2.0. Introduction

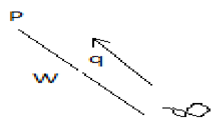
In the previous chapter, we studied **electric force** and **electric field**, which describe how charges exert forces on one another.

However, in many practical situations, it is more convenient to describe an electric field in terms of **energy** rather than force. This leads to the concepts of **electrostatic potential** and **potential difference**.

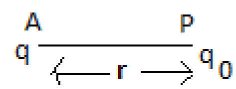
Just as a body possesses **gravitational potential energy** due to its position in a gravitational field, a charge possesses **electrostatic potential energy** due to its position in an electric field.

2.1. ELECTROSTATIC POTENTIAL:

1. Electric Potential is defined as the work done to bring a unit positive charge from infinity to a point against the electric field. It can also be defined as the potential energy per unit test charge at a point. It is scalar quantity. Its SI unit is volt (V). **Fig.2.1-1**



$$\boxed{i \xrightarrow{W} f = Q\left(V_f - V_i\right)}$$



$$\boxed{V_P = Lim_{q_0 \to 0} \frac{U}{q_0}}$$

2.2. RELATION BETWEEN POTENTIAL DIFFERENCE, WORK DONE AND POTENTIAL ENERGY:

1. By the conservative force: work done to move a point charge q from initial point i to final point j:

$$\boxed{W_{i \to f} = q\left(V_i - V_f\right) = U_i - U_f = -\Delta U}$$

2. Against the conservative force or by the external force: work done to move a point charge q from initial point i to final point j:

$$\boxed{W_{i \to f} = q\left(V_f - V_i\right) = \left(U_f - U_i\right) = \Delta U}$$

3. If nothing is given to find the work done by the conservative force or by the external force, it is simply asked to find or calculate the work done, then apply the formula mentioned above in point 3).

Example-1:

Calculate the work done to move a charge of 1 μC from infinity to a point which has the potential of 12 V.

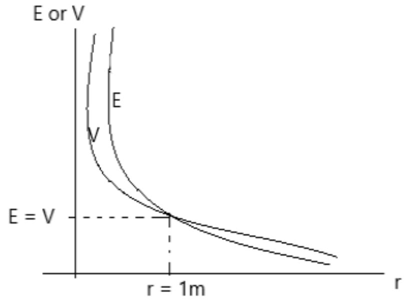
Solution:

Since, $W_{\infty \to A} = q(V_A - V_\infty) = 1 \times 10^{-6}(12 - 0) = 12 \times 10^{-6} = 1.2 \times 10^{-5}$

Answer: $\boxed{1.2 \times 10^{-5} V}$

2.3. Electric Potential and Electric Field due to a point Charge at distance r:

Graph-



2.4. POTENTIAL DUE TO A POINT CHARGE:

1. Electric potential at a point (sat at point P) **due to a positive point charge** q located at a point (say at A) is given as:

$$\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q}{r}\right)}$$

Where r = distance AP. whereas, **due to a negative point charge**, potential at distance r from point P is:

$$\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{-q}{r}\right)}$$

2. As distance increases, potential decreases due to positive point charge, and potential increases due to negative point charge. In the figure below, AB = r_1 and AC = r_2; since $r_2 > r_1$ therefore, due to + q charge, $V_C < V_B$ and due to –q charge, $V_C > V_B$.

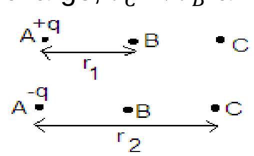


Fig.2.3-2

3. Graph between V and r (both are inversely proportional) due to positive point charge and negative point charge, is given below: $V \propto \left(\frac{+q}{r}\right)$ and $V \propto \left(\frac{-q}{r}\right)$

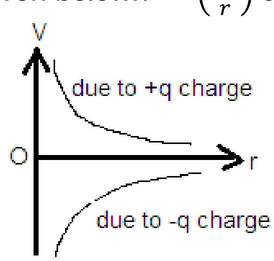


Fig. 2.3-3

4. Electric potential at infinity as well as potential energy at infinity is equal to zero. The Electric potential is a scalar quantity, so, it has no direction. and its SI unit is volt. 1 V = 1 J/s.

2.5. POTENTIAL DUE TO A SYSTEM OF CHARGES:

1. Electric potential at a point due to several charges q_1, q_2, q_3 ………..which are at distances r_1, r_2, r_3, ..respectively is given as

$$\boxed{V=\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}+-------+\frac{q_n}{r_n}\right)}$$

$$\boxed{V=\left(\frac{1}{4\pi\epsilon_0}\right)\sum_{i=1}^{n}\frac{q_i}{r_i}}$$

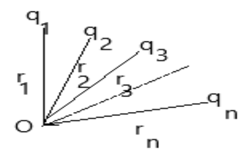


Fig.2.4.1-4: Electrical Potential at point O due to several charges

2. Electric potential at a point (say at P) due to several charges $(q_1, q_2, q_3, \ldots.)$ which are at same distance from point P, say at distance r in the case of a square, is given as

$$V=\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_{net}}{r}\right) \ where\ q_{net}=q_1+q_2+q_3+\cdots---(8)$$

3. Electric potential at a point (say at point P) at position vector $\vec{r}$ due to charges $q_1, q_2, q_3, \ldots.$. which are placed at position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots$respectively, given as:

$$V=\sum_{i=1}^{n}\frac{q_i}{|\vec{r}-\vec{r}_i|}--(9)$$

Example-2:

The four charges, +q, -q, + q –q are located at each vertex of a square ABCD respectively. Calculate the electric potential at a point O of a square of each side L.

Answer:

Since, electric potential at a point at distance r due to a positive charge q is given as: $V=\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q}{r}\right)$.

So, $V_O=V_{OA}+V_{OB}+V_{)C}+V_{OD}=\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{\sqrt{2}q}{a}+\frac{\sqrt{2}(-q)}{a}+\frac{\sqrt{2}q}{a}+\frac{\sqrt{2}q}{a}\right)=0$

2.6. POTENTIAL DUE TO AN ELECTRIC DIPOLE:

1. Suppose a dipole (+q and –q) is separated by a distance 2L and we have to find the electric potential at distance r from the centre of dipole and inclined an angle θ from the dipole axis, then it is given as:

$$V=\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p\cos\theta}{r^2}\right)$$

Derivation:

As shown in the figure2.5.1-5,

Let AP = r_1, PB = r_2; $AP \approx PC\ and\ PD \approx PB$

$r_1 = PO + OC = r + l\cos\theta;$

$r_2 = PO - O\,D = r - l\cos\theta,$

Hence, Electric Potential at point P due to dipole is

$V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{-q}{r_1} + \frac{q}{r_2}\right)$

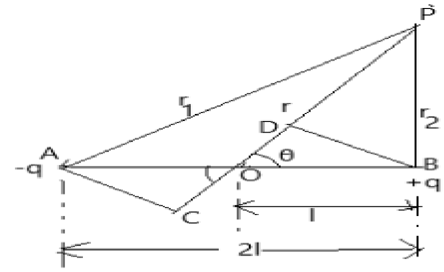


Fig.2.5.1-5: Potential due to a dipole

$$Or, V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{-q}{r + l\cos\theta} + \frac{q}{r - l\cos\theta}\right)\ Or, \qquad V = \left(\frac{q}{4\pi\epsilon_0}\right)\left(\frac{2l\cos\theta}{r^2 - l^2 cos^2\theta}\right)$$

Since, dipole moment p = 2lq and for r >> l, $l^2 cos^2\theta$ is neglected. So, the expression of electric potential due to dipole at distance r from the centre of dipole, makes an angle θ is,

$$\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p\cos\theta}{r^2}\right) - -Derived}$$

From the above equation,

2. Axial Position of Dipole:

The electric potential at Axial position of dipole is: $\theta = 0^0\ Or, 180^0$

$$\boxed{V_{axial} = \pm\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p}{r^2}\right)}$$

3. Equatorial Position of Dipole: $\theta = 90^0$, $\cos 90^0 = 0$

$$\boxed{V_{equatotial} = 0}$$

2.7. POTENTIAL ENERGY OF A SYSTEM OF TWO CHARGES WHEN NO EXTERNAL FIELD:

1. Suppose two charges $q_1$and q_2 are initially at infinity and determine the work done by an external agency to bring the charges to the locations of position vectors $\vec{r}_1\ and\ \vec{r}_2$ relative to some origin.

Derivation:

Suppose the first charge q_1 is brought from infinity to a point of position $\vec{r}_1$. There is no external field against which work to be done. So, work done in bringing q_1 from infinity to $\vec{r}_1$is zero. So potential at point P (position vector $\vec{r}_2$) at distance r_{12} from point charge q_1 (position vector $\vec{r}_2$) is:

$$V_1 = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1}{r_{12}}\right)$$

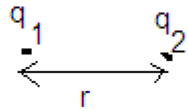
Now, work done to bring charge q_2 from infinity to point P whose position vector $\vec{r}_2$ is:

$$W = q_2 V_1 = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1 q_2}{r}\right) \; where \; r = r_{12}$$

Since, electric force is a conservative force, so, this work is stored as potential energy. Hence,

$$U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r}\right) \quad \dots(\boldsymbol{Derived})$$

So, potential energy is directly proportional to the product of two charges and inversely proportional to the separation between two charges.



$$U = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1 q_2}{r}\right)$$

2. When a charge q is brought near to another positive charge Q, its potential energy increases to overcome the repulsive force as shown in the above equation and its kinetic energy decreases as the total energy is conserved.
3. The charge which is brought from infinity to a point against the electric force has no acceleration. This means it is brought with constant velocity.
4. Potential Energy of the system due to various charges $q_1, q_2, q_3, ..$ at positions $r_1, r_2, r_3, ..$is given as:

$$\boxed{U = \sum_{i=1\, i<j}^{n} \frac{q_j q_i}{r_j i}}$$

Example-3

Three charges of $1\mu C$ *are placed* at the vertex A, B, C of an equilateral triangle of side 10 cm. Find the total potential energy of the system.

Solution:

Since,

$$U = 9 \times 10^9 \left(\frac{q_3 q_2}{r_{32}} + \frac{q_3 q_1}{r_{31}} + \frac{q_2 q_1}{r_{21}}\right)$$

$$= 9 \times 10^9 \left(\frac{1 \times 10^{-6} \times 1 \times 10^{-6}}{10 \times 10^{-2}} + \frac{1 \times 10^{-6} \times 1 \times 10^{-6}}{10 \times 10^{-2}} + \frac{1 \times 10^{-6} \times 1 \times 10^{-6}}{10 \times 10^{-2}}\right)$$

$$= 9 \times 3 \times 10^9 \times 10^{-12} \times 10^1 = 0.12\, J$$

2.8. POTENTIAL ENERGY OF A SINGLE CHARGE IN AN EXTERNAL FIELD:

Potential energy of charge q at position $\vec{r}$ in an external electric field is:

$$U = qV_{(\vec{r})}$$

Where $V_{\vec{r}}$ is the external potential at the point $\vec{r}$.

2.9. POTENTIAL ENERGY OF A SYSTEM OF TWO POINT CHARGES IN AN EXTERNAL ELECTRIC FIELD:

To find the potential energy of charges q_1 and q_2 located at positions $\vec{r}_1$ and $\vec{r}_2$ respectively in an external field.

Derivation:

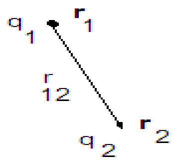


Fig.2.8-6

Work done to bring charge q_1 from infinity to a point A at position vector $\vec{r}_1$in the external electric field is:

$$W_1 = q_1 V_{(\vec{r}_1)} - - - - - (i)$$

Work done to bring the charge q_2 from infinity to a point B at position vector $\vec{r}_2$ in the external electric field = Work done on charge q_2 against the external electric field + work done on q_2 against the electric field due to charge q_1.

Now, work done on charge q_2 against the external electric field:

$$W_2 = q_2 V_{(\vec{r}_2)} - - - - - - (ii)$$

And, work done on q_2 against the electric field due to the charge q_1:

$$W_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}}\right) - - - - - (iii)$$

Where, r_{12} = distance between charge q_1 and q_2.

Now, potential energy of the system = Total work done in assembling the configuration.

So,

$$U = W_1 + W_2 + W_3 = q_1 V_{(\vec{r}_1)} + q_2 V_{(\vec{r}_2)} + \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1 q_2}{r_{12}}\right) - - - (\boldsymbol{Derived})$$

Example-4: (NCERT Example 2.5)

(a) Determine the electrostatic potential energy of a system consisting of two charges $7\ \mu C$ and -2μC with no external field placed at $(-9\ cm, 0, 0)$ and $(9\ cm, 0, 0)$respectively.

(b) How much work is required to separate tow charges infinitely away from each other?

(c) Suppose that the same system of charges is now placed in an external electric field $E = A(1/r^2)$; $A = 9 \times 10^5 NC^{-1}m^2$, what would be electrostatic energy of the combination be?

Solution:

(a) Electrostatic potential energy,

$$U = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1 q_2}{r}\right)$$

Here, r $=r_2 - r_1 = 9 — 9 = 18\ cm,\ q_1 = 7\mu C = 7 \times 10^{-6} C\ and\ q_2 = -2\mu C$

$= -2 \times 10^{-6}$C, therefore,

$$U = 9 \times 10^9 \times \frac{7 \times 10^{-6} \times (-2) \times 10^{-6}}{18 \times 10^{-2}} = -0.7\ J$$

(b) $W = U_f - U_i = 0 - (-0.7\ ) = 0.7\ J$

(c) If both charges are in the external electric field, then potential energy,

$$U = q_1 V_{\vec{r}_1} + q_2 V_{\vec{r}_2} + \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_2 q_1}{r}\right)$$

Here, $V_{\vec{r}_1} = E(r_1) = A\left(\frac{1}{{r_1}^2}\right)(r_1) = 9 \times 10^5 \times \frac{1}{r_1} = 9 \times 10^5 \times \frac{1}{9 \times 10^{-2}} = 10^7 V$

$$V_{\vec{r}_2} = E(r_2) = A\left(\frac{1}{{r_2}^2}\right)(r_2) = 9 \times 10^5 \times \frac{1}{r_2} = 9 \times 10^5 \times \frac{1}{9 \times 10^{-2}} = 10^7 V$$

Therefore,

$$U = 7 \times 10^{-6} \times 10^7 + (-2) \times 10^{-6} \times 10^7 + 9 \times 10^9 \times \left(\frac{(-2 \times 10^{-6})(7 \times 10^{-6})}{18 \times 10^{-2}}\right)$$

$$= 70 - 20 - 0.7 = 50 - 0.7 = 49.3\ J$$

2.10. POTENTIAL ENERGY OF A DIPOLE IN AN EXTERNAL ELECTRIC FIELD:

1. The potential energy due a dipole of dipole moment $\vec{p}$ placed in an external electric field $\vec{E}$ and its dipole axis making angle θ (angle between dipole moment and electric field) is given as:

$$\boxed{U = -pE\cos\theta}$$

From the above formula, mentioned in equation, it can be explained the followings:

a) Potential energy will be zero if a dipole is placed perpendicular to the electric field, θ = 90^0.

b) Potential energy in **stable equilibrium position** (dipole moment vector and electric field vector parallel to each other, θ = 0^0) is minimum and its value is –PE.

c) Potential energy in **unstable equilibrium position** (dipole moment vector and electric field vector anti- parallel to each other, θ = 180^0) is maximum and its value is +PE.

d) Work done to rotate a dipole from angle θ_1 to angle $\theta_2$by an external agent (if not mentioned, apply this formula) as:

$$\boxed{W = pE(\cos\theta_1 - \cos\theta_2)}$$

e) Work done to rotate a dipole from angle θ_1 to angle $\theta_2$by conservative force is:

$$W = pE(\cos\theta_2 - \cos\theta_1)$$

2. Potential energy due to electric dipole in vector form in the external uniform electric field can be expressed as:

$$\boxed{U = -\vec{p} \cdot \vec{E}}$$

2.11. RELATION BETWEEN ELECTRIC FIELD AND ELECTRIC POTENTIAL:

1. The electric field $\vec{E}$ and potential gradient dV/dr is related as:

   $$dV = -\vec{E} \cdot d\vec{r}$$

   From the above formula mentioned in equation (20), if potential at point 1 is V_1 and at point 2, it is V_2, while the point 1 has position vector $\vec{r}_1$ and point 2 has position vector $\vec{r}_2$, then from equation (19), the potential difference between point 1 and point 2, if any plane (such as square, rectangle or triangle) placed in the uniform electric field $\vec{E}$ can be written as:

   $$\int_{V_2}^{V_1} dV = -\int_{r_2}^{r_1} \vec{E} \cdot d\vec{r}\,, where\ d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$$

   Where,

   $$\boxed{\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k}}$$

2. From the expression in equation (20), negative sign shown that electric potential decreases in the direction of electric field.
3. If potential at point 1 is V_1 and at point 2, it is V_2 and distance between these two points is d, then,

   $$\boxed{V_1 - V_2 = Ed}$$

2.12. EQUIPOTENTIAL SURFACE AND ITS PROPERTIES:

1. A surface on which potential is equal at every point is known as equipotential surface. Equipotential surfaces may be a straight line in the case of dipole at the equatorial line and may be a curve in case of dipole.

2.12.1. Properties of equipotential surface:

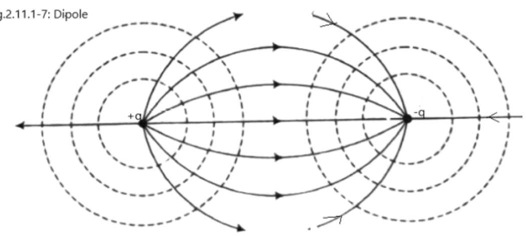
(i) Work done to move a charge on equipotential surface ΔV is zero because potential difference between any two points is zero as: $W = q\Delta V$

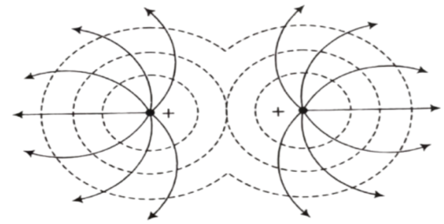
(ii) Electrical field is perpendicular to the equipotential surface. As $dV = 0$ and $dV = -\vec{E} \cdot d\vec{r}$. Hence, $\vec{E} \cdot d\vec{r} = 0$. That is, $\vec{E}$ is perpendicular to $d\vec{r}$.

(iii) Two equipotential surfaces never cross each other. If they do so, then at the point of intersection, there will be two potential at the same point and this is not possible.

(iv) Spacing between two equipotential surfaces decreases in the region of strong electric field (attractive force in case of dipole) and increases outside in the weaker field. Because, for the same potential difference, $r \propto \frac{1}{E}$, where r is the spacing between two surfaces.

Fig.2.11.1-7: Dipole





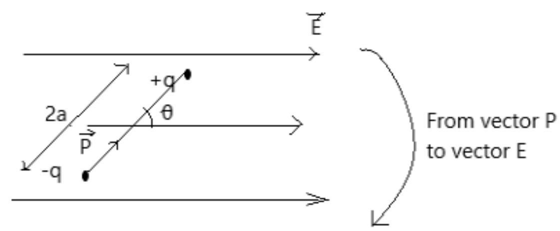
(v) Electric potential decreases along the direction of electric field, because, $dV = -\vec{E}\cdot d\vec{r}$.

2. Equipotential surfaces are planar for the uniform electric field.
3. For a single isolated charge, equipotential surfaces are spherical in shape and spacing increases.

2.13. To Derive Electric Potential Energy due to a Dipole:

As shown in the figure, a small work done to rotate a dipole against the torque,

$dW = -\tau d\theta - - - - (i)$



Since,

$\tau = pE\sin\theta - - - - (ii)$

Hence,

$dW = -pE\sin\theta\, d\theta - - - (iii)$

The work done by a conservative force is,

$dW = -dU - -(iv)$

Integrating equation (iv) both sides, we have,

$$U = \int_{90^0}^{\theta} pE\sin\theta\, d\theta = -PE\cos\theta - - - (v)$$

Note: At, 90^0, the electric potential energy is zero.

Hence,

$$\boxed{U = -pE\cos\theta - - - Derived}$$

2.14. ELECTROSTATICS OF CONDUCTORS:

1. Electric field is zero inside a metallic conductor because there is no charge resides inside a conductor.
2. Electric potential is same throughout inside a conductor and its value is equal to the potential at the surface.
3. Electric field is perpendicular on surface of a conductor irrespective of its shape and size.
4. Electric field at the surface of a conductor is: $E = \frac{\sigma}{\epsilon_0}$, where $\sigma = \frac{q}{A}$ = surface charge density.

5. Electric charge resides on the surface of a conductor.
6. Any cavity in a conductor remains shielded from outside electric influence. The field inside the cavity is always zero. This is known as electrostatic shielding.

2.15. DIELECTRICS AND POLARISATION:

1. Dielectrics are non-conducting substances. They have no or negligible charge carriers.
2. The charges are induced inside the dielectric due to external field and this induced charge opposes the external field. The opposing field so induced does not exactly cancel the external field, it only reduces it. The extent of the effect depends on nature of the dielectric.
3. To understand this effect, we need to look at the distribution of charges of dielectric at the molecular level.
4. The molecules of a substance may be polar or non-polar.
5. **In a non-polar molecule**, the centres of positive and negative charges coincide. The **molecule has no permanent dipole moment** because of their symmetry. Examples of non-polar molecules are oxygen O_2, H_2, CO_2.
6. **In a polar molecule**, centre of positive and negative charges are separated, even when there is no external field. Such molecules have a **permanent dipole moment**. The examples of polar molecule are HCl and Water (H_2O).
7. In an external field, the positive and negative charges of a non-polar molecule are displaced in opposite directions. They develop induced dipole moment and dielectric is said to be polarized by the external electric field.
8. Whether polar or non-polar, a dielectric develops net dipole moment in the external field.
9. The dipole moment per unit volume is called polarisation (denoted by $P^{\rightarrow}$). Induced dipole moment proportional to electric field is called linear isotropic dielectrics. Hence, polarisation index,

$$\vec{P} = \epsilon_0 \chi \vec{E}$$

   Where χ = Electrical susceptibility
10. **Dielectric constant K** is the ratio of external electric field to the net electric field inside a dielectric. It is also defined as the ratio of capacitance with dielectric between two plates to the capacitance with vacuum between two plates.
11. The maximum electric field that can exist in a dielectric without causing the breakdown of its insulating property is called **dielectric strength**.
12. Electrical susceptibility is related with relative permittivity (ϵ_r) as:

$$\boxed{\epsilon_r = 1 + \chi}$$

2.16. CAPACITORS AND CAPACITANCE:

1. A two conductor device of equal and opposite sign charges, separated by some distance and field with a dielectric is known as capacitor. A capacitor stores much energy in comparison to a single conductor.

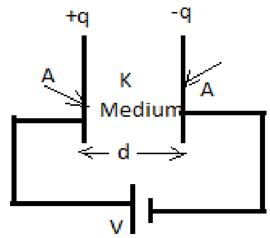
2. A capacitor stores the charge as well as potential energy in the electrostatic field. The capacity of a capacitor is called its capacitance.
3. A capacitance is defined as the ratio of charge stored (q) to the potential difference (V) across it.

$$q \propto V \ Or, q = CV$$

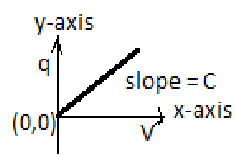
$$\boxed{C = \frac{q}{V} = Constant}$$

Here, proportionality constant C is known as capacitance.

4. The SI unit of capacitance is Farad and 1 F = 1 C/V
5. The capacitance C does not depend upon charge and potential difference.



6. The capacitance depends upon:
   (i) Area (A) of plates, $C \propto A$,
   (ii) Separation (d) between the plates, $C \propto \frac{1}{d}$
   (iii) Permittivity $(\epsilon_0 r)$of the medium, $C \propto \epsilon_r \ Or, C \propto K$, where K = dielectric constant and equals to relative permittivity
7. For n parallel plates, equivalent capacitance will be $(n-1)C$.
8. The graph between q and V is a straight line and its slope = C if q on y-axis and V on x-axis as shown below.



9. If charge on positive plate of the capacitor is q_1 and on negative plate, it is q_2, then its capacitance will be:

$$\boxed{C = \frac{q_1 - q_2}{2V}}$$

10. Actually, capacitance on positive plate is taken as the charge on capacitor.

2.17. PARALLEL PLATE CAPACITOR:

1) The capacitance of a parallel plates of plate area A, separated by distance d and filled with air (air capacitor with dielectric constant K = 1) is given by:

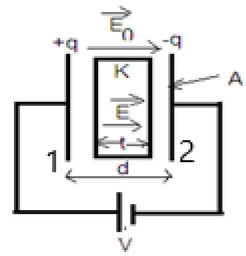
$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

2) The capacitance of a parallel plates of plate area A, separated by distance d and filled with dielectric slab of dielectric constant K and thickness t, (partially inserted) is given by

$$C = \frac{\epsilon_0 A}{(d-t) + \frac{t}{K}}$$

Derivation:

Suppose, A capacitor C of plate area A, separated by distance d
and a dielectric slab of dielectric constant K is partially filled between the gap of a parallel plate capacitor, having thickness t.
As shown in the arrangement, the potential difference V
Is given as,



$$V = V_1 - V_2 = Et + E_0(d-t) - - - (i)$$

Since, dielectric constant,

$$K = \frac{E_0}{E} - - - (ii)$$

Where,
E_0 = Electric field in the air and E = Net electric field inside the dielectric.
$From\ equation\ (i) and\ (2)$, we get,

$$V = \frac{E_0 t}{K} + E_0(d-t) - - - (iii)$$

By definition,

$$V = \frac{q}{C} - - - - (iv)$$

From equation (iii) and (iv), we have,

$$\frac{q}{C} = E_0\left[(d-t) + \frac{t}{K}\right] - - - - (v)$$

Now,
Electric field in the air,

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0} - - - (vi)$$

Where,
$\sigma = Surface\ charge\ density$, $\epsilon_A = permittivity\ of\ free\ space$
So, from equation (v) and (vi), we get

$$\frac{q}{C} = \frac{q}{A\epsilon_0}\left[(d-t) + \frac{t}{K}\right] - - - - (vii)$$

Hence, from equation (vii), we get the capacitance of a parallel plate capacitor filled with dielectric of constant K is,

$$\boxed{C = \frac{\epsilon_0 A}{(d-t) + \frac{t}{K}}}$$

3) From the above equation of Parallel Plates Capacitor,

(i) If dielectric slab is fully inserted between the plates, i. e. d = t, then its capacitance will be as:

$$\boxed{C = \frac{\epsilon_0 K A}{d}}$$

(ii) If a conductor is completely inserted between the plates, i. e. d = t, and K =∞ $\left(K = \frac{E_0}{E} \text{ and } E = 0 \text{ inside a conductor}\right)$then its capacitance will be as:

$$\boxed{C = \infty}$$

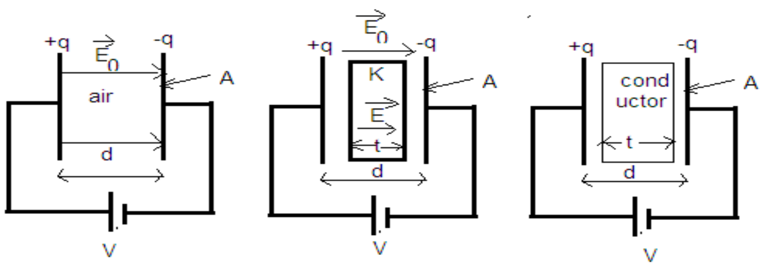
(iii) If a conductor is partially inserted between the plates, i. e. $d \neq t$, and K =∞, then

$$\boxed{C = \frac{\epsilon_0 A}{(d - t)}}$$

(iv) If two dielectric slabs of dielectric constant $K_1$and K_2 with their respective thickness t_1 and t_2 are partially inserted, then its capacitance will be

$$\boxed{C = \frac{\epsilon_0 A}{[d - (t_1 + t_2)] + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2}\right)}}$$

The arrangements of parallel plate capacitors are shown below:



2.18. EFFECT OF DIELECTRIC ON CAPACITANCE:

1) When a single capacitor of same plate area A, separated by distance d and has charge equal and opposite on plates, is first connected with a battery and then after some time, **battery is disconnected** and a dielectric slab of dielectric constant K is completely inserted, then its new parameters will be as (Initial values are: initial capacitance, C_0, initial potential difference V_0, initial electric field E_0, initial charge q_0, initial potential energy U_0 and initial energy density u_0): (**Shown in Fig-1 and Fig-2 below**)

   (i) capacitance, $C = C_0 K$ (**increases by K times**)

   (ii) potential difference across plates will be, $V = \frac{V_0}{K}$ (**decreased and becomes 1/K times**)

   (iii) electric field,$E = \frac{E_0}{K}$ (**decreased and becomes 1/K times)**

   (iv) electric charge, $q = q_0$(**remains same**)

   (v) potential energy, $U = \frac{U_0}{K}$ (**decreased and becomes 1/K times**)

(vi) energy density, $u = u_0/K$ (**decreased and becomes 1/K times**)

2) When a single capacitor is first connected with a battery and then after some time, a dielectric slab of dielectric constant K is completely inserted, **when battery remains connected**, then its new parameters will be as (Initial values are: initial capacitance, C_0, initial potential difference V_0, initial electric field E_0, initial charge q_0, initial potential energy U_0 and initial energy density u_0).(**Shown in Fig-1 and Fig-3 below)**
   (i) capacitance, $C = C_0K$ (**increases by K times**)
   (ii) potential difference across plates will be $V = V_0$
   (iii) electric field, $E = E_0$
   (iv) electric charge, $q = q_0K$
   (v) potential energy, $U = U_0K$
   (vi) energy density,$u = u_0K$

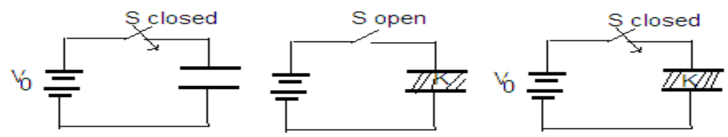


Fig-1: Initially Closed

Fig-2: Disconned and slab is fully inserted

Fig-3: Remains connected with battery and dielectric is fully inserted

Note: Initially, $C_0 = \frac{\epsilon_0 A}{d}$; $V_0 = \frac{q_0}{C_0}$; $E_0 = \frac{q_0}{A\epsilon_0}$;Finally: $V = \frac{q}{C}$, $E = \frac{q}{A\epsilon}$, $\epsilon = \epsilon_0 K$; $U = \frac{q^2}{2C} = \frac{CV^2}{2}$; $u = \frac{\epsilon E^2}{2}$

2.19. COMBINATION OF CAPACITORS-SERIES:

Let two unequal capacitors of capacitances C_1 and C_2 are connected in series and the combination is connected with a dc battery of potential difference V as shown below Fig-1.To know its equivalent capacitance and potential difference across each capacitor, the following steps must be followed:

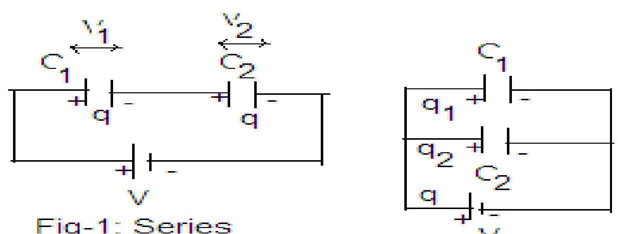


Fig-1: Series Connection

Fig-2: Parallel Connection

(i) The charge stored by each capacitor will be equal and
(ii) The potential difference across each capacitor will be unequal.
$C = \frac{q}{V}$:and q = constant so, $V \propto \frac{1}{C}$

2.19.1. Equivalent Capacitance (C_{eq}) in Series Combination of Two capacitors:

As shown in the Fig-1:

$$V = V_1 + V_2 \Rightarrow \frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} ----(i)$$

(i) Hence, if two capacitors are connected in series, then its equivalent capacitance will be:

$$C_{eq} = \frac{Product\ of\ Two\ capacitors}{Sum\ of\ Two\ capacitors} = \frac{C_1C_2}{C_1 + C_2} --- (ii)$$

(ii) If more than two capacitors are connected in series and the combination is connected with a battery of potential difference V, then its equivalent capacitance will be as:

$$\boxed{\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}}$$

Potential Difference across each capacitor:

$$V_1 = \frac{q}{C_1} \quad and\ V_2 = \frac{q}{C_2}; also, q = C_{eq}V, therefore,$$

$$V_1 = V\left(\frac{C_2}{C_1 + C_2}\right)\ and\ V_2 = V\left(\frac{C_1}{C_1 + C_2}\right)$$

In series combination, a graph is plotted between V and C, which is rectangular hyperbola shape with q = constant and shown in Fig-3 below.

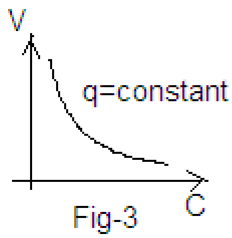


Fig-3

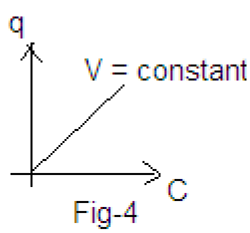


Fig-4

2.20. Equivalent Capacitance (C_{eq}) in Parallel Combination of Two capacitors:

When two capacitors of capacitance C_1 and C_2 are connected in parallel and the combination is connected with a battery of potential difference V across it as shown in the Fig-2 above. Then its equivalent capacitance will be found as:

(i) The charge stored by each capacitor will be unequal and

(ii) The potential difference across each capacitor will be same.

Since, $C = \frac{q}{V}$:and V = constant so, $q \propto C$ and the graph plotted between q and C will be a straight line as shown in Fig-4 above also.

Equivalent Capacitance (C_{eq}) in Parallel Combination of Two capacitors:

As shown in the Fig-2.:

$$q = q_1 + q_2 \Rightarrow C_{eq}V = C_1V + C_2V \Rightarrow C_{eq} = C_1 + C_2 --- (i)$$

(i) Hence, if two capacitors are connected in parallel, then its equivalent capacitance will be

$$C_{eq} = sum\ of\ two\ capacitances = C_1 + C_2 - - - -(ii)$$

(ii) If more than two capacitors are connected in parallel and the combination is connected with a battery of potential difference V, then its equivalent capacitance will be as:

$$\boxed{C_{eqv} = \sum_{i=1}^{n} C_i}$$

Charge stored by each capacitor:

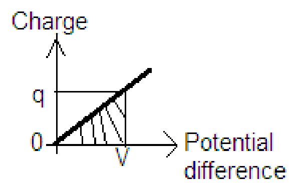
$$q_1 = C_1V\ and\ q_2 = C_2V\ \Rightarrow q_1 = q\left(\frac{C_1}{C_1 + C_2}\right)\ and\ q_2 = q\left(\frac{C_2}{C_1 + C_2}\right)$$

2.21. ENERGY STORED IN A CAPACITOR (U) and Energy Density (u):

(i) Energy stored in a parallel plate capacitor is:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{1}{2}\frac{q^2}{C}$$

The area under the graph between q and V gives the potential energy.



(ii) Energy density (u) 9s defined as the energy stored in the parallel plate capacitor per unit volume. That is,

$$u = \frac{Energy\ Stored}{Volume} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\left(\frac{\epsilon_0 A}{d}\right)(Ed)}{2Ad} = \frac{1}{2}\epsilon_0 E^2$$

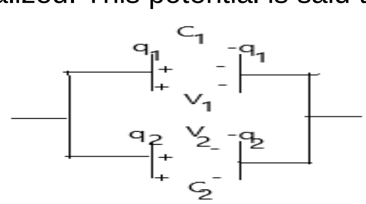
Energy density in electrostatic field is,

$$\boxed{u_E = \frac{1}{2}\epsilon_0 E^2}$$

(iii) SI unit of energy density is J/m^3.

2.22. Loss of Electrostatic Energy in the Capacitor-

Suppose a capacitor of capacitance C_1 is charged to a potential difference of V_1 by a battery. Another capacitor of capacitance C_2 also charged by another battery after disconnection from each battery, they connected in parallel with C_1, then the capacitor C_1 will supply the charge to C_2 until the potential is equalized. This potential is said to be **Common Potential (V).**



Let initially, a capacitor C_1 has charge q_1, potential V_1 and another capacitor C_2 has potential V_2. Finally, both are disconnected from battery and connected with wire in parallel, then charge is transferred from one capacitor to another until potentials are equalized. The common potential will be

$$\boxed{V_{com} = \frac{Net\ Charge}{Net\ Capacitance} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}} - - - (i)$$

The loss in electrostatic energy,

$$\boxed{U_i - U_f}$$

Hence,

$$U_i - U_f = \frac{1}{2}C_1{V_1}^2 - \frac{1}{2}(C_1 + C_2){V_{com}}^2 - - - -(ii)$$

Putting the value of V_{com} from equation (i) in equation (ii), we get,

$$U_i - U_f = \frac{1}{2}C_1{V_1}^2 - \frac{1}{2}(C_1 + C_2){V_{com}}^2 = \frac{1}{2}C_1{V_1}^2 - \frac{1}{2}(C_1 + C_2)\left(\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2\right) - - - (iii)$$

By solving the above equation (iii), we have,

$$\boxed{loss\ in\ Energy = U_i - U_f = \frac{(C_1C_2)(V_1 - V_2)^2}{2(C_1 + C_2)}}$$

Example-5: (NCERT Exercise2.11)

A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in this process?

Solution:

We know that loss in electrostatic energy is given by,

$$U_i - U_f = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}$$

Given, $C_1 = 600pF = 6 \times 10^{-10} = C_2$, $V_1 = 200\,V$, $V_2 = 0$, hence,

$$U_i - U_f = \frac{6 \times 10^{-10} \times 6 \times 10^{-10} \times (200 - 0)^2}{2(6 \times 10^{-10} + 6 \times 10^{-10})} = \frac{36 \times 4 \times 10^{-16}}{24 \times 10^{-10}} = 6 \times 10^{-6} J - Ans.$$