Chapter-4: Moving Charges and Magnetism

All magnetic effects are associated with the motion of electric charges or current elements.

Magnetic field-The region or space surrounding the moving charges or current carrying conductor in which its effects can be experienced, is called magnetic field.

Oersted concluded that moving charges or currents produced a magnetic field in the surrounding space.

Magnetic Force:

There are three ways in which force due to magnetic fields can be experienced:

a) Due to moving charged particle in a magnetic field $(\vec{B})$
b) On a current element in an external magnetic field
c) Between two current elements

Force on a moving charged particle:

The electric force on a stationary or moving electric charge (Q) in an electric field is given by

$$\vec{F_e} = Q\vec{E} - - - - (1)$$

If Q is positive, Fe and E have the same direction.

While magnetic field can exert force only on a moving charge and it is experimentally found that

$$\vec{F}_m = Q\vec{v}x\vec{B} - - - - (2)$$

Equation (3) shows that $\vec{F}_m$ *is perpendicular to both* $\vec{v}$*and* $\vec{B}$.

The following points are worth noting regarding equation (2) mentioned above:

a) **Magnitude of $\overrightarrow{F_m}$** is , $F_m = QvB \sin\theta - - - - (3)$

   Where θ is the angle between $\vec{v}$ *and* $\vec{B}$ and drawn from v to B clockwise.

b) $\vec{F}_m$ **is zero when**,
   - B=0, i. e. no magnetic field is present
   - Q=0, i. e. particle is neutral.
   - v=0, i. e. charged particle is at rest.
   - θ=0^0 or 180^0, i. e. $\vec{v} \uparrow\uparrow \vec{B}$ or $\vec{v} \uparrow\downarrow \vec{B}$

c) $\vec{F}_m$ is maximum when θ=90^0 and this maximum value is QvB.

d) The unit of $\vec{B}$ is Tesla (has the same unit as of F/Qv: Ns/C-m: SI unit) in the honour of Nikola Tesla, the prominent Serbian-American Scientist and inventor. Thus,

$$1\ tesla = 1\ T = \frac{1N-s}{C-m} = \frac{1N}{A-m}$$

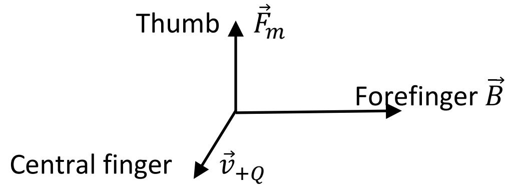
e) The CGS unit of $\vec{B}$ is Gauss (1T=10^4 G) and also in common use.

   In equation (3), Q to be substituted with sign, if Q is positive, magnetic force is along $\vec{v}x\vec{B}$ and if Q is negative, the magnetic force is in opposite direction to $\vec{v}x\vec{B}$.

f) Direction of $\vec{F}_m$: From the property of cross product we can infer that $\vec{F}_m$ is perpendicular to both $\vec{v}x\vec{B}$ or it is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$. The exact direction of $\overrightarrow{F_m}$ can be given by any of the following methods:

g) Direction of $\vec{F}_m$: (sign of Q) (direction of $\vec{v}x\vec{B}$) or $\vec{F}_m \uparrow\downarrow \vec{v}x\vec{B}$ if Q is positive and $\vec{F}_m \uparrow\downarrow \vec{v}x\vec{B}$ if Q is negative.

h) Fleming's left hand rule: According to this rule, the forefinger, the central finger and the thumb of the left hand are stretched in such a way that they are mutually perpendicular to each other. If the central finger shows the direction of velocity of positive charge $(\vec{v}_{+Q})$ and forefinger shows the direction of magnetic field $(\vec{B})$, then the thumb will give the direction of magnetic force $(\vec{F}_m)$.



i) Right hand rule: Wrap the fingers of your right hand around the line perpendicular to the plane of $\vec{v}$ and $\vec{B}$ as shown in the figure so that they curl around with sense of rotation from $\vec{v}$ to $\vec{B}$ through the smaller angle between them. The thumb will point the direction of force on positive charge. Or

j) $\vec{F}_m \perp \vec{v}$ or $\vec{F}_m \perp \frac{\vec{ds}}{dt}$. Therefore, $\vec{F}_m \perp \overrightarrow{ds}$ or the work done by the magnetic force in a static (constant or uniform) magnetic field is zero. $W_{\vec{F}_m} = 0$

So, from work energy theorem, KE and hence the speed of charged particle remains constant in a magnetic field. The magnetic force can change the direction only. It cannot increase or decrease the speed or KE of the particle.

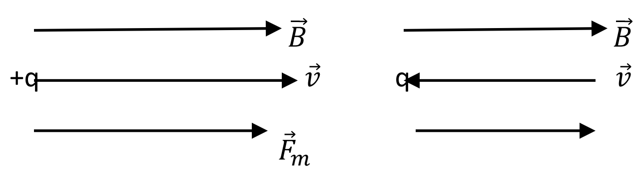
Path of a charged particle in uniform magnetic field:

The path of a charged particle in uniform magnetic field depends upon the angle θ (the angle between $\vec{v}$ and $\vec{B}$.

Depending on different values of θ, following three cases are possible.

Case I. When θ=0^0 or 180^0 i. e. $\vec{F}_m = 0$ hence **path is a straight line** (undeviated when it enters parallel or antiparallel the magnetic field.

When angle between $\vec{v}$ and $\vec{B}$ is zero, that is, $\vec{v} \uparrow\uparrow \vec{B}$ i. e. $\vec{v}$ and $\vec{B}$ are in same direction $(\vec{v} x \vec{B} = 0)$ So, $\vec{F}_m$ will be also in same direction.



(When θ=0^0) (When θ=180^0)

Case II. When θ=90^0, $\vec{v} \perp \vec{B}$ that is, $\vec{v}.\vec{B} = 0$ and magnetic force is maximum and maximum value of magnetic force is $F_m = qvB \sin 90^0 = qvB$. Further, $\vec{F}_m \perp \vec{v}$ at every instant. Hence **path is a circle**. The necessary centripetal force is provided by the magnetic force. If r be the radius of circle, then

- Radius of circular Path: $m\frac{v^2}{r} = qvB \Rightarrow r = \frac{mv}{qB}$
- Momentum of the particle: P=mv=rqB
- Kinetic energy of the particle: If the charged particle is accelerated by a potential difference V volts, it acquires a kinetic energy given by K=qV

Relation between momentum and Kinetic energy:

In terms of momentum (P) of a particle and Kinetic energy (K), radius r can be written as

$r = \frac{P}{qB} = \frac{\sqrt{2Km}}{qB}$ and hence $P = \sqrt{2Km}$

- Time period (T) of circular path: $T = \frac{Length\ or\ perimeter\ of\ circular\ path}{Velocity\ of\ the\ particle} = \frac{2\pi r}{v} = 2\pi\frac{\frac{mv}{qB}}{v} = \frac{2\pi m}{qB}$
- Angular speed of the particle: $\omega = \frac{2\pi}{T} = \frac{qB}{m}$
- Frequency of rotation of the particle: $f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$

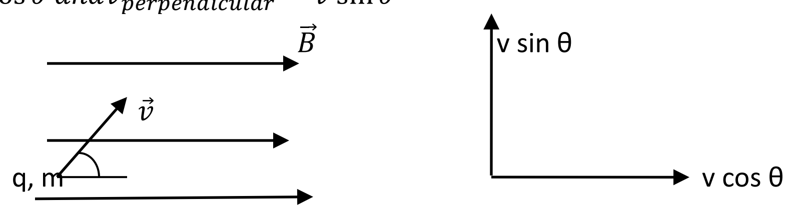
Following points are worth noting regarding a circular path:

- ➢ The plane of the circle is perpendicular to magnetic field. If the magnetic field is along Z-direction, the circular path is in X-Y plane. The speed of the particle does not change in magnetic field. If $\vec{v}$ of particle given by $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$ then speed of the particle will be $v_0 = v_x^2 + v_y^2$.
- ➢ T, f and ω are independent of *v* while the radius is directly proportional to v.

Specific charge (α) : Charge per unit mass (q/m) is known as specific charge.

Case III. When θ other than 0^0 or 180^0 0r 90^0

In this case, velocity can be resolved into two components, one along $\vec{B}$ and another perpendicular to $\vec{B}$. So, $v_{parallel} = v\cos\theta\ and v_{perpendicular} = v\sin\theta$



The component perpendicular to field gives a circular path and component parallel to field gives a straight line path. The **resultant path is a helix**.

The radius of helical path: $r = \frac{mv\sin\theta}{Bq} = \frac{mv_{perpendicular}}{Bq}$

Time period and frequency do not depend upon velocity so they remains same as mentioned in case II.

Pitch (p) of the helical path: Pitch is defined as distance travelled along magnetic field in one complete cycle. $pitch\ (p) = v_{parallel} x Time\ period = v\cos\theta\left(\frac{2\pi m}{Bq}\right)$

Important points in helical paths (i) The plane of the circle of helix is perpendicular to the magnetic field. (ii)The axis of the helix is parallel to the magnetic field.

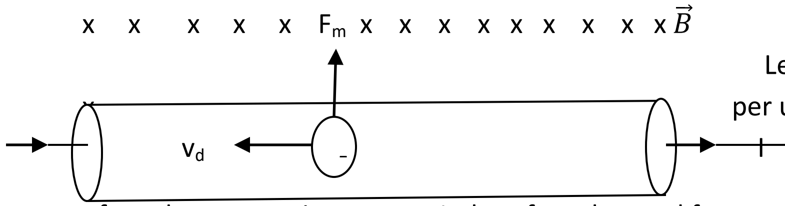
Lorentz Force:

The total force on a charge Q moving in the presence of both electric and magnetic field is given by

$\vec{F} = \vec{F_e} + \vec{F_m} = Q(\vec{E} + \vec{v}X\vec{B}) ----(4)$

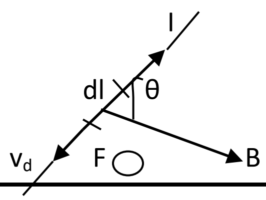
This equation (4) is known as Lorentz force equation.

Magnetic Force on a current carrying conductor



Let n be the number of electrons per unit volume, A the area of cross section and l be length of conductor carrying current I, therefore the total force on wire is given by the equation (2) as $\vec{F}_m = Q(\vec{v}X\vec{B})$, therefore, $\vec{F}_m = -e(nAl)(\vec{v}_d X\vec{B})$ Since $I = neAv_d$, hence

$$\vec{F}_m = I(\vec{l}X\vec{B}) ----(5)$$



Here I and electron drift velocity (v_d)is opposite to the direction of current I.

In magnitude form, $F_m = IlB\sin\theta$

From equation (5), $\vec{l}$ is vector that points in the direction of current I and has a magnitude equal to the length.

Note: The net magnetic force acting on any closed current loop in a uniform magnetic field is zero.

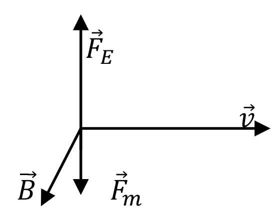
Motion in combined electric and magnetic fields-

Velocity Selector or filter: A charged particle moving with velocity v in the presence of both electric and magnetic fields experiences a force given by-

$$\vec{F} = q(\vec{E} + \vec{v}x\vec{B}) = \vec{F}_E + \vec{F}_m$$

If we consider $\vec{E}$ and $\vec{B}$ are perpendicular to each other and also perpendicular to the velocity of the particle, then, $\vec{E} = E\hat{j}, \vec{v} = v\hat{\imath}, \vec{B} = B\hat{k}$ so $\vec{E}$

$$\vec{F} = q(E\hat{j} + vB\hat{\imath}x\hat{k}) = q(E - vB)\hat{j}$$



Thus, electric and magnetic field forces are in opposite directions.

If the values of E and B are adjusted in such a way that their

Magnitudes are equal, then total force on charge is zero and charge will move in the electric and magnetic fields undeflected. This happens when qE=qvB or, $v = \frac{E}{B}$

This condition ($\vec{E} \perp \vec{B}$) can be used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds (irrespective of their charge and masses).

So, crossed E and B fields serve as velocity selector. Only particles with speed E/B pass undeflected through the region of crossed fields. This velocity selector principle is also employed in Mass Spectrometer (a device that separates charged particles, usually ions, according to their charge to mass ratio).

Magnetic Dipole:

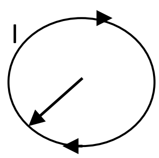
Every current carrying loop is a magnetic dipole. It has two poles, south(S) and north pole (N). This is similar to a bar magnet. Magnetic field lines emanate from North pole forming a closed path terminate on South Pole. Each magnetic dipole has some magnetic moment ($\vec{M}$). The magnitude of $\vec{M}$ is given by $M = NIA$

Where, N=Number of turns in the loop, I=Current in the loop and A=Area of cross section of the loop.

Unit of magnetic dipole moment: A-m^2

Direction of $\vec{M}$: First Method: As in the case of electric dipole, the direction of electric dipole moment $\vec{P}$ is from negative charge to positive charge, likewise, direction of magnetic dipole moment $\vec{M}$ is from South Pole to North Pole. The south and North Pole can be identified by sense of current.

The side where from current seems to be clockwise becomes South Pole and where it seems anticlockwise becomes North Pole.



Fig, 1.

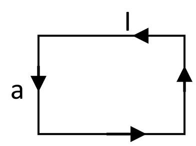


Fig. 2.

In Fig. 1. Current appears to be clockwise from outside the paper and becomes South Pole. From the back of the paper, it seems anti clockwise; hence this side becomes north pole. As the magnetic moment is directed from south to North Pole, it is perpendicular to the paper inwards.

Magnitude of dipole moment is $M = IA = \pi R^2 I$

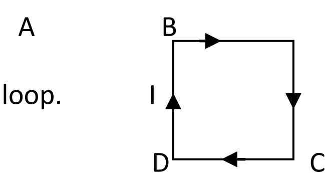
In Fig. 2. In front of paper, North pole and back of the paper, South pole, so direction of magnetic dipole moment is perpendicular to the paper in outward direction.

Second method: Right hand curl rule

The curl fingers show the direction of current in a loop and right hand thumb points the direction of dipole moment.

Calculation of magnetic dipole moment ($\vec{M}$): The methods discussing here is in addition to mentioned above.

Method 1: This method is useful for a rectangular or square loop.



$$\vec{M} = I(\overrightarrow{AB} x \overrightarrow{BC}) = I(\overrightarrow{BC} x \overrightarrow{CD}) = I(\overrightarrow{CD} x \overrightarrow{DA}) = I(\overrightarrow{DA} x \overrightarrow{AB})$$

Here, cross product of any two consecutive sides (taken in order) gives the area as well as the correction direction of $\vec{M}$ also.

Torque in magnetic field:

So, $\vec{\tau} = \vec{M} x \vec{B} ----(6)$

Note:

This formula is derived for a rectangular loop but it is true for shape of loop. Following points are worth noting regarding the torque acting on the loop in uniform magnetic field.

Magnitude of $\vec{\tau}$ is MB Sin θ. Here θ is the angle between $\vec{M}$ and $\vec{B}$. Torque is zero when θ=0^0 or 180^0 and it is maximum when θ=90^0.

The expression for the magnetic dipole can be obtained from the expression for the electric dipole by replacing $\vec{P}$ by $\vec{M}$ and $\varepsilon_0 by$ $\frac{1}{\mu_0}$. Here μ_0 is the permeability of free space. It is related with ε_0 and speed of light c as, $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}, where, \mu_0 = 4\pi x 10^{-7}$ T-m/A.

Biot Savart law: As in electrostatic field, there are two methods to calculate electric field intensity at a point. One is Coulomb's law and another method is Gauss's law which is useful for calculating the electric field of a highly symmetric configuration of charge. Similarly in magnetic, there are two basic methods, one is Biot Savart law and another is Ampere's law which is also useful in calculating the magnetic field of a highly symmetric configuration carrying a steady current.

On experimental observations, Biot and Savart found that magnetic field $\overrightarrow{dB}$ at a point P associated with a length element $\overrightarrow{dl}$ of a wire carrying a steady current I is as-

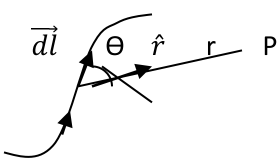
The vector $\overrightarrow{dB}$ is perpendicular to both $\overrightarrow{dl}$ (which points in the direction of current) and unit vector $\hat{r}$ directed from $\overrightarrow{dl}$ to P.

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi} I \left(\frac{\overrightarrow{dl} x \vec{r}}{|\vec{r}|^3}\right) = \frac{\mu_0}{4\pi} I \left(\frac{\overrightarrow{dl} x \hat{r}}{r^2}\right)$$

And magnitude of $\overrightarrow{dB}$ is $|\overrightarrow{dB}|$ or $dB = \frac{\mu_0}{4\pi}\left(\frac{Idl\sin\theta}{r^2}\right)$ here $\frac{\mu_0}{4\pi} = 10^{-7}$ T-m/A

Here $\overrightarrow{dB}$ is field created by the current in only a small length element $\overrightarrow{dl}$ of the conductor. Total field created by current of all elements in a finite size of conductor, $\vec{B}$ is calculated as-

$$\vec{B} = \frac{\mu_0}{4\pi} I \int \frac{\overrightarrow{dl} x \hat{r}}{r^2}$$

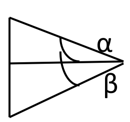


Direction of $\overrightarrow{dB}$:

a) $\overrightarrow{dB}$ ↑↑ $(\overrightarrow{dl} x \hat{r})$ so, $\overrightarrow{dB}$ is along $\overrightarrow{dl} x \hat{r}$

b) If $\overrightarrow{dl}$ is in the plane of paper, $\overrightarrow{dB} = 0$ at all points lying on the straight line passing through $\overrightarrow{dl}$(above and below $\overrightarrow{dl}$).

Magnetic field at a point at distance x due to a finite conductor carrying current I is given by



$$B = \frac{\mu_0 I}{4\pi x}(\sin\alpha + \sin\beta) - - - - - (7)$$

Special Case:

- For an infinitely long conductor, putting α=β=90^0 in equation (12) and (12) becomes as-Magnetic field for an infinitely long conductor, P lies near the middle of conductor at a distance x, the magnitude of $\vec{B}$ is as-

$$B = \frac{\mu_0 I}{2\pi x} - - - - (8)$$

- Direction of $\vec{B}$: The direction of magnetic field can be determined by Right Hand Curl Rule as:

The right hand curl fingers point the direction of field and the thumb of right hand will be in the direction of current. (as in Fig. 1.a. direction of field at point P is perpendicular and going into the plane of paper $(-\hat{k})$.

- If the one end of the conductor is infinitely long but the other end is finite, then α=90^0 and β=0^0 and magnitude of magnetic field B becomes as-

$$B = \frac{\mu_0 I}{4\pi x} - - - - (9)$$

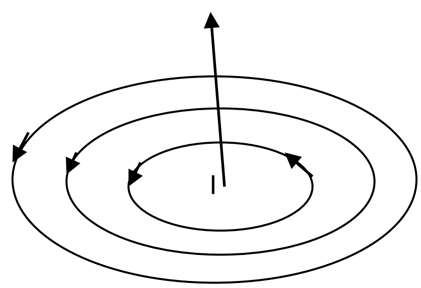
Direction of Magnetic Field:

For an infinitely long conductor, the magnitude of magnetic field $\vec{B}$ is given as-

$$B = \frac{\mu_0 I}{2\pi x}$$

Clearly, magnitude of magnetic field will be same at all points located at same distance from the conductor. Hence magnetic lines of force of a straight current carrying conductor are concentric

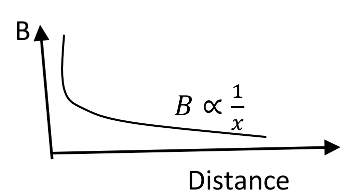
circle and in a plane perpendicular to the wire.



Rules for finding direction of magnetic field:

- Right hand thumb rule-If we held the straight conductor in the grip of right hand in such a way that extended thumb points in the direction of current, then the direction of curl of fingers will give the direction of magnetic field.
- Maxwell's cork screw rule-If a right handed screw to be rotated along the wire in the direction of current, then the direction in which the thumb rotates, gives gives the direction of magnetic field.

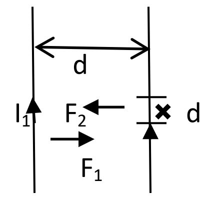
Variation of magnetic field with distance from infinite straight carrying conductor:



Derivation-1: Force between parallel conductors carrying current I_1 and I_2

Suppose two wires W_1 and W_2 are carrying current I_1 and I_2 and both are separated by distance d and placed in vacuum. It is being derived the force on conductor W_1 due conductor W_2 and vise-versa.

W_1 W_2



Magnetic field on $\overrightarrow{dl}$ at wire W_2 due to wire W_1 is given by

$\vec{B}_2 = \frac{\mu_0 I_1}{2\pi d}$

Hence, force at element dl on W_2 is $\overrightarrow{dF_2} = I_2 \overrightarrow{dl} x \overrightarrow{B_2}$

The direction of force is pointing towards W_1 as Fleming Left hand rule.

Magnitude of force on element dl due to wire W_1 is given by-($\theta = 90^0$ as $\overrightarrow{dl} \perp \vec{B}$. So,

$$dF = \frac{\mu_0}{2\pi}\left(\frac{I_1 I_2 dl}{d}\right) \Rightarrow \frac{dF}{dl} = \frac{\mu_0}{2\pi}\left(\frac{I_1 I_2}{d}\right) ----(10)$$

If we take an element dl on wire W_1 and calculating magnetic force per unit length of wire W_1 due to wire W_2, we will get the same result as given by equation (16).

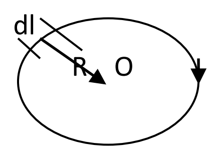
Note:

- When two parallel wires carrying current in same direction, the magnetic force acting between them is attractive force and
- When two parallel wires carrying current in opposite direction, the magnetic force acting between is repulsive force. [applying Fleming left hand rule]

Definition of Ampere: If two parallel long wires kept 1 m apart in vacuum, carry equal current in same direction and there is a force of attraction of $2x10^{-7}$ Nm^{-1} of each wire, the current in each wire is said to be 1 A.

The magnitude of magnetic field at distance x from long conductor is given as,

$B = \frac{\mu_0 I}{2\pi x}$



Derivation -2:Magnetic field at the centre of circular loop:

Let the radius of circular loop be r, and taking small current element dl on the periphery of single circular loop. Now,

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi} I \frac{\overrightarrow{dl} x \vec{r}}{r^3}$$

Since, $\overrightarrow{dl} \perp \vec{r}$, so magnitude of $\overrightarrow{dB}$ at the centre of loop due to $\overrightarrow{dl}$ is given as,

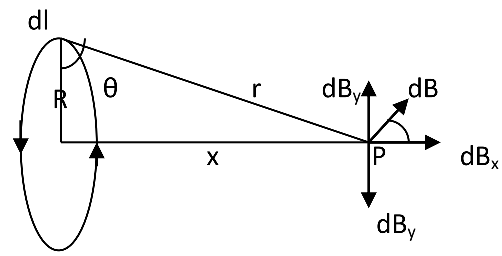
$$dB = \frac{\mu_0 I}{4\pi}\left(\frac{dl}{r^2}\right) \Rightarrow B = \frac{\mu_0 I}{4\pi r^2}\int_0^{2\pi r} dl = \frac{\mu_0 I}{4\pi r^2}(2\pi r) = \frac{\mu_0 I}{2r}$$

If instead of single loop, there are coils of N turns, all wound over one another, then

$$B = \frac{\mu_0 N I}{2r}$$

The direction of field is perpendicular to the plane of paper and going in to it.

Derivation-3: Magnetic field on the axis of circular loop of radius R at distance x:



$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\left(\frac{I\overrightarrow{dl}x\vec{r}}{r^3}\right)$$

In magnitude form, $dB = \frac{\mu_0}{4\pi}\left(\frac{Idl\sin\theta}{r^2}\right) = \frac{\mu_0}{4\pi}\left(\frac{Idl}{r^2}\right)$ [As θ=90^0]

As $\overrightarrow{dB} \perp \vec{r}$ and θ is the angle between $\overrightarrow{dl}\ and\ \vec{r}$, the vertical components of $\overrightarrow{dB}$ due to current element $\overrightarrow{d\,l}$ on top and bottom portion of ring cancels each other as shown in the figure. So, magnitude of horizontal component of dB is dB cos θ and this dB cos θ is the field to be derived.
So, magnitude of magnetic field on axial position at distance X from the centre of ring is given by

$$dB_x = \frac{\mu_0 I}{4\pi}\left(\frac{dl}{r^2}\right)\cos\theta ----(1)$$

Since, $r = \sqrt{R^2 + x^2}$ and $\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{R^2+x^2}}$. So, from equation (1),

$dB_x = \frac{\mu_0 I}{4\pi}\left(\frac{Rdl}{(R^2+x^2)^{\frac{3}{2}}}\right)$, So total magnetic field at axial position due to all element of ring is given as,

$B_x = \frac{\mu_0 IR}{4\pi\left[(R^2+x^2)^{\frac{3}{2}}\right]}\int_0^{2\pi R} dl = \frac{\mu_0 IR^2}{2(R^2+x^2)^{\frac{3}{2}}}$, S0, $B_x = \frac{\mu_0 IR^2}{2(R^2+x^2)^{\frac{3}{2}}} ----(2)$, B_y =0, B_z=0

From equation (2), we see that magnetic field at the centre of ring is, $B_o = \frac{\mu_0 I}{2R} ----(3)$
For N circular loops, $B_0 = \frac{\mu_0 NI}{2R}$ and the direction of field is going into the plane of paper.
When x>>R, from equation (2), for N circular loops,

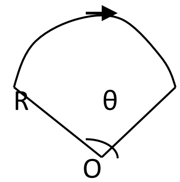
$$B_x = \frac{\mu_0 NIR^2}{2x^3} = \frac{\mu_0 NI}{4\pi}\left(\frac{2\pi R^2}{x^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{2NIA}{x^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{2M}{x^3}\right) ----(4)$$

Where, magnetic moment of the circular loop is, M=NIA, ($A = \pi R^2$)

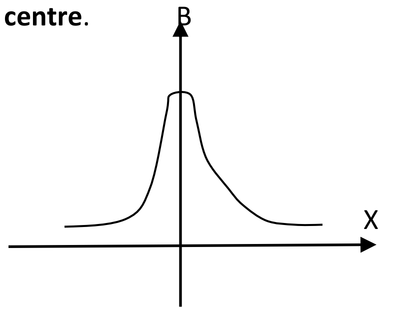
Magnetic field due to an arc of a circle at the centre is given by-

$$B = \left(\frac{\theta}{2\pi}\right)\frac{\mu_0 I}{2R} - - - - - (5)$$

Note: Magnetic field is maximum at the centre and decreases when we move away from the centre.



The B-x graph is shown as-



Magnetic Field along the axis of a solenoid:

Magnetic field at the centre, (L>>R), $B(centre) = \mu_0 nI$

Magnetic field at the end of solenoid is, (L>>R), $B(end) = \frac{1}{2}\mu_0 nI$

Ampere Law or Ampere Circuital Law:

It states that line integral of $\vec{B}$ around a closed path is equal to μ_0 times current enclosed by that path, provided that electric field inside the loop remains constant. That is,

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 I_{net} - - - - - (1)$$

If the path does not enclose the current, then $\oint \vec{B}.\overrightarrow{dl} = 0$

We know that work done by the static electric force around any closed path is zero.

$$Bl = \mu_0 I_{net} - - - - - - - (2)$$

This equation (1) can be used only under following conditions.

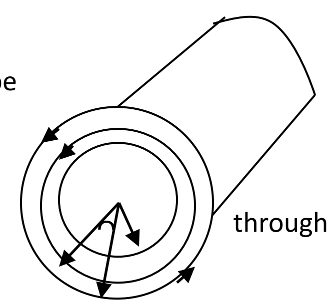
- At every point of closed path, $\vec{B} \uparrow\uparrow \overrightarrow{dl}$.
- Magnetic field has the same magnitude B at all places on the closed path.

Derivation-4: Magnetic field created by a long current carrying wire

Suppose a long straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire.

- When r >R,

From, symmetry, $\vec{B}$ must be constant in magnitude And parallel to $\overrightarrow{dl}$. The total current passing through the plane of the circle Is I. Applying Ampere's law,



$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 I_{net}$$

So, $B(l) = B(total\ length\ of\ Ampere\ circular\ path) = B(2\pi r) = \mu_0 I$ or, $B = \frac{\mu_0 I}{2\pi r}$

- When r = R, $B = \frac{\mu_0 I}{2\pi R}$
- When r < R, $B(2\pi r) = \mu_0 I' - - - - - (1)$

Here $\frac{I'}{I} = \frac{\pi r^2}{\pi R^2} \Rightarrow I' = I\frac{r^2}{R^2}$

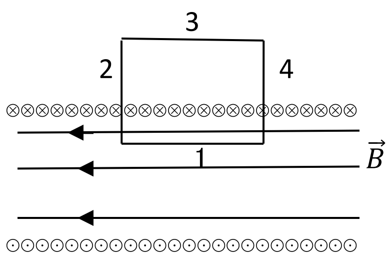
Hence, from equation (1),

$$B = \frac{\mu_0 I r}{2\pi R^2} - - - - (2)$$

This relation (2) is similar to electric field inside a uniformly charged sphere.

Derivation-5: Magnetic field of a solenoid using Ampere circuital law:

A solenoid is a long wire wound in the form of helix. One end the solenoid behaves like a north pole and other end behaves like a south pole. As the length of solenoid increases, interior field becomes more uniform. An ideal solenoid is approached when turns are closely spaced and the length is much greater than the radius of turns. In this case external field is zero and the interior field is uniform over a great volume. Consider a rectangular path of length l and width w. Applying Ampere's law to this path,



For side 2 and 4, $\vec{B} \perp \vec{dl}$

For side 3, B=0 as side 3 is outside solenoid. Now, magnetic field due to side 1 is given by-

$\oint \vec{B}.\vec{dl} = \mu_0 I = Bl$ (as $\vec{B}$ is uniform and parallel to $\vec{dl}$.

If the solenoid has n turns per unit length, then number of turns in length l of the solenoid=nl, so,

$Bl = \mu_0 nlI \Rightarrow B = \mu_0 nI$, Hence, magnetic field inside a solenoid is given by-

$$B = \mu_0 nI - - - - - (1)$$

Toroid: A device which is often used to create almost an uniform magnetic field in some enclosed area. The device consists of a conducting wire wrapped around a ring (a torus) made of a non conducting material. For a toroid having N closely spaced turns of wires, it is being calculated the magnetic field in the region, occupied by the torus, a distance r from centre.

Derive-6: Magnetic field in the toroid: Using Ampere circuital law,

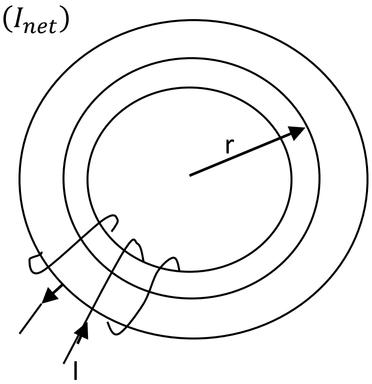
$\oint \vec{B}.\vec{dl} = Bl = B(2\pi r) = \mu_0(I_{net})$

Furthermore, the circular

Path encloses N turns, so

$B(2\pi r) = \mu_0(NI)$, or

$B = \frac{\mu_0 NI}{2\pi r}$



Here, $B \propto \frac{1}{r}$ and hence it is nonuniform in the region occupied by torus. However, if r is very large compared with cross sectional radius of torus, then the field is approximately uniform inside the torus. In that case, $\frac{N}{2\pi r} = n = number\ of\ turns\ per\ unit\ length\ of\ torus$, therefore,

$$B = \mu_0 nI$$

For an ideal toroid, in which turns are closely spaced, external magnetic field is zero. This is because the net current passing through any circular path lying outside the toroid is zero.

Magnetic field of a moving point charge:

$B = \frac{\mu_0}{4\pi}\left(\frac{qv\sin\theta}{r^2}\right)$.

Here, θ is the angle between $\vec{v}\ and\ \vec{r}$. It is zero at $\theta=0^0$ and maximum at $\theta = 90^0$.

The direction of $\vec{B}\ is\ along\ \vec{v}x\vec{r}$ if q is positive and opposite to $\vec{v}x\vec{r}$ if q is negative.

Moving coil Galvanometer:

The moving coil is a device used to measure an electric current.

Principle: Action of moving coil galvanometer is based upon the principle that when a current carrying coil is placed in a magnetic field, it experiences a torque whose magnitude depends on the magnitude of current.

Theory: The current to be measured is passed through the galvanometer. As the coil is in the magnetic field, it experiences a torque given by-

$$\tau = MB\sin\theta = (NIA)B\sin\theta ----(1)$$

N: Total number of turns of the coil, I: current passing through the coil, A: Area of cross section of the coil, B: magnitude of radial magnetic field

The pole pieces are made cylindrical, the magnetic field always remains parallel to the plane of the coil. That is angle between $\vec{M}\ and\ \vec{B}$ always remains 90^0. In equation (1), this torque is balanced by spring in galvanometer, called spring torque or counter torque as in equilibrium,

$$k\varphi = NIAB$$

Here, k is the torsional constant of the spring. So, $I = \left(\frac{k}{NAB}\right)\varphi$

Hence, the current $I \propto \varphi$; $Galvanometer\ constant = \frac{k}{NAB}$

Sensitivity of Galvanometer: A galvanometer is said to be sensitive if it shows large scale deflection even when a small current is passed through it or small voltage is applied across it.

There are two types of sensitivities: (i) Current sensitivity and (ii) Voltage sensitivity

Deflection per unit current is called current sensitivity.

$$current\ sensitivity = \frac{\varphi}{I} = \frac{NAB}{k}$$

The sensitivity of a galvanometer can be increased by-

- Increasing the number of turns (N) in the coil or,
- Increasing the magnitude of magnetic field

Deflection produced in the galvanometer when a unit P.D. is applied across its ends.

$$Voltage\ sensitivity = \frac{\varphi}{V} = \frac{\varphi}{IR} = \frac{NBA}{kR}$$

So, $Current\ sensitivity = R(Voltage\ sensitivity)$

Factors on which sensitivity of moving coil depends upon:

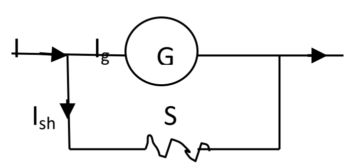
- Number of turns N in its coil
- Magnetic field B,
- Area A of the coil and
- Torsion constant k of the spring and suspension

How moving coil galvanometer can be increased?

- By increasing number of turns of the coil but N cannot be increased beyond a certain limit because the coil makes galvanometer bulky and increase its resistance
- By increasing its magnetic field and this can be done by using a strong horse shoe magnet and placing a soft iron core within the coil
- By increasing area of the coil but beyond certain limit, it will make galvanometer bulky
- By decreasing the value of torsion constant and this can be done by using spring and suspension wire of phosphor bronze.

Conversion of galvanometer in to an ammeter:

$$I = I_g + I_S$$



$$I_g R_g = I_{sh} R_{sh} = (I - I_g) R_{sh} \Rightarrow (R_g + R_{sh}) I_g = I R_{sh} \Rightarrow I_g = I \frac{R_{sh}}{R_{sh} + R_g}$$

Hence an ammeter is a shunted or low resistance galvanometer.

Its effective resistance is, $R_A = \frac{R_g R_{sh}}{R_g + R_{sh}} < R_{sh}$

Shunt is low resistance which is connected in parallel with galvanometer to protect it from strong currents.

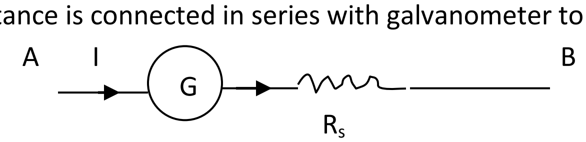
Uses of shunt:

- To prevent a galvanometer from being damaged due to large current
- To convert a galvanometer into ammeter
- To increase the range of an ammeter

Conversion of galvanometer into a voltmeter:

A voltmeter is a device to measure potential difference between two points and connected across the circuit element of which potential difference (or voltage drop) to be measured.AS a result, a small part of the total current passes through the voltmeter and so the current through the circuit element decreases. This decreases the P.D. across the circuit element being measured. To avoid this, a voltmeter is designed to have very high resistance. An ideal voltmeter has infinite resistance.

A high resistance is connected in series with galvanometer to make a voltmeter.



Let $V_A - V_B = V$, then, $V = I(R_g + R_s)$

And total resistance of the circuit (resistance of voltmeter) is given by-

$$R_V = R_g + R_s \gg R_g$$

Difference between Ammeter and Voltmeter-

Ammeter	Voltmeter
It is a parallel combination of galvanometer and shunt resistance (low resistance).	It is a series combination of galvanometer and a very high resistance.
$R_A << R_g$	$R_V >> R_g$
An ideal ammeter has zero resistance.	An ideal voltmeter has very high resistance.
An ammeter is connected in series with the circuit to measure current in the circuit.	A voltmeter is connected in parallel with circuit element of which voltage difference being measured.
The ammeter of lower range has a higher resistance than the ammeter of lower range.	The voltmeter of lower range has lower resistance and higher range has higher res.
The range of an ammeter can be increased but it cannot be decreased.	The range of voltmeter can be both increased and decreased.

Derive-7: Expression of magnetic dipole moment of a revolving electron and thus define Bohr's Magneton

Since, $I = \frac{e}{T}$, since $T = \frac{2\pi r}{v}$, so, $I = \frac{ev}{2\pi r}$

Where, T is time period, r is radius of orbit, v is the velocity of electron.

The magnetic moment due to the current,

$$M = IA = \frac{ev}{2\pi r}(\pi r^2) = \frac{evr}{2}$$

If the electron revolves in anticlockwise sense, the current will be in clockwise sense. Hence, according to right hand rule, the direction of magnetic moment will be perpendicular to the plane of orbit and directed inward to the plane. (The front side is the direction of current in clockwise (S), so backward side is anticlockwise (N) and hence magnetic moment is from south to north, perpendicular and going into the plane of paper). Again, magnetic moment , $M = \frac{evr}{2} = \frac{mvre}{2m}$

The angular orbital momentum L=mvr, so, $M = \frac{eL}{2m}$, and in vector form, $\vec{M} = -e\frac{\vec{L}}{2m}$

Negative sign indicates that M and L are in opposite direction.

From the Bohr's postulates,

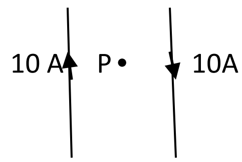
$L = mvr = \frac{nh}{2\pi}$, where, n=1, 2, 3-------, therefore, $M = \frac{e}{2m}\left(\frac{nh}{2\pi}\right) = nM_{min}$

Where, $M_{min} = \frac{eh}{4\pi m}$ *is called Bohr's magneton.*

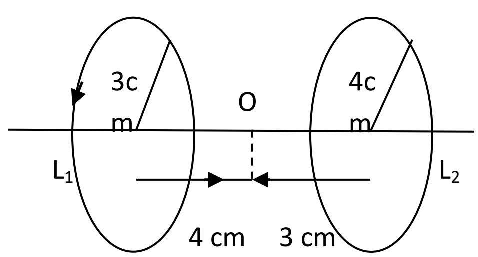
Ch-4: Numerical

1. A charge q is uniformly distributed on a nonconducting disc of radius R. It is rotated with an angular speed ω about an axis passing through the centre of mass of the disc and perpendicular to its plane. Find the magnetic moment of the disc. [$Magnetic\ moment = \frac{q}{2m}(angular\ momentum)$][Ans. $q\frac{R^2\omega}{4}$] [$Angular\ momentum\ L = I\omega = \frac{1}{2}mR^2\omega$]

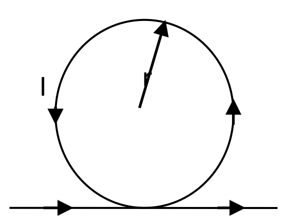
2. As shown below, two long straight wires carrying electric current in opposite directions. The separation between the wires is 5 cm. Find the magnetic field at point P midway between the wires. [Ans. 160 μT]



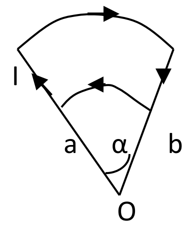
3. Two long straight wires each carrying an electric current of 5 A, are kept parallel to each other at a separation of 2.5 cm. Find the magnitude of magnetic force by 10 cm of a wire. [Ans. 2 x 10^{-5} N]
4. Use Biot Savart law, to obtain an expression for magnetic field at the centre of a coil bent in the form a square of side 2a carrying current I.[Ans. $\frac{\sqrt{2}\mu_0 I}{\pi a}$]
5. A current of 1 A is flowing in the sides of an equilateral triangle of side 45 mm. Find the magnetic field at the centroid of the triangle. [Ans. 4 x 10^{-5} T]
6. A straight wire carries a current of 3 A. Calculate the magnitude of magnetic field at a point 10 cm away from the wire. [CBSE D 96] [Ans. 6 x 10^{-6} T]
7. The magnetic field due to a current carrying circular loop of radius 12 cm at its centre is 0.50 x 10^{-4} T. Find the magnetic field due to this loop at a point on the axis at a distance 5 cm from the centre. [Ans. 3.9 x 10^{-5} T]
8. Two identical circular coil of radius 0.1 m each having 20 turns are mounted co-axially o.1 m apart. A current of 0.5 A is passed through both of them (i) in the same direction, (ii) in the opposite direction. Find the magnetic field at the centre of each coil. [Ans. (i) 8.5 x 10^{-5} T, (ii) 4.06 x 10^{-5} T]
9. Two co-axial circular loops L_1 and L_2 of radii 3 cm and 4 cm are placed as shown. What should be the magnitude and direction of the current in the loop L_2 so that the net magnetic field at the point O is zero? [CBSE SP 08], [Ans. I_2 =0.56 A, clock wise]Given I_1 = 1A



10. A long wire is bent as shown in the figure below. What will be the magnitude and the direction of the field at the centre O of the circular portion, if a current I is passed through the wire? [Ans. $\frac{\mu_0 I}{2r}\left(1+\frac{1}{\pi}\right)$]



11. Figure below shows a current loop having two circular segments and joined by two radial lines. Find the magnetic field at the centre O. [Ans. $\frac{\mu_0 I \alpha (b-a)}{4\pi ab}$]



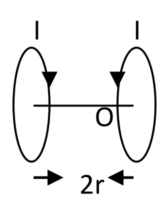
12. The magnetic field B due to a current carrying circular loop of radius 12 cm at its centre is 0.5 x 10^{-4} T. Find the magnetic field due to this loop at a point on the axis at a distance of 5 cm from the centre. [Ans. 3.9 x 10^{-5} T]
13. Two moving coil meters M_1 and M_2 having the following particulars:

R_1=10 Ω, N_1=30, A_1=3.6 x 10^{-3} m^2, B_1=0.25 T, R_2=14 Ω, N_2=42, A_2=1.8 x 10^{-3} m^2, B_2=0.5T

The spring constants are identical for the two meters. Find the ratio of Current sensitivity and [Ans. 1.4] and Voltage sensitivity of M_2 and M_1 [Ans. 1] [NCERT]

14. A galvanometer coil has a resistance of 12 Ω and the meter shows full scale deflection for a current of 3 mA. How will you convert the meter into a voltmeter of range 0 to 18 V? [Ans. By converting 5988 Ω resistance in series with galvanometer, here potential difference is taken as 18 V] [NCERT]
15. A galvanometer coil has a resistance of 15 Ω and the meter shows full scale deflection for a current of 4 mA. How will you convert the meter into an ammeter of range 0 to 6 A? [Ans. By shunting resistance of 0.01 Ω with galvanometer]
16. A solenoid of length 5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of magnetic field inside the solenoid?[Ans. 6.28 x 10^{-3} T][NCERT]
17. A toroid has a core of inner radius 25 cm and outer radius is 26 cm, around which 3500 turns of a wire are wound, if the current is 11 A. What is the magnetic field (i) outside the toroid, (ii) inside the core of toroid, (iii) in the empty space surrounded by the toroid? [NCERT] [Ans. (i) zero, (ii)3.02 x 10^{-2} T, (iii) zero]
18. Two identical circular loops P and Q , each of radius r and carrying equal currents, are kept in the parallel planes having a common axis passing through O. The direction of current in P is clockwise and in Q is anti-clockwise as seen from O. Find the magnitude of net magnetic field at O. [Hint: Loop P as seen from point O becomes south pole, so magnetic field at point O is towards P. Loop Q as seen from point O becomes north pole and magnetic field at O is from Q to O. The net magnetic field at point O is from Q to P]

[Ans. $B = \frac{\mu_0 I}{2^{\frac{3}{2}} r}$]



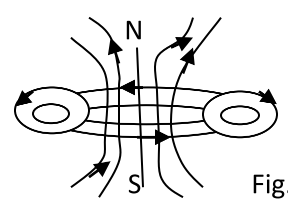


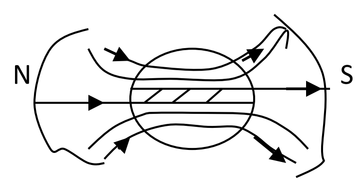
Fig. 1-39

19. Draw the magnetic field lines due to a current carrying loop. [Hint: shown in Fig. 1-39] [Delhi 2013 C]

20. State the underlying principle of cyclotron. Write briefly how this machine is used to accelerate charged particle to high energy. [Delhi 2014]
Answer: **Principle:** It works on the principle that positively charged ions can be accelerated to high kinetic energy by making it pass again and again smaller value of same oscillating electric field by making use of strong perpendicular magnetic field.
Also, the frequency of charged particle must be equal to the frequency of oscillating
In cyclotron, the Lorentz force is utilised. Force is acting perpendicular to both velocity and magnetic field. The frequency of cyclotron frequency, $f = \frac{qB}{2\pi m}$.
The combination of crossed electric and magnetic field is used to increase the energy of the charged particle.
Cyclotron uses the fact that the frequency of revolution of charged particle in a magnetic field is independent of its energy.
Inside the dees, the particle is shielded from the electric and magnetic field acts on the particle and makes it to go round a circular path inside a dee.
Every time the particle moves from one dee to the other it comes under the influence of electric field which ensures to increase the energy of the particle as the sign of the electric field changed alternatively.
21. What is the function of radial magnetic field and cylindrical soft iron?
Principle of moving coil galvanometer: It is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque. Deflecting torque = Restoring torque, that is,

$$NIBA = k\theta \Rightarrow I = \frac{k}{NBA}\theta \Rightarrow I \propto \theta$$

It means that deflection produced is directly proportional to the current flowing through the galvanometer.



Function of radial magnetic field: It is a field in which coil of galvanometer always remain parallel to field even on large deflection and area vector of coil is perpendicular to the magnetic field. It always exerts maximum torque on the coil.
Function of cylindrical soft iron: The core which not only makes the field radial but also increases the strength of the magnet.
22. Why does increasing current sensitivity not necessarily increase voltage sensitivity?[All India 2009]
Answer: Current sensitivity= $\frac{NBA}{k}$, so, current sensitivity can be increased by increasing number of turns, means resistance is increased. But, voltage sensitivity $= \frac{NBA}{kR}$, and increased R by increasing number of turns affect adversely voltage sensitivity(by term resistance (R) in the expression of voltage sensitivity). So, voltage sensitivity may not increase.
23. A straight wire carries a current of 3 A. Calculate the magnitude of magnetic field at a point 15 cm away from the wire. [Ans. 4 x 10^{-6} T]

24. A solenoid of length 1 m and diameter 3 cm has 5 layers of windings of 850 turns each and carries a current of 5 A. What is the magnetic field at the centre of solenoid? Also, calculate the magnetic flux from a cross section of the solenoid at the centre of the solenoid. [Ans. 2.67×10^{-2} T, 1.89×10^{-5} Wb] [All India 2011]
25. A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. If the current carried is 8A, estimate B inside solenoid.[Ans. 2.5×10^{-2} T] [NCERT]