opposes its motion. The oscillations of the coils are damped. This is called electromagnetic damping. The electromagnetic damping can be further increased by winding the coil on a light copper or aluminium frame. As the frame moves in magnetic field, eddy currents are set up in the frame which resist the motion of the coil. This is how a galvanometer rendered dead beat i. e. coil does not oscillate-It deflects and stays in the final position immediately.
3. **Electric brakes**: A strong magnetic field is applied to the rotating drum attached to the wheel. Eddy current sets up in the drum exert a torque on the drum so as to stop the train.
4. **Speedometer:** In a speedometer, a magnet rotates with the speed of the vehicle. The magnet is placed inside an aluminium drum which is carefully pivoted and held in position by a hair spring. As the magnet rotates, eddy currents are set up in the drum which opposes the motion of the magnet. A torque is exerted on the drum in the opposite direction which deflects the drum through an angle depending upon the speed of the vehicle.
5. Induction Motor
6. Electromagnetic shielding
7. **Inductothermy**: Eddy currents can be used to heat localised tissues of the human body. This branch is called Inductothermy.
8. **Energy meters**: Energy meters are used for measuring electric energy. The eddy currents induced in an aluminium disc are made use of.

Electric motor: An electric motor converts electrical energy into mechanical energy. It is based on the fact that current carrying coil in an uniform magnetic field experiences a torque. As the coil rotates in the magnetic field, the flux linked with the rotating coil will change and hence an emf, called back emf is produced in the coil. When the motor is first turned on, the coil is at rest and hence there is no back emf. The starting current can be quite large ($I_{starting}$ = 5-6 times I_{rated}). To reduce starting current, Starter is put in series with the motor for a short period when the motor is started. As the rotation rate increases, back emf increases and hence, current reduces.

Electric generator or dynamo: A dynamo converts mechanical energy (rotational KE) into electrical energy. It consists of a coil rotating in a magnetic field. Due to rotation of the coil, magnetic flux linked with it changes. So, an emf is induced in the coil.



Suppose at time t = 0, the plane of the coil is perpendicular to the magnetic field. The flux linked with it at any time t is

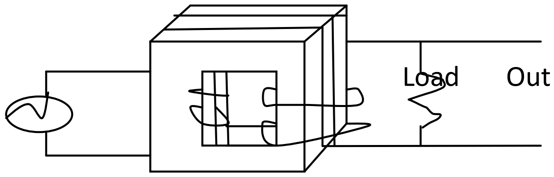
$$\varphi = BA\cos\theta = BA\cos\omega t$$

(Where, N = number of turns in the coil, ω is the angular speed) Therefore, emf induced is

$$e = -N\frac{d\varphi}{dt} = -N\frac{d}{dt}(BA\cos\omega t) = NBA\omega\sin\omega t = e_0\sin\omega t$$

Where, $e_0 = NBA\omega$

Transformer: It is a device (electrostatic) which is used to decrease (step down) or increase (step up) the voltage in ac circuit through mutual induction. A transformer consists of two coils wound over the same core.



The coil connected to input is called primary while the other connected to output is called secondary coil. An alternating current passing through the primary creates a continuously changing flux through the core. This changing flux induces an alternating emf in the secondary.

As magnetic lines of force are closed curves, the flux per turn of primary = flux per turn of secondary. Therefore,

$$\frac{\varphi_p}{N_p} = \frac{\varphi_s}{N_s} \Rightarrow \frac{1}{N_p}\frac{d\varphi_p}{dt} = \frac{1}{N_s}\frac{d\varphi_s}{dt} \Rightarrow \frac{e_p}{N_p} = \frac{e_s}{N_s} \Rightarrow \frac{e_s}{e_p} = \frac{N_s}{N_p}$$

(as $e \propto \frac{d\varphi}{dt}$)

In an ideal transformer, there is no loss of power. That is,

$$ei = constant$$

$$\frac{e_s}{e_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s}$$

Regarding a transformer, **followings are few important points**:

1. In step up transformer, $N_s > N_p$. It increases voltage and reduces current
2. In step down transformer, $N_p > N_s$. It reduces voltage and increase current
3. It works only on AC.
4. A transformer cannot increase or decrease voltage or current simultaneously as ei = constt.
5. Some power is always lost due to eddy current or hysteresis etc.

$$Output\ Power\ of\ transformer = Input\ Power\ x\ Efficiency\ x\ PF = VI\cos\varphi\,\eta$$

Some points regarding induced emf:

1. Induced emf is set up when magnetic flux link with the circuit changes even if the circuit is open. However, induced current only flows when circuit is closed.
2. No emf is induced when a coil and a magnet move with same velocity.
3. No emf is induced when a magnet is rotated about its own axis. EMF is induced when a magnet is rotated about an axis perpendicular to its length.
4. No emf is induced when a closed loop moves totally inside a uniform magnetic field.
5. Just as changing magnetic field produces an electric field, a changing electric field also sets up magnetic field.

In the equation, $E = E_{max} \sin \omega t$, the graph between E and t is a sine curve. Such an emf is a sinusoidal emf. Both the magnitude and direction of this emf change regularly with time.

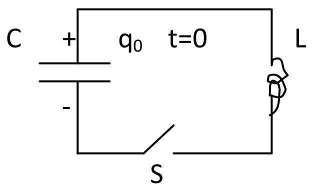
1. At θ = 0^0, π, 2 π, the plane of the coil is perpendicular to $\vec{B}$.
2. At θ = π/2, 3π/2, the plane of the coil is parallel to $\vec{B}$.

Oscillations in L-C circuit:

If a charged capacitor C is short circuited through an inductor L, the charge and current in the circuit start oscillating simple harmonically. If resistance of the circuit is zero, no energy is dissipated as heat. We assume an idealised situation in which energy is not radiated away from the circuit.

Idealisation as; zero resistance and no radiation, the oscillation in the circuit persists indefinitely and energy is transferred from the capacitor's electric field to the inductor magnetic field and back. The total energy associated with the circuit is constant.

Fig. 1 a



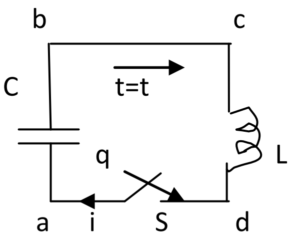


Fig. 1b

A capacitor is charged to a PD $V_0 = q_0/C$, here, q_0 is the maximum charge on the capacitor. At time t =0, it is connected to an inductor through a switch S. At time t = o, switch S is closed. (Fig. 1a)

In fig.1b, when switch S is closed, the capacitor starts discharging. Let at time t, charge on capacitor is q. ($q < q_0$). As charge is oscillating, there may be a situation when q will be increasing but in that case, the direction of current is reversed.

$$-V_{ab} - V_{cd} = 0 \Rightarrow V_b - V_a - V_c + V_d = 0 \Rightarrow V_b - V_a = V_c - V_d$$

$$\frac{q}{c} = L\frac{di}{dt}$$

Now, as the charge is decreasing, $i = -\frac{dq}{dt}$, so,

$\frac{q}{c} = -L\frac{d^2q}{dt^2}$ or, $\frac{d^2q}{dt^2} = -\frac{q}{LC} - - - (i)$. This is the same equation as $\frac{d^2x}{dt^2} = -\omega^2 x$ which is the equation of simple harmonic motion. Here,

$$\omega^2 = \frac{1}{LC}$$

The general solution for the equation (1) is,

$$q = q_0 \cos(\omega t \pm \varphi) - - - (2)$$

At t =0, q = q_0 and ϕ = 0, so, equation (2) becomes as

$$q = q_0 \cos \omega t - - - -(3)$$

So, we can say that charge in the circuit oscillates simple harmonically. Where,

$$\omega = \frac{1}{\sqrt{LC}} \; and \; T = 2\pi\sqrt{LC}$$

The oscillation of electric circuit is an electromagnetic analog to the mechanical oscillations of a block spring system.

Comparison of an oscillation of a mass spring system and an LC circuit:

Mass Spring system	L-C circuit
Displacement x	Charge q
Velocity v	Current i
Acceleration a	Rate of change of current di/dt
$\frac{d^2x}{dt^2} = -\omega^2 x$ where $\omega = \sqrt{\frac{K}{m}}$	$\frac{d^2q}{dt^2} = -\omega^2 q$ where $\omega = \frac{1}{\sqrt{LC}}$
$x = A\sin(\omega t \pm \varphi)\; or\; x = A\cos(\omega t \pm \varphi)$	$q = q_0 \sin(\omega t \pm \varphi)\; or\; q = q_0 \cos(\omega t \pm \varphi)$
$\frac{1}{K}$	Capacitance C
Mass m	Inductance L
$\lvert v_{max}\rvert = A\omega$	$i_{max} = q_0\omega$
Kinetic energy $\frac{1}{2}mv^2$	Magnetic energy $\frac{1}{2}Li^2$
Potential energy $\frac{1}{2}kx^2$	Potential energy $\frac{1}{2}\frac{q^2}{C}$

Differential Equation:

Order of differential equation: Order is the highest order derivative and degree is the power of highest order derivative. e. g. $\frac{dy}{dx}$ is the 1st order and 1st degree; $\left(\frac{dy}{dx}\right)^2$ is the 1st order and 2nd degree.

$\frac{d^2y}{dx^2}$ is the 2nd order and 1st degree. $\left(\frac{d^2y}{dx^2}\right)^2$ is the 2nd order and 2nd degree.

Second order and first degree differential equations:

Solve the differential equations as:

1. $\frac{d^2y}{dx^2} + 4y = 0$

   Solution: Let $m^0 = \frac{d^0y}{dx^0}, m^1 = \frac{dy}{dx}, m^2 = \frac{d^2y}{dx^2}$, then

   $m^2 + 4 = 0 \Rightarrow m = \pm\, \mathbf{2i}$

Note: If α_1 and α_2 are two roots and $\alpha_1 = \beta + i\mu$ and $\alpha_2 = \beta - i\mu$ and all other roots are real and unequal. The general solution will be,

$$y = e^{\beta x}(C_1 \cos \mu x + C_2 \sin \mu x)$$

Here β = 0 and μ = 2 therefore, general solution of the given equation in Q. 1 is,

$$y = C_1 \cos 2x + C_2 \sin 2x$$

Likewise, in the differential equation,

$\frac{d^2q}{dt^2} + \frac{q}{LC} = 0$, $m^2 + 1/LC = 0 \Rightarrow m = \pm j\sqrt{1/LC}$ and general solution is,

$$q = C_1 \cos \omega t + C_2 \sin \omega t = q_0 \cos(\omega t \pm \varphi) = q_0 \sin(\omega t \pm \varphi)$$

2. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$

$$m^2 + 3m + 2 = 0 \Rightarrow (m+1)(m+2) = 0 \Rightarrow m = -1, m = -2$$

Since all roots are real and unequal, so general solution will be as-

If m = α_1 and α_2 are roots of auxiliary equation, then general solution will be

$$y = C_1 e^{\alpha_1 x} + C_2 e^{\alpha_2 x}$$

So, here in this question, the general solution is,

$$y = C_1 e^{-x} + C_2 e^{-2x}$$

3. If $\alpha_1 = \alpha_2 = \alpha$, then general solution of the d.e. is,

$$y = (C_1 + C_2 x)e^{\alpha x}$$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 4y = 0$$

$$m^2 + 4m + 4 = 0 \Rightarrow (m+2)^2 = 0 \Rightarrow m = -2$$

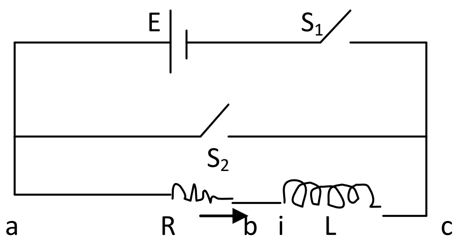
The general solution is,

$$y = (C_1 + C_2 x)e^{-2x}$$

Growth and decay of current in an L-R circuit:

When S_1 is closed,

<u>Growth:</u>



$$E - V_{ab} - V_{bc} = 0 \Rightarrow E - iR - L\frac{di}{dt} = 0 \Rightarrow \frac{di}{E - iR} = \frac{dt}{L} \Rightarrow \int_0^i \frac{di}{E - iR} = \int_o^t \frac{dt}{L}$$

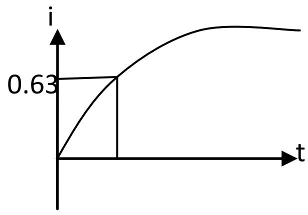
$$\frac{-1}{R} ln(E - iR) + \frac{1}{R} lnE = \frac{t}{L} \Rightarrow \frac{1}{R} ln\left(\frac{E - iR}{E}\right) = -\frac{t}{L} \Rightarrow i = i_0\left(1 - e^{\frac{-t}{\tau}}\right)$$

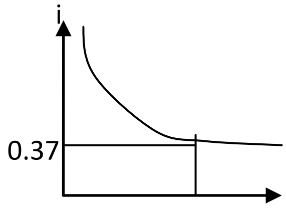
Where, $i_0 = \frac{E}{R}, \tau = \frac{L}{R}$

Decay:

When S_2 is closed,

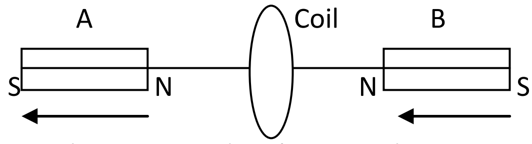
$V_{ab} + V_{bc} = 0 \Rightarrow iR + L\frac{di}{dt} = 0 \Rightarrow \frac{di}{i} = -\frac{R}{L}dt \Rightarrow \int_{i_0}^{i}\frac{di}{i} = \int_0^t -\frac{R}{L}dt \Rightarrow ln\frac{i}{i_0} = -\frac{Rt}{L} \Rightarrow i = i_0 e^{\frac{-t}{\tau}}$





To find induced emf and current in the coil in various arrangements:

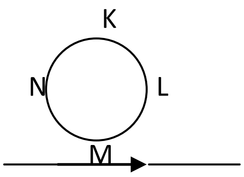
1. [CBSE D 99]



[Ans. zero]

A and B are moved in the same direction with same speed. Find the induced emf in the coil.

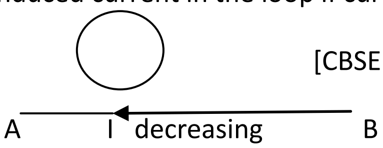
2. Find magnitude of induced current in the circular loop KLMN of radius r if the straight wire PQ carries a steady current of magnitude i.



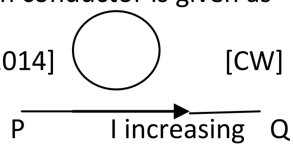
[Ans. zero]

3. Find the induced current in the loop if current in conductor is given as-

[Ans. CW]

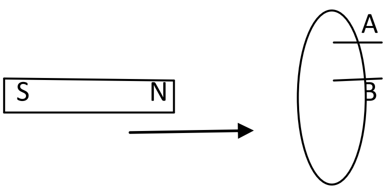


[CBSE OD 2014]



[CW]

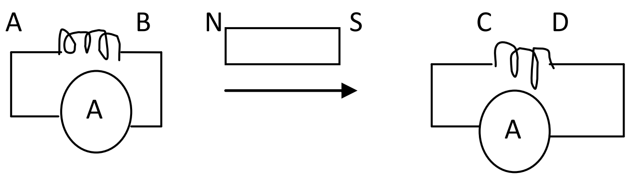
4. Predict the polarity of the plate A of the capacitor, when the magnet is moved towards it.



[CBSE OD 14 C, 15 C]

[Ans. A is positive]

5. A magnet is moved in the direction indicated by arrow between two coils AB and CD as shown in the figure. Suggest the direction of current in each coil.



[CBSE OD 01 C, 12]

[Answer CW]

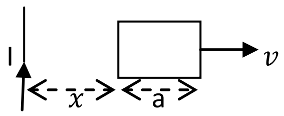
Derivation and general questions asked in CBSE:

1. State the law that gives the polarity of induced emf.[CBSE 2009]
2. A rod of length l is moved horizontally with a uniform velocity v in a direction perpendicular to its length through a region in which a uniform magnetic field is acting vertically downward. Derive the expression for the emf induced across the ends of the rod. How does one understand this motional emf by invoking the Lorentz force acting on the free charge carriers of the conductor? [CBSE 2014]
3. The current flowing through an inductor of self inductance L is continuously increasing. Plot a graph showing the variation of (i) magnetic flux versus current, (ii) induced emf versus $\frac{di}{dt}$, (iii) magnetic potential energy stored versus the current. [CBSE 2014]
4. Define self inductance. Write its SI unit. Derive an expression for self inductance of a long solenoid of length l, cross sectional area A having N number of turns. [CBSE 2009]
5. Define mutual inductance. Write its SI unit. Derive an expression for the mutual inductance of two long co-axial solenoids of same length wound over the other. [CBSE 2015, 2012, 2009, 2008]
6. Two concentric circular coils C_1 and C_2 of radius R_1 and R_2 ($R_1 << R_2$) respectively are placed co-axially. If the current is passed through C_2, then find an expression for mutual inductance between the two coils. [CBSE 2009, 2010, 2011]

$$M = \frac{N_1\varphi_1}{i_2} = \frac{N_1A_1B_2}{i_2} = \left(\frac{N_1\pi {R_1}^2}{i_2}\right)\left(\frac{\mu_0N_2i_2}{2R_2}\right) = \frac{\mu_0\pi {R_1}^2N_1N_2}{2R_2}$$

(Note: No current flows in coil C_1, so no magnetic field in C_1. Only the flux produced in C_2 is linked with C_1 due to mutual induction.)

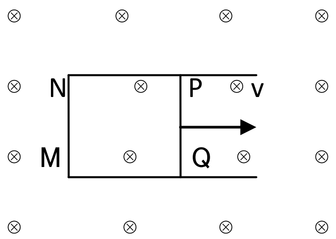
7. A source of emf e is used to establish a current I through a coil of self inductance L. Show that the work done by the source of build up the current I is ½ LI^2. [CBSE 2010, 2014]
8. Obtain the expression of energy density as $\frac{B^2}{2\mu_0}$. [CBSE 2013]
9. Derive the expression for maximum value of induced emf and state the rule that gives the direction of induced emf. [Delhi 2014 C]
10. (i) How can induced emf be increased? State it. (ii) Define Faraday's law of electromagnetic induction and give its two example to know the direction of induced emf. Define also Lenz/s law. (iii) What is the difference between induced emf and motional emf?
11. (a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown below. [NCERT 6.16][$M = \frac{\mu_0 a}{2\pi}\ln\left(1+\frac{a}{x}\right)$]



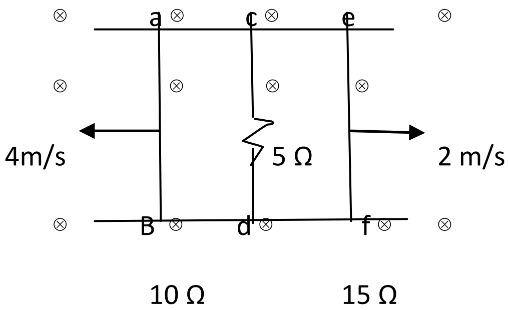
(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, v= 10 m/s. Calculate the induced emf in the loop when x = 0.2 m. Take a = 0.1 m and assume that the loop has a large resistance. [Ans. 1.7 x 10^{-5} V]

Numerical asked in CBSE, NCERT and others:

1. Current in circuit falls steadily from 2.0 A to 0.0 A in 10 ms. If an average emf of 200 V is induced, then calculate the self inductance of the circuit.[Ans. 1 H] [CBSE 2011]
2. An average induced emf of 0.4 V appears in a coil when the current in it is changed from 10 A in one direction to 10 A in opposite direction in 0.40 s. Find the coefficient of self induction of the coil. [Ans. 8 mH] [CBSE Sample paper 11]
3. What is the self inductance of a coil in which magnetic flux of 40 m Wb is produced when 2 A current flows through it. [Ans. 2 x 10^{-2} H] [CBSE F 02]
4. If the rate of change of current of 4 A/s induces an emf of 20 mV in a solenoid, what is the self inductance of the solenoid? [Ans. 5 mH] [CBSE D 96]
5. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?[NCERT Q. 6.3] [Ans. 7.5 x 10^{-6} V]
6. A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 ms^{-1} at right angles to the horizontal component of earth's magnetic field, 0.30 Wb m^{-2}. (a) what is the instantaneous value of the emf induced in the wire?[Ans. 1.5 mV], (ii) What is the direction of the emf? [From West to East], (iii) which end of the wire is at higher electric potential? [East side] [NCERT 6.7]
7. Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self inductance of the circuit. [Ans. 4 H][NCERT 6.8]
8. A pair of the adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage in other coil? [NCERT 6.9] [Ans. 30 Wb]
9. A jet plane is travelling towards west at a speed of 1800 km/hr. What is the voltage difference developed between the ends of the wing having a span of 25 m. If the earth's magnetic field at the location has a magnitude of 5 x 10^{-4} T and the dip angle is 30^0.[NCERT 6.10][Ans. 3.125 V]
10. An air cored solenoid with length 30 cm, area of cross section 25 cm^2 and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3}s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in mag. field near the ends of the solenoid. [NCERT 6.15] [Ans. 6.5 V]
11. A rectangular loop of area 20 cm x 30 cm is placed in a magnetic field of 0.3 T with its plane (i) normal to the field, (ii) inclined 30^0 to the field and (iii) parallel to the field. Find the flux linked with the coil in each case.[Ans. (i) 1.8 x 10^{-2} Wb, (ii) 0.9 x 10^{-2} Wb, (iii) zero]
12. Find the magnetic flux linked with a rectangular coil of size 6 cm x 8 cm placed at right angle to a magnetic field of 0.5 Wb m^{-2}. [Ans. 2.4 x 10^{-3} Wb]
13. A 70 turn coil with average diameter of 0.02 m is placed perpendicular to the magnetic field of 9000 T. If the magnetic field is changed to 6000 T in 3 s, what is the magnitude of induced emf? [Ans. 22 V] [CBSE F 94 C]
14. A rectangular loop PQMN with movable arm PQ of length 10 cm and resistance 2 Ω is placed in a uniform magnetic field of 0.1 T acting perpendicular to the plane of the loop as shown in the figure below. The resistances of the arm MN, NP and MQ are negligible. Calculate the (i) emf induced in the arm PQ and (ii) current induced in the loop when arm PQ is moved with velocity 20 ms^{-1}. [Ans. 0.2 V, 0.1 A] [CBSE D 14 C]

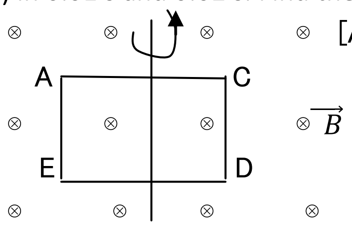


15. Two parallel rails with negligible resistance are 10.0 cm apart. They are connected by a 5.0 Ω resistor. The circuit also contains two metal rods having resistances of 10.0 Ω and 15.0 Ω along the rails. The rods are pulled away from the resistor at constant speed of 4.0 ms^{-1} and 2.0 ms^{-1} respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of rails. Determine the current in the 5.0 Ω resistor. { 8/55 mA from d to c]

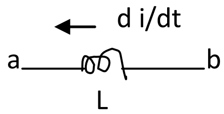


16. A square loop ACDE of area 20 cm^2 and resistance 5 Ω is rotated in a magnetic field $\vec{B}$=2T through 180^0 (a) in 0.01 s and 0.02 s. Find the magnitude of e, i and Δq in both cases.

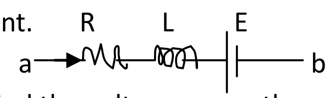
[Ans. $\Delta q = 1.6x10^{-3}$ C, e=0.8 V, i =0/16 A]



17. The inductor shown in figure has inductance 0.54 H and carries a current in the direction shown that is decreasing at a uniform rate di/dt = - 0.03 A/s.
    a) Find the self induced emf [1.62 x 10^{-2} V)
    b) Which end of the inductor a or b is at higher potential?

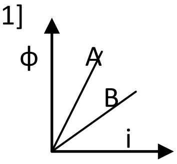


18. In the circuit diagram shown in figure, R = 10 Ω, L = 5 H, E = 20 V, I = 2 A. This current is decreasing at a rate of – 1 A/s. Find V_{ab} at this instant. [Ans. 35 V]



19. The current through an inductor of 1 H is given by $i = 3\ t\ \sin t$. Find the voltage across the inductor. [Ans. $3\ (t\ \cos t + \sin t)$]
20. Calculate inductance of an air core solenoid containing 300 turns if the length of the solenoid is 25 cm and cross sectional area is 4 cm^2. Also calculate the self induced emf in the solenoid if the current through it is decreasing at the rate of 50 A/s. [Ans. 1.81 x 10^{-4} H, 9.05 mV]
21. What inductance would be needed to store 1 kWh of energy in a coil carrying a 200 A current? [Ans. 180 H]

22. Calculate the induced emf in a coil of 10 H inductance in which the current changes from 8 A to 3 A in 0.2 s. [Ans. 250 V]
23. Calculate the mutual inductance between two coils when a current of 2 A changes to 6 A in 2 s and induces an emf of 20 mV in the secondary coil. [Ans. 10 mH] [Punjab, 99]
24. If the current in the primary circuit of a pair of coils changes from 10 A to 0 in 0.1 s, calculate
    a) The induced emf in the secondary if the mutual inductance between the two coils is 2 H and [Ans. 200 V]
    b) The change of flux per turn in the secondary if it has 500 turns.[Ans. -0.04 Wb]
25. A solenoid of length 50 cm with 20 turns per cm and area of cross section 40 cm^2 completely surrounds another co-axial solenoid of the same length, area of cross section 25 cm^2 with 25 turns per cm. Calculate the mutual inductance of the system. [Ans. 7.85 mH] [NCERT]
26. A toroidal solenoid with an air core has an average radius of 15 cm, area of cross section of 12 cm^2 and 1200 turns. Obtain the self inductance of the toroid. (ignore field variation across the cross section of the toroid).
27. Two coils with a coefficient of coupling of 0.5 between them, are connected in series so as to magnetise (a) in the same direction (b) in the opposite direction. The corresponding values of total inductances are for (a) 1.9 H and for (b) 0.7 H. Find the self inductances of the two coils and the mutual inductance between them. [Ans. L_1 = 0.9 H, L_2 =0.4 H, M =0.3 H]
28. The current flowing in the two coils of self inductance L_1 = 16 mH and L_2 = 12 mH are increasing at the same rate. If the power supplied to the two coils are equal, find the ratio of
    a) Induced voltages [Ans. 4:3]
    b) The currents and [Ans. 3:4]
    c) The energies stored in the coil at a given instant [Foreign 2014] [Ans. 3:4]
29. Current in a circuit falls steadily from 2.0 A to 0.0 Ain 10 ms. If an average emf of 200 V is induced, then calculate the self inductance of the current. [Ans. 1 H] [CBSE F 2011]
30. What are eddy currents? Write their two applications. [All India 2012]
31. A plot of magnetic flux Vs. current is shown below.
    Which of the two has larger value of self inductance?



32. A 2.00 H inductor carries a steady current of 0.500 A. When the switch is the circuit is opened, the current is effectively zero after 10.0 ms. What is the average induced emf in the inductor during this time interval? [Ans. 100 V]
33. An emf of 25.0 mV is induced in a 500-turn coil when the current is changing at the rate of 10.0 A/s. What is the magnetic flux through each turn of the coil at an instant when the current is 4.00 A? [20 µT m^2]
34. The current in a coil changes from 3.50 A to 2.00 A in the same direction in 0.500 s. If the average emf induced in the coil is 12 mV, what is the inductance of the coil?[Ans. 4.00 mH]
35. An emf of 96.0 mV is induced in the windings of a coil when the current in a nearby coil is increasing at the rate of 1.20 A/s. Find the mutual inductance of the two coils. [Ans. 80 mH]
36. A solenoid of radius 2.50 cm has 400 turns and a length of 20.0 cm. The current in the coil is changing with time such that to produce an emf of 75.0 mV. Calculate the (i) inductance (L) of the solenoid. [Ans. 2 mH], (ii) rate at which current must change through the solenoid. [Ans. 38.0 mA/s]