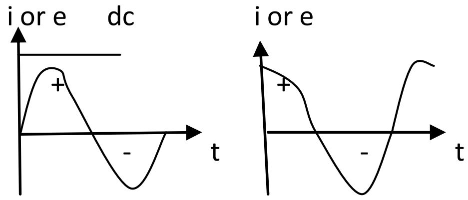
Ch-7: Alternating Current

Thomas Edison favoured direct current (dc), that is , steady current that does not vary with time. George Westinghouse favoured alternating current (ac), with sinusoidally varying voltages and currents. He argued that transformers can be used to step up or step down the voltages with ac but not with dc. Low voltages are safer for consumer use, but high voltages and correspondingly low currents are best for long distances power transmission to minimise I^2R losses in the cables. Eventually, Westinghouse prevailed, and most present day household and industrial power distribution systems operate with alternating currents.

A time varying voltage may be periodic or non-periodic.



If the current or voltage vary periodically as sine or cosine function of time, the current or voltage is said to be sinusoidal.

Alternating current (Average Vale, RMS value and Form Factor):

The basic principle of ac generator is is direct consequence of Faraday's law of induction. When a conducting loop is rotated in a magnetic field at constant angular frequency ω, a sinusoidal voltage is induced in the loop. This instantaneous voltage is,

$$e = E_o \sin \omega t$$

Where, E_0 is the maximum output voltage of ac generator or the voltage amplitude. Suppose this emf is applied to a circuit of resistance R, then,

$$i = \frac{e}{R} = I_0 \sin \omega t$$

Where I = instantaneous value of a.c. at any instant t. $I_0 = \frac{E_0}{R} = peak\ or\ maximum\ value\ of\ ac$ and is called current amplitude (the maximum value attained by an alternating current or voltage in either direction).

Time period (T): It is ratio of angular displacement in a complete cycle to the angular velocity. That is, the time of one cycle is called time period (T).

$$T = \frac{2\pi}{\omega}\ or\ \omega = 2\pi f$$

Where, f = angular frequency

The frequency of ac in India is 50 Hz so, ω=2π f = 314 rad/s

Show that average value of alternating current or voltage over one complete cycle is zero.

If an alternating current is passed through an ordinary ammeter or voltmeter, it will record the mean value for the complete cycle, as the quantity to be measured varies with time. The average value of current for one cycle is,

$$\langle I \rangle = (I_{av})_T = \frac{\int_0^T idt}{\int_0^T dt} = \frac{\int_0^{\frac{2\pi}{\omega}} I_0 \sin \omega t\, dt}{\int_0^{\frac{2\pi}{\omega}} dt} = \frac{\frac{-I_0}{\omega}(\cos \omega t)_0^{\frac{2\pi}{\omega}}}{(t)_0^{\frac{2\pi}{\omega}}} = \frac{\frac{-I_0}{\omega}(\cos 2\pi - \cos 0)}{\frac{2\pi}{\omega}} = 0$$

Thus, $\langle I \rangle_{one\ cycle} = 0$

Similarly, average value of voltage or emf over one cycle is zero. That is,

$$\langle V \rangle_{one\ cycle} = 0$$

Mean or average value of ac:

These averages over one complete cycle is zero. The dc instrument will indicate zero deflection. In ac, the average value of current is defined as its average value taken over half cycle. Hence,

$$\langle i\rangle_{half\ cycle} = (I_{av})_{\frac{T}{2}} = \frac{\int_0^{\frac{T}{2}} i\,dt}{\int_0^{\frac{T}{2}} dt} = \frac{\int_0^{\frac{T}{2}} I_0 \sin\omega t}{\int_0^{\frac{T}{2}} dt} = \frac{I_0 \int_0^{\frac{\pi}{\omega}} \sin\omega t}{\int_0^{\frac{\pi}{\omega}} dt} = \frac{\frac{-I_0}{\omega}(\cos\omega t)_0^{\frac{\pi}{\omega}}}{(t)_0^{\frac{\pi}{\omega}}} = \frac{2I_0}{\pi} = 0.637 I_0$$

$$= I_{av}$$

Similarly, $V_{av}\ or\ E_{av} = \frac{2}{\pi}V_0 = 0.637V_0$ $\quad [\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} \Rightarrow \frac{T}{2} = \frac{\pi}{\omega}]$

A DC meter can be used in an ac circuit if it is connected in the full wave rectifier circuit. The average value of the rectified current is the same as the average current in any half cycle, i. e. 2/π times the maximum current.

Root Mean Square (RMS) or Virtual or Effective value of A. C.:

''Squaring the instantaneous value (i^2), take the mean (average) value of i^2 and finally take the square root of that average'':

$$\langle i^2\rangle_{One\ cycle} = \frac{\int_0^T i^2 dt}{\int_0^T dt} = \frac{\int_0^T (I_0 \sin\omega t)^2 dt}{\int_0^T dt} = \frac{\frac{{I_0}^2}{2}\int_0^{\frac{2\pi}{\omega}}(1-\cos 2\omega t)dt}{(t)_0^{\frac{2\pi}{\omega}}} = \frac{\frac{{I_0}^2}{2}\left[t - \frac{1}{2\omega}\sin 2\omega t\right]_0^{\frac{2\pi}{\omega}}}{(t)_0^{\frac{2\pi}{\omega}}}$$

$$= \frac{{I_0}^2}{2\left(\frac{2\pi}{\omega}\right)}\left[\frac{2\pi}{\omega} - 0\right] = \frac{{I_0}^2}{2}$$

Hence,

$$I_{rms} = \sqrt{\frac{{I_0}^2}{2}} = \frac{I_0}{\sqrt{2}}$$

Form factor: $= \frac{RMS\ value\ of\ current\ or\ voltage}{Average\ value} = \frac{\frac{V_0}{\sqrt{2}}}{\frac{2V_0}{\pi}} = 1.11$

1. The average value of sin ωt, cos ωt, sin 2ωt, cos 2ωt is zero because it is positive for half of the time and negative for rest half of the time.
2. The average value of $\sin^2\omega t$, $\cos^2\omega t$ is ½.
3. Like SHM, general expressions of current, voltage in an sinusoidal ac are-

$$i = I_0 \sin(\omega t \pm \varphi), V = V_0 \sin(\omega t \pm \varphi), i = I_0 \cos(\omega t \pm \varphi), V = V_0 \cos(\omega t \pm \varphi)$$

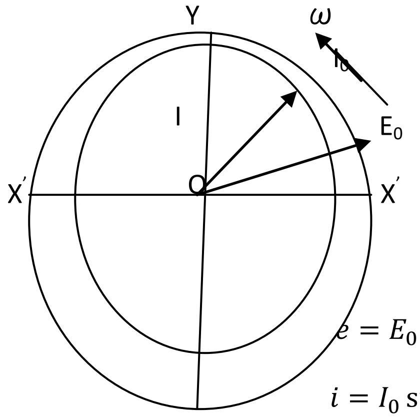
4. Think whether average value of current or voltage over half cycle is zero (this can also be zero).

Phasors: If an ac generator is connected to a series circuit containing resistors, inductors and capacitors, we would like to know the amplitude and time characteristics of the alternating current. To simplify analysis of circuits containing two or more elements, we use graphical constructions called phasor diagram. In these constructions, alternating quantities such as current and voltage are rotating vectors, called phasors.

When projection on to a vertical axis, it is sine function and when projection onto a horizontal axis, it is a cosine function. The vector rotates counter clockwise with constant angular velocity ω.

Phasor diagram:

A diagram that represents alternating current and voltage of the same frequency as rotating vectors (phasors) alongwith proper phase angle between them is called a phasor diagram or Argand diagram.



Here current i leads e by phase angle ϕ.

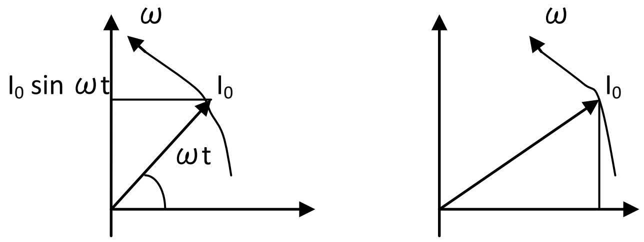
$$e = E_0 \sin \omega t, i = I_0 \sin(\omega t + \varphi)$$

If current i lags e by phase angle ϕ, then

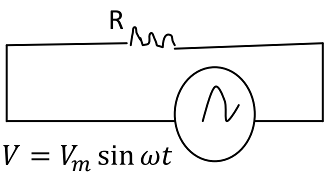
$e = E_0 \sin \omega t$ and

$$i = I_0 \sin(\omega t - \varphi)$$

A phasor is not a real physical quantity with a direction in space such as velocity, momentum or electric field. Rather it is a geometric entity that helps us to describe and analyze physical quantities that vary sinusoidally with time.



Purely Resistive Circuit: A circuit contains only a resistor-



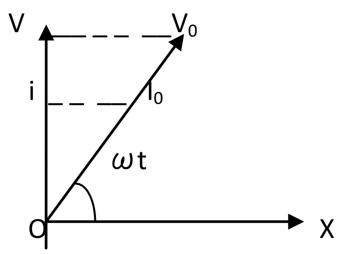
$V = V_m \sin \omega t$

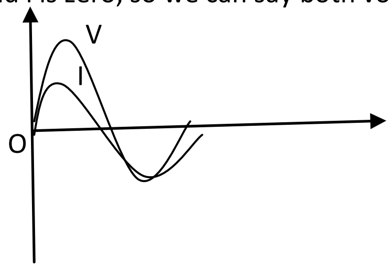
Instantaneous Voltage of source = Instantaneous P.D. across R

$$V_m \sin \omega t = iR \Rightarrow i = \frac{V_m}{R} \sin \omega t \Rightarrow i = I_m \sin \omega t$$

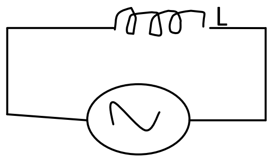
Where, V_m = maximum or peak voltage, I_m = Maximum or peak current

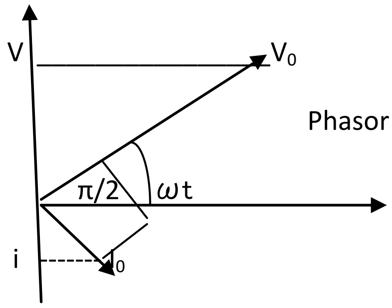
Here phase difference between V and i is zero, so we can say both voltage and current are in same phase. Y





Purely Inductive Circuit:





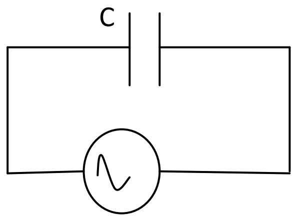
Phasor diagram

$$V = V_m \sin \omega t$$

$$i = \frac{1}{L}\int V\,dt = \frac{1}{L}\int V_m \sin \omega t\,dt = -\frac{V_m \cos \omega t}{\omega L} = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Here, V leads i by 90^0 or i lags V by 90^0. Also, $X_L = \omega L = Inductive\ Reactance$

Purely Capacitive circuit:

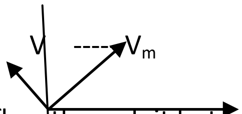


$$V = V_m \sin \omega t$$

$$i = C\frac{dV}{dt} = CV_m\frac{d}{dt}(\sin \omega t) = V_m C\omega \cos \omega t = \frac{V_m}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)$$

Here, $X_c = \frac{1}{\omega C} = Capacitive\ Reactance$

We see that I leads V by 90^0.



For, AC, $X_C \propto \frac{1}{f}$, for dc, f = 0, so, $X_C = \infty$ That is, capacitor allows ac to flow through it but for dc, it offers infinite resistance to flow of dc , that is it blocks dc.

Variation of capacitive reactance with frequency-

Phasor Algebra vectors.: The complex quantities normally employed in AC circuit analysis, can be added or subtracted like coplanar vectors. Such coplanar vectors which represent sinusoidally time varying quantities are known as Phasors.

In Cartesian form, phasor A can be written as,

$$\vec{A} = a + jb$$

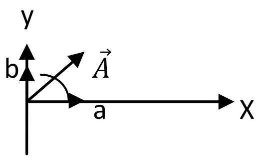
Where, a is the x-component and b is the y-component of phasor $\vec{A}$. The magnitude of $\vec{A}$ is,

$$|\vec{A}| = \sqrt{a^2 + b^2}$$

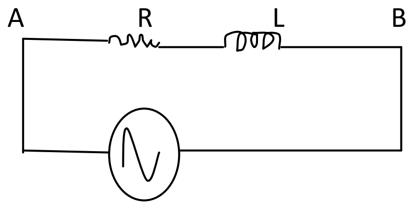
The angle between direction of phasor $\vec{A}$ and positive x-axis is,

$\theta = tan^{-1}\frac{b}{a}$

Here, $j = \sqrt{-1}$



Series R-L Circuit:

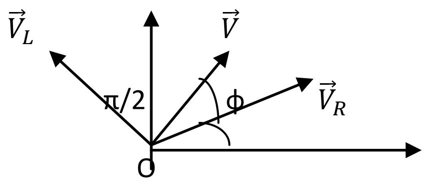


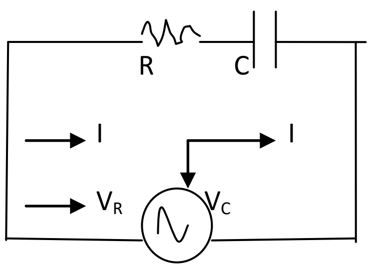
Let $i = I_0 \sin \omega t$, Also the current i lags the applied voltage by an angle ϕ, so
The PD across resistance in ac is in- phase and in inductance; it leads by π/2 with current.

$$\vec{V} = \vec{V}_R + \vec{V}_L = iR + j(iX_L) = i(R + j\omega L) = i\vec{Z}$$

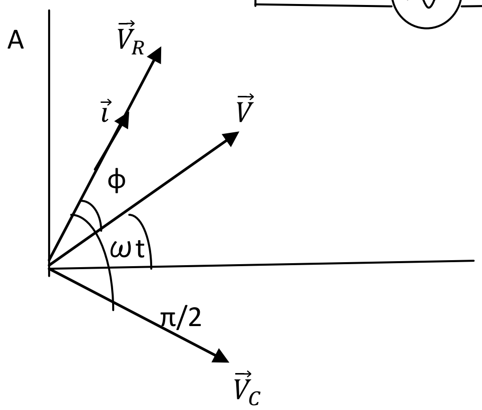
Where, Z = Impedance = $R + j\omega L$
The impedance plays the same role in ac circuit as resistance plays in dc circuit.
Note: The reciprocal of Impedance is Admittance. **The reciprocal of Reactance is Susceptance.**

The unit of admittance or susceptance is Siemen or mho or Ω^{-1}. $Z = \sqrt{R^2 + {\omega_0}^2 L^2}$
The impedance Z is $Z = \sqrt{R^2 + \omega^2 L^2}$
The P. D. Leads the current by an angle ϕ given as $\varphi = tan^{-1}\frac{V_L}{V_R} = tan^{-1}\frac{X_L}{R} = tan^{-1}\frac{\omega L}{R}$
Here, V_L leads V_R by π/2. So, it can be written as , $i = I_0 \sin(\omega t - \varphi)$





R-C Circuit



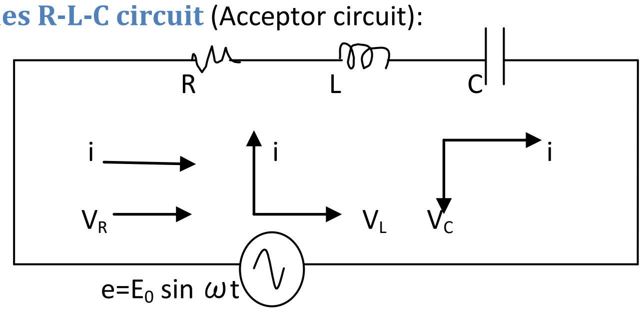
$$\vec{V} = \vec{V}_R + \vec{V}_C = i(R - jX_C) = i\vec{Z}$$

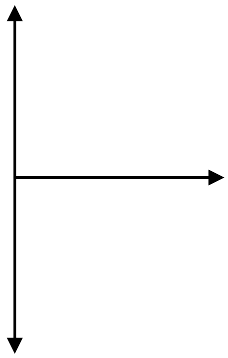
Here, $= \sqrt{R^2 + {X_C}^2}$, $X_C = \frac{1}{j\omega C}$

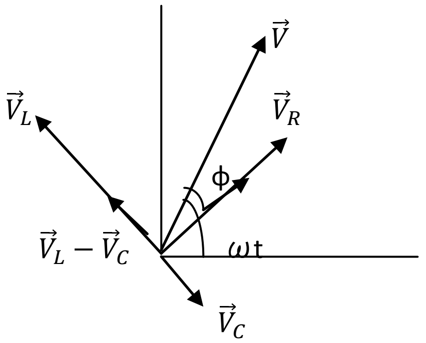
$$\varphi = tan^{-1}\frac{X_C}{R} = tan^{-1}\frac{V_C}{R}$$

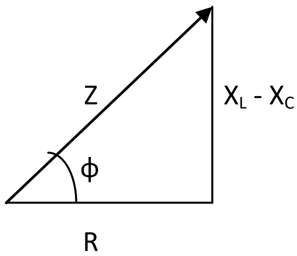
So, in R-C circuit, current leads the voltage by angle ϕ. Hence equation can be written as,
$e = V = V_0 \sin \omega t$ and $i = I_0 \sin(\omega t + \varphi)$

Series R-L-C circuit (Acceptor circuit):









Let, $V_L > V_C$

$$\vec{V} = \vec{V}_R + \vec{V}_L + \vec{V}_C = \vec{I}Z$$

$$\tan\varphi = \frac{X_L - X_C}{R} = \frac{V_L - V_C}{R}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$$

Special cases:

a) When $V_L > V_C$, voltage leads the current by an angle ϕ (Inductive circuit)
$V = V_0 \sin\omega t$ and $i = I_0 \sin(\omega t - \varphi)$, $\tan\varphi = \frac{X_L - X_C}{R}$, $pf = \cos\varphi = \frac{R}{Z}$

b) When $V_L < V_C$, Current leads the voltage by an angle ϕ (capacitive circuit)
$V = V_0 \sin\omega t, i = I_0 \sin(\omega t + \varphi)$, $\tan\varphi = \frac{X_C - X_L}{R}$, $\cos\varphi = \frac{R}{Z}$

c) When $V_L > V_C$, voltage and current are in same phase. The series LCR circuit is said to be purely resistive. $I_0 = \frac{V_0}{Z}$, or $I_{rms} = \frac{V_{rms}}{Z}$

Resonance condition in series LCR circuit:

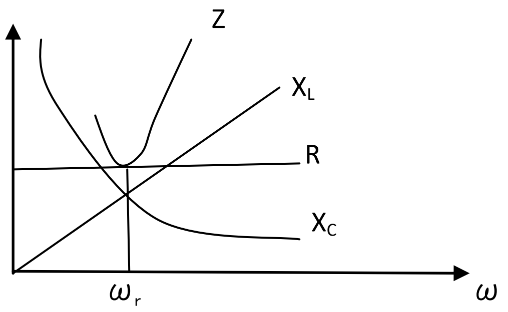
A series LCR circuit is said to be in resonance condition when the current through it has its maximum value. Clearly I_0 becomes maximum (becomes 0 when ω→0 or ω→∞) when ωL = 1/ωC

Resonance condition in series: In series L-C-R circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

If $X_L = X_C$, Z = R, further, $\omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$

So, in series L-C-R circuit, resonance occurs when (i) $\omega = \omega_r$ (ii) $X_L = X_C$ (iii) Z = minimum value = R (iv) $I_{rms} = Maximum\ value = \frac{V_{rms}}{Z_{min}} = \frac{V_{rms}}{R}$ (iv) $Power\ factor = \cos\varphi = 1$ (v) Phase angle is 0^0.

Graph between Z vs. ω



$X_L = \omega L \Rightarrow X_L \propto \omega$ and $X_C = \frac{1}{\omega C} \Rightarrow X_C \propto \frac{1}{\omega}$

R does not depends upon ω.

1. For $\omega > \omega_r$, $X_L > X_C$ or $V_L > V_C$ and voltage leads the current or current lags the voltage and the circuit is inductive.
2. For $\omega < \omega_r, X_L < X_C\ or\ V_L < V_C$ and voltage lags the current or current leads the voltage and the circuit is capacitive.

At $\omega = \omega_r, X_L = X_C$, the voltage and current are in same phase and the circuit is resistive.

Power factor (pf): Definitions

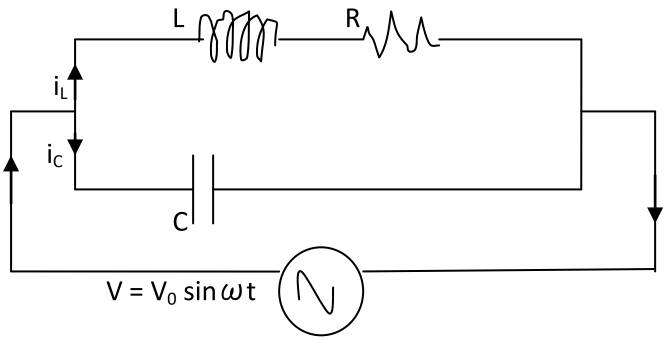
1. It is the cosine of angle between voltage and current or,
2. It is the ratio of actual power to the apparent power, or
3. It is the ratio of R and Z,

that is, $pf = \cos\varphi = \frac{Actual\ power\ (kW)}{Apparent\ power\ (kVA)} = \frac{R}{Z}$, It has limit as 0 ≤ pf ≤ 1.

Note: In a L-C-R circuit, whenever voltage across various elements are asked, find rms value unless stated in the questions for the peak or instantaneous value. $V_R = I_{rms}R, V_L = I_{rms}X_L, V_C = I_{rms}X_C, V_{applied} = \sqrt{V_R{}^2 + (V_L - V_C)^2}$

Parallel Circuit (Rejector circuit):

Let us consider an alternating source connected across an inductance L in parallel with a capacitor C. Every inductor has some non zero internal resistance while in case of a capacitor, it can be neglected. Let R be connected in series with L.



Let instantaneous value of emf applied be V and corresponding currents are i, i_L and i_C, then

$$i = i_L + i_C \Rightarrow \frac{V}{Z} = \frac{V}{R + j\omega L} + j\omega CV \Rightarrow Y = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C = \frac{R + j(\omega CR^2 + \omega^3 L^2 C - \omega L)}{R^2 + \omega^2 L^2}$$

For admittance to be minimum, imaginary part should be zero. So,

$$\omega CR^2 + \omega^3 L^2 C - \omega L = 0 \Rightarrow \omega^2 L^2 C = L - CR^2 \Rightarrow \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} --- (1)$$

It is the frequency at resonance. At resonance admittance is minimum and impedance is maximum. The parallel circuit does not allow this frequency from the source to pass in the circuit.
If R = 0, resonance frequency is same in the acceptor circuit.

The reciprocal of admittance is called parallel resistor or dynamic resistance.

$$Dynamic\ resistance = \frac{R^2 + \omega^2 L^2}{R}$$

Putting ω from equation (1), dynamic resistance is

$$Dynamic\ resistance = \frac{L}{RC}$$

Q-factor:

$$Q - factor = \frac{Peak\ current\ through\ capacitor}{Peak\ current\ through\ supply} = \frac{\omega CV_0}{\frac{V_0}{Dynamic\ resistance}} = \frac{\omega L}{R}$$

This is basically the measure of current magnification.
In view of Euler's identity,

$$e^{j\theta} = \cos\theta + j\sin\theta$$

There are three equivalent notations for a phasors.

1. Polar form: $\vec{V} = V\angle\theta$
2. Rectangular form: $\vec{V} = V(\cos\theta + j\sin\theta)$
3. Exponential form: $\vec{V} = Ve^{j\theta}$

Impedance (Z): If impedance is given as

$$Z = \frac{V\angle\varphi}{I\angle\theta}$$

1. If impedance angle is positive ($\phi > \theta$), current lags the voltage and circuit is R-L circuit.
2. If impedance angle is negative ($\phi < \theta$), current leads the voltage and circuit is R-C circuit.
3. If impedance angle is 0^0, purely resistive circuit
4. If impedance angle is $\pi/2$, current lags the voltage by $\pi/2$, the circuit is purely inductive.
5. If impedance angle is $-\pi/2$, current leads the voltage by $\pi/2$, the circuit is purely capacitive.

Leads/Lags mean:

1. If voltage maximum comes first than that of current at given time, voltage leads current. The circuit is R-L circuit.
2. If current maximum comes first than that of voltage, current leads the voltage. The circuit is purely capacitive.
3. If voltage and current maximum comes at same time, voltage is in phase with current and circuit is purely resistive.

Advantage of A.C. over D.C:

1. The generation of ac is more economical over dc
2. The alternating voltage can be easily stepped up or step down by using a transformer
3. The alternating current can be reduced by using a choke coil without any significant wastage of energy
4. AC can be easily converted in to dc by using rectifier

Disadvantages of ac over dc

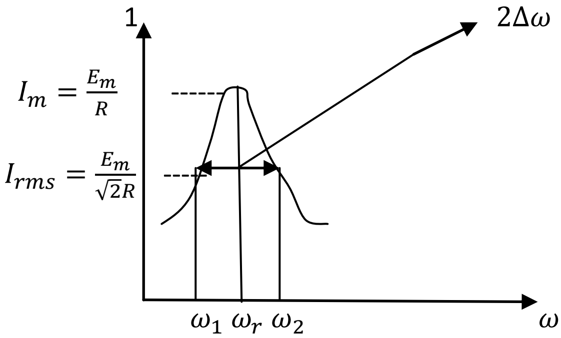
1. Peal value of ac is high. It is dangerous to work with ac
2. In phenomena like electroplating, electrotyping, ac cannot be used.
3. AC is transmitted more from the surface of conductor than from inside. This is called skin effect. Therefore, several fine insulated wires are required for the transmission of ac.

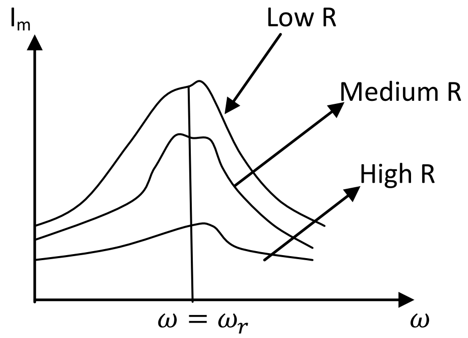
Sharpness of resonance in a series LCR circuit:

Quality factor Q is defined as

$$Q = \frac{Resonant\ frequency}{Bandwidth} = \frac{\omega_r}{2\Delta\omega} = \frac{\omega_r}{\omega_2 - \omega_1}$$

$$\omega_2 - \omega_r = \Delta\omega, \omega_r - \omega_1 = \Delta\omega, or\ \omega_2 - \omega_1 = 2\Delta\omega$$





Current amplitude has peak at the resonant frequency $\omega_r = \frac{1}{\sqrt{LC}}$ and falls to zero in either direction. The resonant frequency is independent of R, but the sharpness of peak depends on R. The peak is higher for smaller value of R and resonance is sharp for small R and flat for large R.

The smaller the bandwidth, the larger the Q and sharper is the peak in the current. Q-factor is a dimensionless quantity.

At $\omega_1\ and\ \omega_2, Z = \frac{E_m}{I_m} = \sqrt{2}R$

Expression for Q-factor:

At ω_r, the impedance is equal to R but at $\omega_1\ and\ \omega_2, Z = \sqrt{2}R$.

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R \Rightarrow R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2 \Rightarrow \omega L - \frac{1}{\omega C} = \pm R$$

So, it can be written as,

$$\omega_2 L - \frac{1}{\omega_2 C} = +R - - - - - (1)$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R - - - - - - (2)$$

Adding equation (1) and (2),

$$(\omega_1 + \omega_2)L - \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)\frac{1}{C} = 0 \Rightarrow (\omega_1 + \omega_2)L = \frac{\omega_1+\omega_2}{\omega_1\omega_2}\left(\frac{1}{C}\right) \Rightarrow \omega_1\omega_2 = \frac{1}{LC} - - - - (3)$$

Subtracting equation (2) from equation (1),

$$(\omega_2 - \omega_1)L + \frac{1}{C}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) = 2R \Rightarrow \omega_2 - \omega_1 = \frac{R}{L}\ \ from\ equation\ (3)$$

So, quality factor is

$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r L}{R}$$

This quality factor can also be written as,

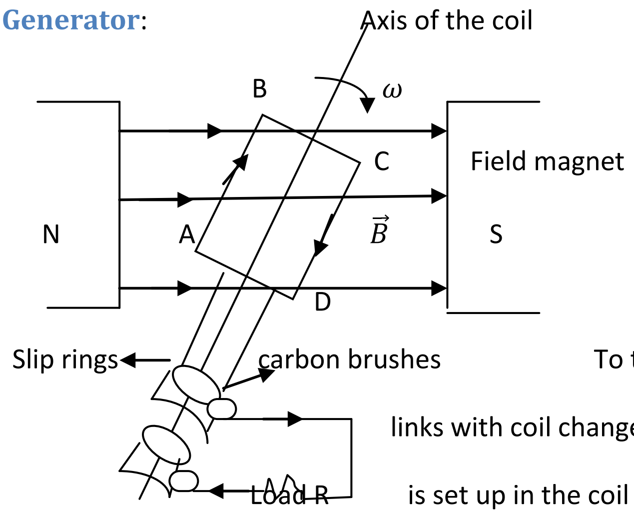
$$Q = \frac{\omega_r(LI_{rms})}{RI_{rms}} = \frac{Voltage\ applied\ across\ L\ or\ C}{Applied\ voltage}$$

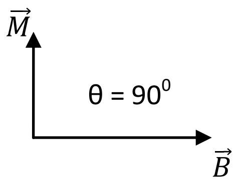
Since resonant frequency is, $\omega_r = \frac{1}{\sqrt{LC}}$

Hence, quality factor can be in another form as, $Q = \frac{1}{R}\left(\sqrt{\frac{L}{C}}\right)$

This means that Q is large if R is small or L is large, this means resonance is sharp or series resonance circuit is more selective.

A.C. Generator:





Here, axis of the coil is perpendicular To the magnetic field, the magnetic flux links with coil changes, an induced emf and hence a current is set up in the coil and torque (CW) is produced.

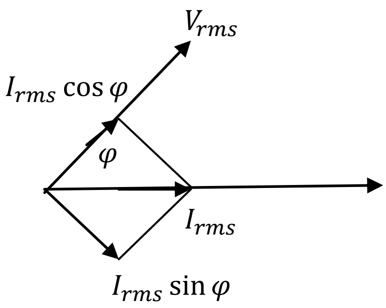
Direction of Torque:

When magnetic field direction is from north to south pole, this means field (uniform) is perpendicular to the axis of the coil and going in to the plane of paper. The direction is shown in the figure. In the rectangular coil ABCD, the direction of current is from A to B in the side AB. The direction of current in the side CD is from C to D. It means the current in the coil is in the clockwise direction. It further means that front face of the coil is South pole. Hence, the direction of magnetic moment is from south pole to north pole, that is, coil rotates from $\vec{M}$ to $\vec{B}$, that is coil rotates in clockwise direction.

Wattless Current:

The current is ac circuit is said to be wattless if the average power consumed in the circuit is zero.

The average power in ac circuit is $P_{av} = V_{rms}I_{rms}\cos\varphi$



Here, $V_{rms}I_{rms}\sin\varphi = Wattless\ current$

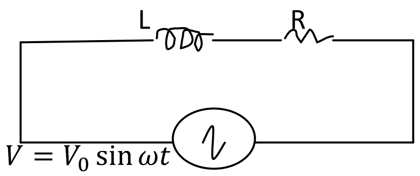
Because, the circuit consume no power and this happens only in inductive or capacitive circuit.

For example, if the secondary of transformer is open, the current in the primary is almost wattless.

Choke coil:

It is simply an inductor with large inductance which is used to reduce current in ac circuit without much loss of energy.

Principle: I lags V by 90^0.



$$P_{av} = V_{rms}I_{rms}\cos\varphi$$

$$\cos\varphi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \omega^2L^2}}$$

Since, $\omega L \gg R$ then, PF = o

Average power:

Average power associated with purely resistive circuit-

Let $V = V_0\sin\omega t$ then $I = I_0\sin\omega t$ in purely resistive circuit.

Instantaneous power P is

$$P = VI = V_0I_0 sin^2\omega t$$

So small amount of work done in small time dt is

$$dW = Pdt = V_0I_0 sin^2\omega t\ dt = V_0I_0\frac{(1 - \cos 2\omega t)}{2}dt$$

So, total work done or energy spent in maintaining current over one cycle is

$$W = \int dW = \int_0^T \frac{V_0 I_0}{2}(1 - \cos 2\omega t)dt = \frac{V_0 I_0}{2}\int_0^T (1 - \cos 2\omega t)\, dt$$

$$= V_{rms} I_{rms}\left[\int_0^T dt - \int_0^T \cos 2\omega t\, dt\right] = V_{rms} I_{rms}[t]_0^T - 0 = V_{rms} I_{rms} T$$

Average power supplied to R over a complete cycle is

$$P_{av} = \frac{W}{T} = \frac{V_{rms} I_{rms} T}{T} = V_{rms} I_{rms} = Virtual\ voltage\ x\ Virtual\ current$$

RMS value is also called virtual value.

Power in AC circuit:

In an AC , the voltage is leading the current by an angle ϕ, then it can be written as,

$V = V_0 \sin \omega t$ and $i = I_0 \sin(\omega t - \varphi)$

The instantaneous value of power is

$$P = Vi = (V_0 \sin \omega t)\,(I_0 \sin(\omega t - \varphi)) = \frac{V_0 I_0}{2}[\cos \varphi - \cos(2\omega t - \varphi)]$$

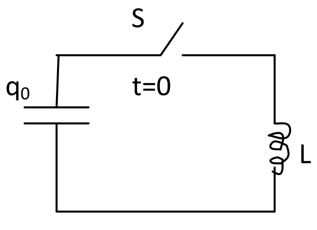
The average power over one cycle is given by,

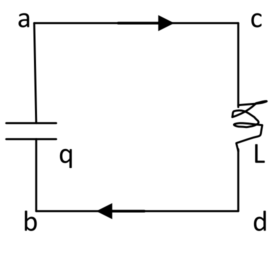
$$\langle P \rangle = P_{av} = \frac{\int_0^T P dt}{\int_0^T dt} = \left(\frac{V_0 I_0}{2}\right)\frac{\int_0^{\frac{2\pi}{\omega}}[\cos \varphi - \cos(2\omega t - \varphi)]dt}{(t)_0^{\frac{2\pi}{\omega}}}$$

$$= \left(\frac{V_0 I_0}{2}\right)\frac{\left(\frac{2\pi}{\omega}\cos \varphi\right) - \frac{1}{2\omega}[\sin(2\omega t - \varphi)]_0^{\frac{2\pi}{\omega}}}{\frac{2\pi}{\omega}} = \left(\frac{V_0 I_0}{2}\right)\frac{\left[\frac{2\pi}{\omega}\cos \varphi - 0\right]}{\frac{2\pi}{\omega}}$$

$$= V_{rms} I_{rms} \cos \varphi$$

L-C oscillations: Qualitative approach





If a charged capacitor is short circuited through an inductor L, the charge and current in the circuit start oscillating simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat. We also assume that energy is not dissipated as heat.

With zero resistance and no radiation, the oscillations in the circuit persist indefinitely and energy is transferred from the capacitor electric field to the inductor magnetic field and back. The total energy associated with the circuit is constant.

When switch is OFF:

Initially the charge on capacitor is maximum (switch S is open) and electrical energy stored in the capacitor is

$$U_E = \frac{1}{2}\frac{{q_0}^2}{C}$$

The energy in the inductor is zero.

When switch S is closed:

The moment switch is ON, the charge on the capacitor starts decreasing and the current in the circuit increases. This current produces magnetic field in the inductor. The magnetic energy stored in the inductor for a maximum current is,

$$U_B = \frac{1}{2}L{I_0}^2$$

As there is no further change in current through the inductor, the magnetic field starts decreasing with time. This decreasing magnetic field induces a current in the circuit. The current starts charging the capacitor but with reversed polarity. Once the capacitor the fully charged w. r. t. To previous states, the whole process repeats itself. Thus, the energy in the system oscillates between the capacitor and the inductor. But in practice, this is never achieved as there is some loss in the circuit due to a resistance in the circuit and due to the radiations in the form of electromagnetic waves.

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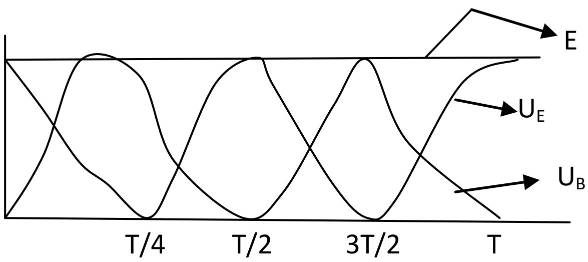
*A capacitor is fully charged. It is allowed to discharge through an inductor . Draw graph showing variations of (i) energy stored in the capacitor, (ii) energy stored in the inductor and (iii) the total energy with time.

Let $q = q_m \cos \omega t$, $i = \frac{dq}{dt} = -q_m \omega \sin \omega t$, Also, $T = \frac{2\pi}{\omega}$

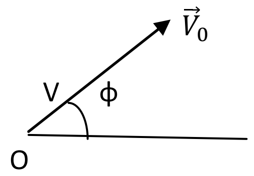
Energy stored by a capacitor is, $U_E = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{{q_m}^2}{C} cos^2\omega t,$

Energy stored by inductor is, $U_B = \frac{1}{2}Li^2 = \frac{1}{2}L{q_m}^2\omega^2 sin^2\omega t = \frac{{q_m}^2}{2C} sin^2\omega t$, Since, at resonance,

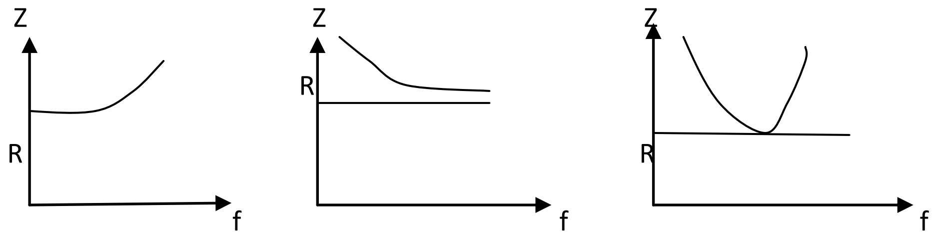
$\omega^2 = \frac{1}{LC}$, so total energy $E = \frac{{q_m}^2}{C}$



If $V = V_0 \sin(\omega t + \varphi)$, then $\vec{V}_0 = V_0 \angle\varphi$



Impedance Vs. frequency of applied voltage in R-L, R-C, R-L-C circuit:



$Z = \sqrt{R^2 + (2\pi f L)^2}$, $Z = \sqrt{R^2 + \frac{1}{(2\pi f C)^2}}$, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Time period (T): Time taken by an alternating quantity to complete one cycle is called time period.

Frequency (f): The number of cycles per second is called the frequency of alternating quantities.

Amplitude: The maximum value positive or negative of an alternating quantity is called amplitude.

Root mean square value: The r.m.s. value of an alternating current is given by that steady current (dc) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

It is the square root of the mean of the squares of the instantaneous current or voltage.
The rms value of a complex current wave is equal to the square root of the sum of the squares of the rms values of its individual components.

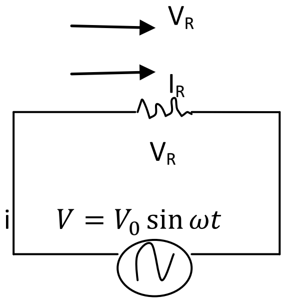
Crest or peak or amplitude factor:

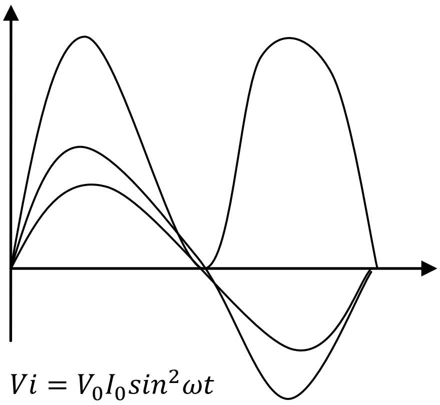
$$K_a = \frac{Maximum\ value}{RMS\ value} = \frac{I_0}{\frac{I_0}{\sqrt{2}}} = \sqrt{2} = 1.414\ (for\ sinusoidal\ ac\ only)$$

It is of importance in dielectric insulation testing where dielectric stress is proportional to peak value applied voltage.

The knowledge is also necessary when measuring iron losses, because iron loss depends upon the value of maximum flux.

AC through pure ohmic resistance $V = V_0 \sin \omega t$, $i = I_0 \sin \omega t$,



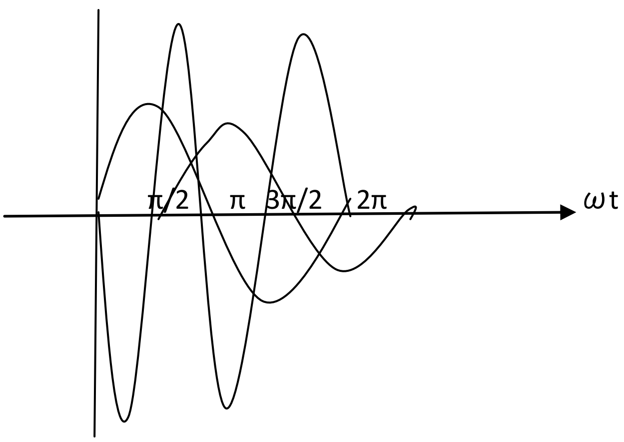


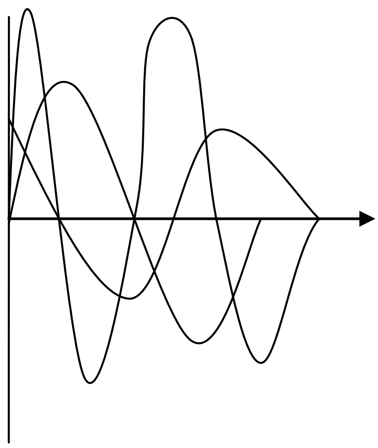
$$p = Vi = V_0 I_0 sin^2 \omega t$$

Purely inductive circuit:

Let $V = V_0 \sin \omega t$, then $i = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$, so instantaneous power in purely inductive circuit is

$$p = Vi = V_0 I_0 \sin \omega t \sin\left(\omega t - \frac{\pi}{2}\right) = \frac{V_0 I_0}{2}\left[\cos\frac{\pi}{2} - \cos\left(2\omega t - \frac{\pi}{2}\right)\right] = \frac{-V_0 I_0}{2} \sin 2\omega t$$





Purely capacitive circuit:

$$p = V_0 I_0 \sin \omega t \sin\left(\omega t + \frac{\pi}{2}\right) = \frac{V_0 I_0}{2}\left[\cos\frac{-\pi}{2} - \cos\left(2\omega t + \frac{\pi}{2}\right)\right] = \frac{V_0 I_0}{2} \sin 2\omega t$$

Chapter-7: Numerical and Derivations

1. If the current in AC circuit is represented by the equation $i = 5\sin\left(300t - \frac{\pi}{4}\right)$, here t in seconds and i in Ampere. Calculate
   a) Peak and rms value of current, [Ans. 5 A, 3.54 A]
   b) Frequency of AC, [Ans. 150/π or 47.75 Hz]
   c) Average current, [Ans. 10/π or 3.18 A]
2. A 100 Ω resistor is connected in series with a 4 H inductor. The voltage across the resistor is, $V_R = 2.0\,V\sin(10^3 rad/s)t$.
   a) Find the expression of circuit current, [$2\,x10^{-2}\sin(10^3)t$
   b) Find the inductive reactance, [Ans. 4 x 10^3 Ω]
   c) Derive an expression for the voltage across the inductor, [Ans. $80\sin\left(10^3 t + \frac{\pi}{2}\right)$
3. A resistance and inductance are connected in series across a voltage $V = 283\sin 314\,t$. The current is found to be $4\sin\left(314t - \frac{\pi}{4}\right)$. Find the values of the inductance and the resistance.

   Solution: Impedance of the circuit is

   $$\vec{Z} = \frac{V_0}{I_0\angle -\frac{\pi}{4}} = \frac{283\angle\frac{\pi}{4}}{4} = \frac{283}{4}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) = R + jX_L$$

   So, Resistance R and inductance reactance $X_{L\,is}$ as

   $R = \frac{283}{4}\cos 45^0 = 50\,\Omega$ and $X_L = \frac{283}{4}\sin 45^0 = \omega L = 314\,L \Rightarrow L = 0.159\,H$ Answer
4. Find the voltage across the various elements, i. e. resistance, capacitance and inductance which are in series and having values as 1000 Ω, 1 μF and 2 H respectively, given emf as, $V = 100\sqrt{2}\sin 1000t$ volt. [Ans. 70.7 V, 141.4 V, 70.7 V]

Solution:

[Since, $\vec{Z} = R + j(X_L - X_C) \Rightarrow |\vec{Z}| = \sqrt{R^2 + (X_L - X_C)^2}$

Also, ω=1000 rad/s, V_0 = 100√2 volt, so

$X_L = \omega L = 1000\,x\,2 = 2000\,\Omega, X_C = \frac{1}{\omega C} = \frac{1}{1000\,x\,10^{-6}} = 1000\,\Omega$

$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100$ volt, Now, $I_{rms} = \frac{V_{rms}}{Z}$

Here, $Z = \sqrt{(1000)^2 + (2000 - 1000)^2}$=1000√2 Ω

Then, $I_{rms} = \frac{100}{1000\sqrt{2}} = \frac{1}{10\sqrt{2}}$ ampere, Hence,

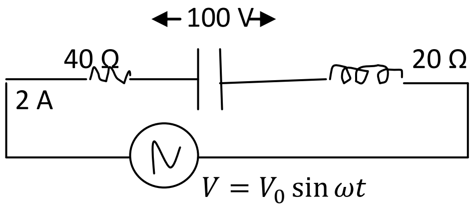
P. D. Across resistor = I_{rms} x R = 100/√2 =70.7 V,

P. D. Across inductor = I_{rms} x X_L = 200/√2 = 141.4 V

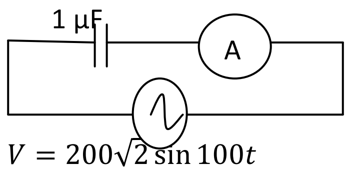
P. D. Across capacitor = I_{rms} x X_C =100/√2 = 70.7 V]

5. A sinusoidal voltage of frequency 60 Hz and peak value 150 V is applied to a series L-R circuit, where R = 15 Ω, L = 40 mH.
   a) Compute T, ω X_L, Z and ϕ
   b) Compute the amplitude of current, V_R and V_L

6. In the circuit shown in the figure, find



   a) V_R [Ans. 80 V]
   b) V_L [Ans. 40 V]
   c) X_C [Ans. 50 Ω]
   d) V_0 [Ans. 60 √10 V]
7. In a series C-R circuit, the applied voltage is 10 V and the voltage across capacitor is found to be 8 V. Find the voltage across R and the phase difference between current and applied voltage.[Ans. 6V, $\tan^{-1}$ (4/3)]
8. A 50 Hz AC source of 20 V is connected across R and C in series. The voltage across R is 10 V. Find the voltage across C. [Ans. 16 V]
9. In the circuit shown, the reading of AC ammeter is –



   a) 20√2 mA
   b) 40√2 mA
   c) 20 mA [Ans.]
   d) 40 mA
10. A coil has a resistance of 10 Ω and an inductance of 0.4 H. It is connected to an ac source of 6.5 V, 30/π Hz. Find the average power consumed in the circuit. [5/8 W]
11. A series Ac circuit contains an inductor of 20 mH, a capacitor of 100 μF, a resistor of 50 Hz and an AC source of 12 V, 50 Hz. Find the energy dissipated in the circuit in 1000 s. [Ans. 2.3 x 10^3 J]
12. A transformer has 50 turns in the primary and 100 turns in the secondary. If primary is connected to 220 V DC supply, what will be the voltage across secondary? [Ans. Zero]

CBSE Questions Last Years-

13. A step down transformer operated on a 2.5 kV line. It supplies a load with 20 A. The ratio of primary winding to the secondary is 10:1. If the transformer is 90% efficient, calculate
   a) the power output [Ans. 45 kW]
   b) the secondary voltage and {Ans. 250 V]
   c) the current in the secondary coil [Ans. 180 A] [CBSE F 2010]

   [Given, I_P = 20 A, N_P /N_S =10:1, η = 90%, V_P = 2.5 kV = 2500 V

   We know that

   $$\frac{V_P}{V_S} = \frac{N_P}{N_S} \Rightarrow V_S = 2500\ x\ \frac{1}{10} = 250\text{ V},$$

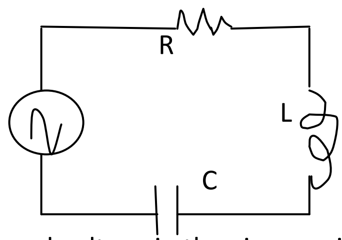
   So, Power output = Power Input x efficiency = V_P I_P x 0.90 =2500 x 20 x 0.9 = 45 kW

   Power output = V_S I_S or, I_S = 45000/250 = 180 A]

14. A resistor of 400 Ω, an inductor of 5 /π H and a capacitor of 50/π μF are connected in series across a source of alternating voltage of 140 sin 100 π t V. Find the voltage (rms) across the resistor, inductor and capacitor. [Ans. 80 V, 100 V and 40 V] [CBSE F 10]
15. A series L-C-R circuit is connected to a 220 V variable frequency AC supply. If L = 220 mH, C = (800/π^2) μF and R = 110 Ω. [CBSE D 2010 C]

a) Find the frequency of the frequency for which average power absorbed by the circuit is maximum [Ans. f_0 = 125 Hz]
b) Calculate the value of the maximum current amplitude [Ans. 2√2 A]

16. An AC voltage $V = V_0 \sin \omega t$ is applied across a pure inductor L Obtain an expression for the current I in the circuit and hence obtain the
    a) Inductive reactance of the circuit and
    b) The phase of the current flowing with respect to the applied voltage
17. An AC voltage $V = V_0 \sin \omega t$ is applied across a pure capacitor C Obtain an expression for the current I in the circuit and hence obtain the
    c) Capacitive reactance of the circuit and
    d) The phase of the current flowing with respect to the applied voltage
18. The figure shows a series L-C-R circuit with L = 10 H, C = 40 μF, R = 60 Ω connected to a variable frequency 240 V source. Calculate
    a) The angular frequency of the source which drives the circuit at resonance
    b) The current at the resonating frequency
    c) The rms P.D. across the inductor at source



19. Determine the value of phase difference between current and voltage in the given series circuit of L-C-R. R = 400 Ω, C = 2 μF, L = 100 μH, $V = V_0 \sin(1000t + \varphi)$ [Ans. -π/4] Calculate the value of additional capacitor which may be joined suitably to the capacitor C that would make the power factor of the circuit unity. [Ans. 8 μF] [CBSE D 2015]
20. An alternating voltage given by V = 70 sin 100 πt is connected across a pure resistor of 25 Ω. Find [All India 2012]
    a) The frequency of the source [Ans. 50 Hz]
    b) The rms current through the resistor [Ans. 1.98 A]
21. Calculate the quality factor of a series L-C-R circuit with L = 2 H, C = 2 μF and R = 10 Ω. Mention the significance of quality factor in L-C-R circuit. [Ans. 100] [CBSE F 2012] [Higher value of Q-factor means energy loss is at lower rate relative to the energy stored i.e. oscillations will die slowly and damping will be less.]
22. In series L-C-R circuit connected to a variable frequency 250 V source with L = 50 mH, C = 80 μF and R = 40 Ω. Determine [All India 2014]
    a) The source frequency which drives the circuit in resonance [80 Hz]
    b) The quality factor of the circuit [Ans. 0.625]
23. A circuit containing a 80 mH inductor and a 60 μC in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible. (NCERT)
    a) Obtain the current amplitude and rms values
    b) Obtain the rms values of PD across each element
    c) What is the average power transferred to the inductor?
    d) What is the average power transferred to the capacitor?
    e) What is the total average power absorbed by the circuit?

24. A light bulb is rated 150 W for 220 V AC supply of 60 Hz. Calculate
    a) the resistance of the bulb
    b) the rms current through the bulb [All India 2012]
25. A coil of 0.01 H inductance and 1 Ω resistance is connected to 200 V, 50 Hz AC supply. Find the impedance of the circuit and time lag between maximum alternating voltage and current. [3.3 Ω, 1/250 s]
    [Time lag $\Delta t = \frac{\varphi}{\omega}$, ϕ = $\tan^{-1}$ (314) = 72^0 = 72 π/180, here ϕ in rad.]
26. How can you improve the quality factor of a series resonance circuit?
    [Ans. to make the ohmic resistance as small as possible, $Q = \frac{\omega_r L}{R} = \frac{1}{R}\left(\sqrt{\frac{L}{C}}\right)$]
27. Can the instantaneous power output of an AC source be negative? [Ans. yes] Can the average power output be negative? [Ans. No, as it is average]
28. If L-C circuit is considered analogous to a harmonically oscillating spring block system, which energy of L-C circuit would be analogous to the potential energy and which one analogous to the kinetic energy? [Ans. $\frac{1}{2}CV^2$ *is PE and* $\frac{1}{2}LI^2$ is analogous to KE.]
29.
    a) For a given AC, $i = I_m \sin\omega t$, show that average power dissipated in a resistor over a complete cycle is $\frac{1}{2}{I_m}^2 R$.
    b) A light bulb is rated at 100 W for a 220 V AC supply. Calculate the resistance of the bulb. [Ans. $R = \frac{V^2}{P} = 484\ \Omega$]

    Let $V = V_m \sin\omega t$ the for a purely resistive circuit,
    $i = I_m \sin\omega t$
    The average power over one cycle is,

$$[P_{av} = \frac{\int_0^T Vidt}{\int_0^T dt} = \frac{\int_0^{\frac{2\pi}{\omega}} V_m I_m sin^2\omega t\, dt}{[t]_0^{\frac{2\pi}{\omega}}} = \frac{V_m I_m\left[\frac{1}{2}\int_0^{\frac{2\pi}{\omega}}(1-\cos 2\omega t)dt\right]}{\frac{2\pi}{\omega}} =$$

$$\frac{V_m I_m\left[\frac{2\pi}{\omega} - \int_0^{\frac{2\pi}{\omega}} \cos 2\omega t dt\right]}{2\left(\frac{2\pi}{\omega}\right)} = \frac{V_m I_m}{2} = \frac{{I_m}^2 R}{2}]$$

30. A transformer is used to step down AC voltage. What devices do you use to step down DC voltage? [Ans. an ohmic resistance is used to step down DC voltage, for exp. Potential divider arrangements}
31. A transformer on a pole near a factory steps the voltage down from 3600 V to 120 V. The transformer is to deliver 1 MW to the factory at 90% efficiency. Find
    a) The power delivered to the primary [Ans. 1.1 MW]
    b) The current in the primary and [3.1 x 10^2 A]
    c) The current in the secondary [8.3 x 10^3 A]
32. A step up transformer operates on a 220 V line and supplies a load of 5 A. The ratio of primary to the secondary winding is 1: 20. Calculate
    a) The voltage across secondary [Ans. 4400 V]
    b) Current in the primary [Ans. 100 A]
    c) The power output [Ans. 22 kW]

33. When an alternating current is passed through a moving coil galvanometer, it shows no deflection. Why?
[Ans. A moving coil galvanometer measures an average value of current, which is zero in ac.]
34. In series LCR circuit, $V_L = V_C \neq V_R$, what is the value of power factor? [Ans. 1]
35. Can ever the rms value be equal to the peak value of an ac?
[Yes, when ac is a square wave.]
36. Which is more dangerous 220 V ac or 220 V dc and why?
[Ans. 220 V ac as its peak value is $220\sqrt{2} = 311$ V > 220 V dc
37. Does a step down transformer violet the principle of conservation of energy?
[Ans. No, as voltage is decreased, current is increased in the same ratio and the product VI remains same.]
38. In a circuit, the applied voltage is found to lag the current by 30^0.
    a) Is the power factor leading or lagging? [Ans. leading]
    b) What is the value of power factor? [Ans. 0.86 (lead)]
    c) Is the circuit inductive or capacitive?[Ans. Capacitive]
39. A series R, an inductance L =0.01 H and a capacitance C are connected in series. When a voltage $V = 400\ \cos(3000t - 10^0)\ volt\ is\ applied\ to\ the\ series\ combination,$ the current flowing is $10\sqrt{2}\cos(3000t - 55^0)$ amperes. Find R and C. [Ans. 20 Ω, 33 μF]
40. In a circuit, applied voltage is 100 V and is found to lag the current of 10 A by 30^0.
    a) Is the power factor leading or lagging?
    b) What is the value of pf?
    c) Is the circuit inductive or capacitive?
    d) What is the value of active and reactive power in the circuit? [Ans. 866 W, 500 VAR]
41. In a series LCR circuit, obtain the condition under which (i) the impedance of the circuit is minimum and (ii) wattless current flows in the circuit.
42. In a series LCR circuit with an ac source of effective voltage 50 V, frequency f = 50/π Hz, R = 300 Ω, C = 20 μF and L = 1 H.
    a) Find the rms current in the circuit [Ans. 0.1 A]
    b) Find the voltage across R, inductance and capacitance
    c) Draw phasor diagram
    d) Draw wave form of instantaneous voltage, instantaneous current
    e) Fins average power
    f) Find power factor (lead or lag)
43. Define capacitive reactance. Write its SI unit.
[Ans. It is the effective resistance offered by capacitance to the flow of ac. Its SI unit is Ohm.]
44. A circuit is set up by connecting inductance L = 100 mH, resistor R = 100 Ω and a capacitor of reactance 200 Ω in series. An alternating emf of 150 √2 V, 500/π Hz is applied across this series combination. Calculate the power dissipated in the resistor. [Ans. 225 W]
45. A voltage $V = V_0 \sin \omega t$ is applied to a series LCR circuit. Derive the expression for the average power dissipated over a cycle.
Under what condition is (i) no power dissipated even though the current flows through the circuit, (ii) maximum power dissipated in the circuit?

46. A series LCR circuit is connected to an ac source. Using the phasor diagram, derive the expression for the impedance of the circuit. Plot a graph to show the variation of the current with frequency of the source, expressing the nature of its variation.
47. An ideal capacitor having a charge $q = q_0 \cos \omega t$ is connected across an ideal inductor L through a switch S. On closing the switch, show that sum of energies in the capacitor and inductor is constant in time in the free oscillations of the LC circuit.