Chapter 5: Magnetism and Matter

Magnet: A magnet is a material that is both attractive and directive properties.

Magnetism: It attracts small pieces of iron, nickel, cobalt etc. This properties of attraction is called magnetism.

When suspended freely, a thin long piece of magnet comes to rest nearly in the geographical north-south direction. This is due to a hypothetical conception that earth itself has magnet whose north pole is towards south and south pole is towards north.

Artificial magnets are bar magnet, magnetic needle, horse shoe magnet, ball ended magnet etc.

Bar magnet-It is a bar of circular or rectangular cross section.

Magnetic needle-It is a thin magnetised steel needle having pointed end and pivoted at its centre so that it is free to rotate in a horizontal plane.

Ball ended magnet-It is a thin bar of circular cross section ending in two spherical balls.

Basic properties of magnet:

1. Attractive properties, 2. Directive properties, 3. Like poles repel and unlike poles attract, 3. Magnetic poles always exits in pair, 5. Magnetic induction-A magnet induces magnetism in a magnetic substance placed near it. This phenomenon is called magnetic induction. When a north pole of powerful magnet is placed close to a soft iron bar, the closer end of the bar becomes south pole and farther ends north pole. As a result, the magnet attracts the iron bar. Thus induction precedes attraction.

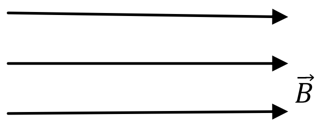
Magnetic field: The space around a magnet within which its influence can be experienced is called magnetic field.

Uniform magnetic field: If it has same magnitude and same direction at all points of that region, is called uniform magnetic field.

The field due to a bar magnet is not uniform.

Examples of uniform magnetic field:-

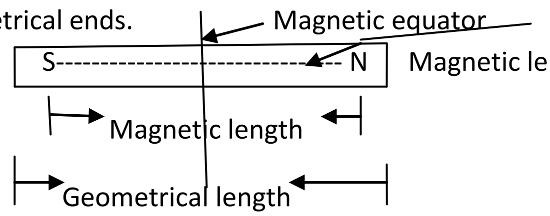
1. At a given place, magnetic field of the earth can be considered uniform.
2. Magnetic field acting in plane of paper is represented by equidistant parallel lines.



3. Magnetic field acting perpendicular to the plane of paper and directed outwards is represented by dot. ⊙ (North pole) ⊙ ⊙ ⊙ ⊙

   Magnetic field acting perpendicular to the plane of paper and directed inwards is represented by cross. (south pole) ⊗ ⊗ ⊗ ⊗

Magnetic poles: These are the regions of apparently concentrated magnetic strength in a magnet where the magnetic attraction is maximum. The poles of a magnet lie somewhat inside the magnet and not at its geometrical ends.



Magnetic length = 0.84 (Geometrical length)

Bar magnet as a dipole:

a) A small current loop carrying a current I, produces a magnetic field at an axial point is-

$$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{2\vec{M}}{x^3}\right)$$

Here, $\vec{M} = I\vec{A}$ is the magnetic dipole moment of the current loop.

b) A current loop placed in a magnetic field $\vec{B}$ experiences a torque

$$\vec{\tau} = \vec{M}x\vec{B}$$

There are two types of magnetic charges, positive magnetic charge and negative magnetic charge.

c) A magnetic charge m placed in a magnetic field $\vec{B}$ experiences a force

$$\vec{F} = m\vec{B}$$

The force on a positive magnetic charge is along the field and force on a negative magnetic charge is opposite to the field.

d) A magnetic charge m produces a magnetic field

$$B = \frac{\mu_0}{4\pi}\left(\frac{m}{r^2}\right)$$

e) Coulomb's law of magnetic force

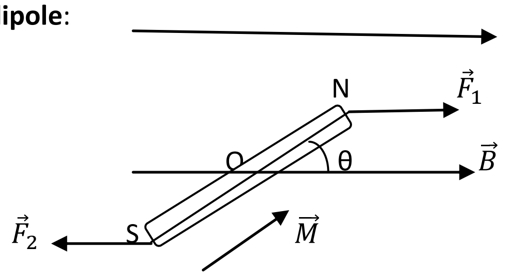
$$F = \frac{\mu_0}{4\pi}\left(\frac{m_1 m_2}{r^2}\right)$$

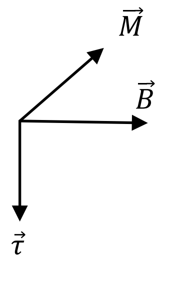
Magnetic dipole and magnetic dipole moment: An arrangement of equal and opposite magnetic poles separated by a small distance is called a magnetic dipole.

-m 2 l +m $\vec{M} = m(\overrightarrow{2l})$

Where m is the pole strength and 2l is the magnetic length. It is a vector quantity and represented from south to north pole.

Torque on a dipole:





Torque of $\vec{F}_1$ about O is $\vec{\tau}_1 = \overrightarrow{OA}x\vec{F}_1 = m(\overrightarrow{OA}x\vec{B})$, and

torque of $\vec{F}_2$ about O is $\vec{\tau}_2 = \overrightarrow{OB}x\vec{F}_2 = -m(\overrightarrow{OB}x\vec{B}) = m(\overrightarrow{BO}x\vec{B})$, so, the net torque acting on the dipole about O is,

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 = m(\overrightarrow{OA}x\vec{B} + \overrightarrow{BO}x\vec{B}) = m(\overrightarrow{BO} + \overrightarrow{OA})x\vec{B} = m(\overrightarrow{BA}x\vec{B}) = m(2\vec{l}x\vec{B}) = \vec{M}x\vec{B}$$

The magnitude of this torque is given by $\tau = MB\sin\theta$

This torque is maximum when θ = 90^0 and $\tau = MB$ and minimum when θ =0^0 or 180^0 and $\tau = 0$.

Potential energy of a magnetic dipole:

Work done in turning a dipole from small angle dθ is $dW = \tau d\theta = -MB\sin\theta\, d\theta$

Negative sign shows that rotation dθ is opposite to torque.

The change in electric potential energy of the dipole is

$$dU = MB\sin\theta\, d\theta$$

If dipole is rotated from initial position θ = θ_1=90^0 to θ = θ_2, then change in potential energy will be

$$\int_{\theta_1}^{\theta_2} dU = U(\theta_2) - U(\theta_1) = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta = -MB(\cos\theta)_{\theta_1}^{\theta_2} = -MB(\cos\theta_2 - \cos\theta_1)$$

If $\theta_1=90^0$, $\theta_2=\theta$, $U = -MB\cos\theta$, In vector form,

$U = -\vec{M}.\vec{B}$ = -MB cos θ

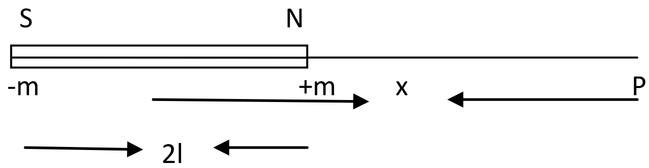
So, work done in rotating a magnetic dipole from θ_1 to θ_2 (against the torque) is

$$W = \Delta U = U_f - U_i = -MB\cos\theta_2 - (MB\cos\theta_1) = MB(\cos\theta_1 - \cos\theta_2)$$

Special cases:

1. When θ = 0^0, that is, $\vec{M} \uparrow\uparrow \vec{B}$ potential energy of a dipole is minimum and in this state, the magnetic dipole is in **stable equilibrium**. The minimum value of U = -MB
2. When θ = 90^0, Potential energy is zero., U= 0
3. When θ = 180^0, that is, $\vec{M} \uparrow\downarrow \vec{B}$, the potential energy of the dipole is maximum and in this state, the dipole is in **unstable equilibrium**. The maximum value of potential energy is U=MB

Magnetic field of a bar magnet at an axial point (end on position):-



Due to +m, magnetic field at P is,

$B_1 = \frac{\mu_0}{4\pi}\left(\frac{m}{(x-l)^2}\right)$, directed along NP (at P, away from P)

Due to –m, magnetic field at P is,

$B_2 = \frac{\mu_0}{4\pi}\left(\frac{m}{(x+l)^2}\right)$, directed along PS (at P, towards S)

Net field at P is,

$B_{axial} = B_1 + B_2 = \frac{\mu_0}{4\pi}\left(\frac{4xml}{(x^2-l^2)^2}\right)$, if x>>l,

$$B_{axial} = \frac{\mu_0}{4\pi}\left(\frac{2M}{x^3}\right)$$

In vector for, it can be written as,

$$\vec{B}_{axial} = \frac{\mu_0}{4\pi}\left(\frac{2\vec{M}}{x^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{4ml}{x^3}\right)\hat{\imath}$$

Similarly, one can deduce the magnetic field at the equatorial position (Broadside-on position):-

$$\vec{B}_{equitorial} = \frac{\mu_0}{4\pi}\left(\frac{\vec{M}}{y^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{2ml}{y^3}\right)(-\hat{\imath})$$

In magnitude form, It is directed along PS and is given by,

$$B_{equitorial} = \frac{\mu_0}{4\pi}\left(\frac{M}{y^3}\right)$$

Magnetic lines of force:

It is defined as the curve the tangent to which at any point gives the direction of the magnetic field at that point.

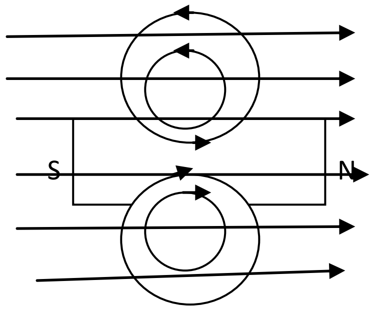
Properties of lines of force:

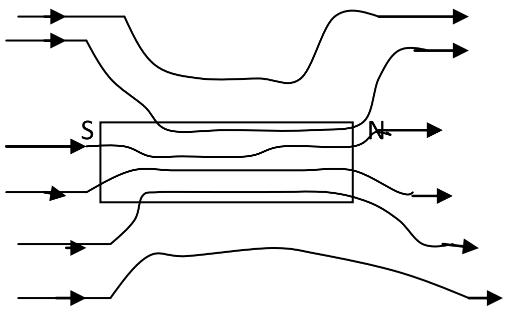
1. Magnetic lines of force are closed curves which start in air from the north pole and ends on south pole and then return to the north pole through the interior of the magnet.

2. The lines of force never cross each other (why?). If they do so, its means there would be two direction of magnetic field at the point of intersection, which is impossible.
3. They start on the surface of the magnet normally.
4. The lines of force have a tendency to contract lengthwise and expand sidewise. This explains attraction between unlike poles and repulsion between like poles.
5. The relative closeness of lines of force gives a measure of the strength of the magnetic field which is maximum at the poles.

Some important terms used in Magnetism:-

1. Magnetic induction $\overrightarrow{(B)}$: When a piece of any substance is placed in an external magnetic field, the substance becomes magnetised. If an iron bar is placed in a uniform magnetic field, the magnetised bar produces its own magnetic field in the same direction as those of the original field inside the bar. The magnetic flux density within the bar is increased whereas it becomes weak at certain places outside the bar.





''The number of magnetic lines of induction inside a magnetic substance crossing unit area normal to their direction is called the magnitude of magnetic induction, or magnetic flux density inside the substance.'' It is denoted by $\vec{B}$. Its unit is tesla (T) or weber/meter^2 (Wb/m^2). The CGS unit is gauss (G). 1 Wb/m^2 = 1 T =10^4 G

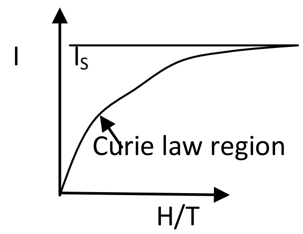
2. **Intensity of Magnetisation**$(\vec{I})$: It is defined as magnetic moment per unit volume of the magnetised substance. This basically represents the extent to which the substance is magnetised. Thus, $I = \frac{M}{V}$, The S I unit is Amp/m.
3. **Magnetic Intensity or magnetic field strength**$(\vec{H})$: When a substance is placed in an external magnetic field, the actual magnetic field inside the substance is the sum of external magnetic field and the field due to its magnetisation (or intensity of magnetisation).
   The capacity of magnetising field to magnetise the substance is expressed by means of a vector$(\vec{H})$, called the magnetic intensity of the field.
   $$\frac{\vec{B}}{\mu_0} = \vec{H} + \vec{I} \Rightarrow \vec{B} = \vec{B}_0 + \vec{B}_M \Rightarrow \vec{B} = \mu_0\vec{H} + \mu_0\vec{I}$$
   The SI unit is same as that of I that is A/m. The CGS unit is Oersted.
4. **Magnetic permeability**$(\boldsymbol{\mu})$: It is defined as the ratio of magnetic induction $(\vec{B})$ inside the magnetised substance to the magnetic intensity $(\vec{H})$ of the magnetising field. $\mu = \frac{B}{H}$
   It is measure of the extent to which material can be penetrated or permeated by a magnetic field. It is basically a measure of conduction of magnetic lines of force through it. The SI unit of magnetic permeability is Wb/A-m.

5. **Relative magnetic permeability(μ_r)**: Permeability of various substances can be compared with one another in terms of relative permeability. It is defined as ratio of permeability of the medium to the permeability of the free space. $\mu_r = \frac{\mu}{\mu_0}$
For vacuum, $\mu_r = 1$, It is also defined as ratio of magnetic field B in the substance when placed in magnetic field B_0. Thus, $\mu_r = \frac{B}{B_0}$
6. **Magnetic susceptibility (χ_m):** It is a measure of how easily a substance is magnetised in a magnetic field. It is defined as the ratio of intensity of magnetisation to the magnetic intensity of the magnetising field. $\chi_m = \frac{I}{H}$
Relation between relative permeability and magnetic susceptibility:
$$B = \mu_0(H + I) = \mu H \Rightarrow H(\mu - \mu_0) = \mu_0 I \Rightarrow \mu_0 H(\mu_r - 1) = \mu_0 I \Rightarrow \frac{I}{H} = \mu_r - 1$$
Since, $\frac{I}{H} = \chi_m$ so, $\mu_r = 1 + \chi_m$

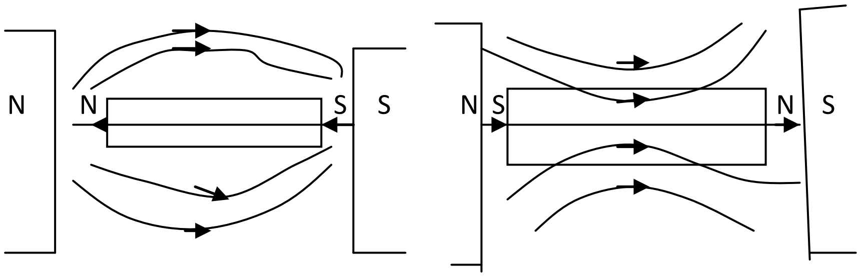
Magnetic substances: There are three types of magnetic substances.
1. Paramagnetic substances
2. Diamagnetic substances
3. Ferromagnetic substances

Paramagnetic substances: These substances which are feebly magnetised in the direction of magnetic field when placed in strong magnetic field are called paramagnetic substances.
Examples: Aluminium, platinum, chromium, manganese, sodium, Cucl2, oxygen, etc.
Properties: (a) These substances are attracted towards strong magnetic field in a non-uniform magnetic field. (b) The magnetism of these substances decreases with increase in temperature.
Diamagnetic substances: These substances which are feebly magnetised in the opposite direction of magnetic field when placed in strong magnetic field are called diamagnetic substances.
Examples: Gold, silver, copper, zinc, tin, mercury, hydrogen, water, air, alcohol, N_2 at STP, $Nacl_2$, bismuth, lead
Properties: (a) These substances are attracted towards weak magnetic field in a non-uniform magnetic field. (b) The magnetism produced in these substances does not change with increase or decrease in temperature.
Ferromagnetic substances: These substances which are strongly magnetised in the direction of magnetic field when placed in it, are called ferromagnetic substances.
Examples: Iron, cobalt, Nickel, Fe_2O_3, gadolinium, alloys like alnico etc.
Properties: (a)The magnetism produced in these substances decreases with increase in temperature. (b) Above a certain temperature, called Curie temperature, ferromagnetic substance becomes paramagnetic substance.
Curie temperature for Cobalt, 1394 K, iron is 1043 K and for Nickel, 631 K, Gadolinium, 317 K (lowest)
Curie's Law: According to Curie's law, magnetic susceptibility of a paramagnetic substance is inversely proportional to the absolute temperature T. $\chi_m \propto \frac{1}{T}$ Or, $\chi_m = \frac{C}{T}$
1. Directly proportional to the magnetising field intensity (H) because the latter tends to align the atomic dipole moments

2. Inversely proportional to the absolute temperature T, because the latter tends to oppose the alignment of the magnetic dipole moments.
   At low H/T values, $I \propto \frac{H}{T} \Rightarrow I = C\frac{H}{T} \Rightarrow \frac{I}{H} = \frac{C}{T} \Rightarrow \chi_m = \frac{C}{T}$
   Beyond the saturation value of I, Curie's law is not valid.



How to identify magnetic substances:



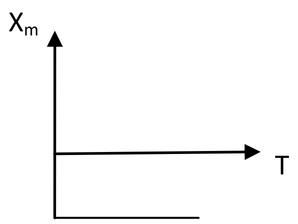
Diamagnetic Paramagnetic

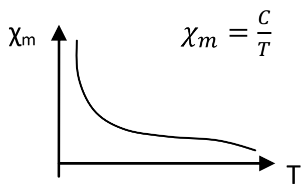
A. When a rod of diamagnetic material is placed in a non uniform magnetic field,
1. Poles are induced on it in a direction opposite to the inducing field
2. Lines of force prefer to pass through the surrounding air than to pass thr.material itself
3. The lines of force gets repelled
4. The magnetic induction B inside the material becomes less than magnetising field
5. Substance moves from stronger to the weaker part of field

B. When a rod of diamagnetic substance is suspended freely in a strong magnetic field, it aligns itself perpendicular to the magnetising field.
C. Atoms of these substances have even number of electrons which form pairs. The net magnetic moment of an atom is thus zero.
D. The susceptibility of diamagnetic substance is independent of magnetising field and the temperature.

A. When a rod of paramagnetic material is placed in a non uniform magnetic field,
1. Poles are induced in it in the same direction of inducing field
2. Lines of force prefer to pass through it than through the surrounding air
3. Lines of force get slightly more concentrated
4. Magnetic induction inside the rod becomes slightly greater than magnetising field
5. Substance moves from weaker to the stronger part of the field.

B. When a rod of paramagnetic substance is suspended freely in a strong magnetic field, it aligns itself parallel to the magnetising field.
C. Atoms or molecules have excess of electrons spinning in the same direction. Atoms of these substances have a permanent magnetic moment.
D. The susceptibility of paramagnetic substance is dependent of magnetising field and inversely proportional the temperature.





Comparative study of properties of para, dia and ferromagnetic materials

Parameters	Paramagnetic	Diamagnetic	Ferromagnetic
Effects of magnets	Feebly attracted by magnets	Feebly repelled by magnets	Strongly attracted by magnets
In external magnetic field	Acquires feeble magnetism in the direction of magnetising field	Acquires feeble magnetism in the opposite direction of magnetising field	Acquires strong magnetism in the direction of magnetic field
In non uniform magnetic field	Tends to move slowly from weaker to stronger	Tends to move slowly stronger to weaker	Tends to move quickly from weaker to stronger
In uniform magnetic field	Freely suspended rod aligns itself parallel to field	Freely suspended rod aligns perpendicular to field	Freely suspended rods aligns parallel to field
μ_r	>1	< 1 or $0 \le \chi_m < 0$	>> 1 or $\mu_{r>} > 1000$
X_m	Slightly positive	Slightly negative, or $-1 \le \chi_m < 0$	Positive and very large, or $\chi_m > 1000$
μ	$\mu > \mu_0$	$\mu < \mu_0$	$\mu >> \mu_0$
Effect of temperature	Inversely as temp.	Independent of temp.	$\chi_m \propto \frac{1}{T - T_C}$, $T > T_C$
Removal of magnetising field	As soon as magnetising field is removed, magnetism is lost	Magnetisation lasts as long as magnetising field is applied	Magnetisation is retained even after magnetising field is removed
Variation of I with H	I changes linearly with H and attains saturation	I changes linearly with I	I changes non linearly with H and ultimately attains saturation
Hysteresis effect	Shows no hysteresis	Shows no hysteresis	Shows hysteresis
Physical state of material	Solid, liquid or gas	Solid, liquid or gas	Normally solids only

Hysteresis: The phenomenon of lagging of magnetic induction (B) behind the magnetising field (H) is called Hysteresis. It is a Greek word, meaning delayed.

Hysteresis Loop: The closed curve ABCDEFA which represents a cycle of magnetisation of a ferromagnetic sample is called its hysteresis loop.

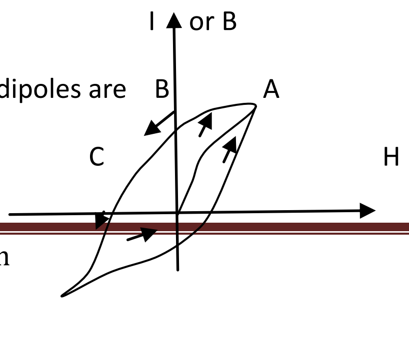
Significance of the area of hysteresis loop: The product B-H has the dimension of energy per unit volume. $BH = B\left(\frac{B}{\mu}\right) = \frac{B^2}{\mu_0 \mu_r} = B^2 v^2 = \frac{F^2}{q^2 v^2} v^2 = \frac{Energy}{Volume}$

Hence, area within the B-H loop represents energy dissipated per unit volume in materials when it is carried through a cycle of magnetisation.

Practical importance of hysteresis loop: It provides us information about retentivity, coercivity and hysteresis loss of a magnetic material. This helps us proper selection of materials in designing cores of transformers, electromagnets and making permanent magnets.

Description: In ferromagnetic materials, distinguishing feature is that I is not directly proportional to magnetising field H. If a gradually increasing magnetic field H is applied to an unmagnetised piece of iron, its magnetisation increases non-linearly until it reaches a maximum value.

OA is called magnetisation curve. At this stage, all the dipoles are



Aligned and I has reached a maximum or saturated value.

If the magnetic field H is now decreased, I does not return along magnetisation curve but follows path AB. At $\vec{H} = 0, \vec{I}$ does not come to its zero value but its value is still near the saturated value. The value of $\vec{I}$ at this point (i.e. OB) is known as remanence, remanent magnetisation or retentivity.

It is also called residual induction. On applying a reverse field, the value of I finally becomes zero. OC represents the reverse magnetic field needed to demagnetise the specimen. This is known as coercivity of the material.

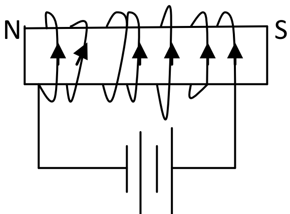
1. The retentivity of a substance is a measure of the magnetisation remaining in the substance when the magnetising field is removed.
2. The coercivity of a substance is a measure of the reverse magnetising field required to destroy the residual magnetism of the substance.
3. The energy lost per unit volume of a substance is in a complete cycle is equal to the area of the hysteresis loop.

Demagnetisation: From the hysteresis loop, we see that intensity of magnetisation does not reduce to zero on removing the magnetising field. At this point, magnetic induction is not zero and the specimen is not demagnetised.
To demagnetise a substance, it is subjected to several cycle of magnetisation. Each time with decreasing magnetising field and finally the magnetising field is reduced to zero H). In this way, the size of hysteresis curve goes on decreasing and finally, the area reduces to zero.

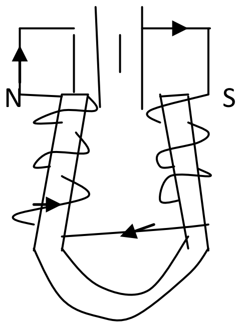
Demagnetisation is best obtained by:

1. Placing the specimen in an alternating field of continuously diminishing amplitude.
2. Heating ferromagnetic materials, as it becomes practically non-zero at sufficiently high temperatures.

Electromagnet: We know that a current carrying solenoid behaves like a bar magnet. If we place a soft iron rod in the solenoid, the magnetism of the solenoid increases hundreds of times and the solenoid is called an electromagnet. It is a temporary magnet.
An electromagnet is made by winding closely a number of turns of insulated copper wires over a soft iron straight rod or a horse shoe rod. On passing current through a solenoid, a magnetic field is produced in the space within the solenoid.



Straight rod



Horse shoe

Curie-Weiss law:
The susceptibility of ferromagnetic material decreases with temperature according to-

$$\chi_m = \frac{C}{T - T_C}$$

Types of ferromagnetic materials:

1. **Soft ferromagnetic materials or soft ferromagnets**: These are materials in which magnetism disappears on removal of the external magnetising field. Such materials have narrow hysteresis loop, consequently, they have
    i. Low retentivity
    ii. Low coercivity
    iii. Low hysteresis loss
    iv. But, they have high relative magnetic permeability
    v. They are used as cores of transformers and solenoids.
    vi. Examples: Soft iron, mu metal, stalloy, etc.
2. **Hard ferromagnetic materials**: These materials retain magnetisation even after the removal of the external magnetising field. Such materials have wide hysteresis loop, consequently, these materials have
    i. High retentivity
    ii. High coercivity
    iii. Large hysteresis loss
    iv. Examples: steel, alnico, loadstone, ticonal, etc.

Comparison of magnetic properties of soft iron and steel:

a) Soft iron has greater permeability (μ) i. e. (B) than steel
b) Soft iron has greater susceptibility ($\chi_m = \mu_r - 1$)
c) Retentivity of soft iron is greater than that for steel
d) Coercivity of soft iron is less than that of steel
e) Soft iron has less hysteresis loss than steel.

How to select magnetic materials to making permanent magnet, electromagnet and cores of transformers:

The materials used for making permanent magnet have the following characteristics:

a) High retentivity, so that it produces a strong magnetic field
b) High coercivity, so that its magnetisation is not destroyed by stray magnetic fields (, temperature variations or minor mechanical damage.
c) High permeability (steel is used to make permanent magnet)

The materials used for making electromagnets have characteristics as:

a) High initial permeability so that magnetisation is large even for a small magnetising field
b) Low retentivity so that the magnetisation is lost as the magnetising current is switched off.

Soft iron is more suitable than steel for cores of electromagnets.

Transformer cores: Materials used for making transformer cores have following characteristics:

a) High initial permeability
b) Low hysteresis loss
c) Low resistivity to reduce eddy current losses

Soft iron is preferred to make transformer cores and telephone diaphragms.

Methods to make permanent magnets:

By placing a steel rod in a solenoid and pass a strong current, the rod gets magnetised due to the magnetic field of the solenoid.

By holding the steel rod in north- south direction and hammering it repeatedly.

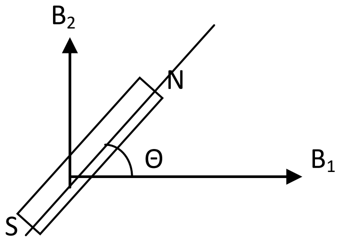
Uses of electromagnets:

1. In electric bells, loudspeaker, in telephone diaphragm, transformer core
2. In hospitals, electromagnets are used to iron or steel bullets
3. In cranes, it is used to lift heavy machinery

Tangent galvanometer:

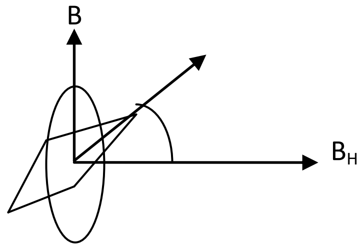
Tangent law: This law states that if a freely suspended small magnet is acted upon by two uniform mutually perpendicular magnetic fields B_1 and B_2 simultaneously, then the magnets comes to rest in such a position that the tangent of the angle θ that the magnet makes with B_1 is equal to the ratio B_2 /B_1 of the two fields. That is,

$\tan\theta = \frac{B_2}{B_1}$



It is a device used to measure very small current. It ia moving magnet type galvanometer. Its working is based on tangent law. When no current is passed through it, the magnetic needle is only influenced by B_H. When current I is passed, there is a magnetic field B along the axis of the coil, perpendicular to the B_H. The magnetic needle is influenced by two perpendicular magnetic fields and it comes to rest at an angle θ with B_H such that $B = B_H \tan\theta$

This relation is known as tangent law.



The magnetic field at the centre of the coil

$$B = \frac{\mu_0 NI}{2R} = B_H \tan\theta \Rightarrow I = \frac{2RB_H \tan\theta}{\mu_0 N}$$

Where, N and R are number of turns and radius of the coil and the Reduction factor K for the tangent galvanometer given as-

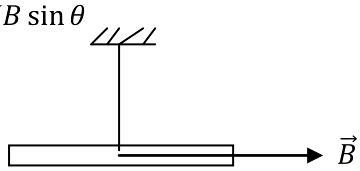
$$K = \frac{2RB_H}{\mu_0 N}$$

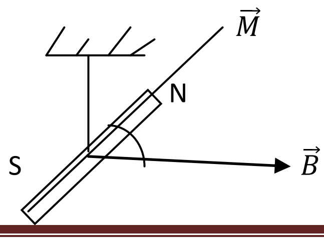
Oscillations of a freely suspended magnet:

Show that the oscillation of a freely suspended magnet is simple harmonic. Hence deduce an expression for its time period.

In the position of equilibrium, magnetic dipole lies along $\vec{B}$. When it is slightly rotated from this position and released, it begins to vibrate about field direction under the restoring torque,

$\tau = -MB\sin\theta$





For small angular displacement, sin θ = θ, hence, $\tau = -MB\theta$ and if I is the moment of inertia of the magnet, the deflecting torque on the magnet is,

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

In the equilibrium condition, deflecting torque = restoring torque, so,

$$I\frac{d^2\theta}{dt^2} = -MB\theta \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{MB}{I}\theta = -\omega^2\theta$$

This shows that angular acceleration is directly proportional to angular displacement θ. Where $\omega = \sqrt{\frac{MB}{I}}$ and time period of oscillation of freely suspended magnet is

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

Vibration magnetometer: It is an instrument used to compare the magnetic moment of two magnets or to determine the horizontal component of earth's magnetic field at a place.

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

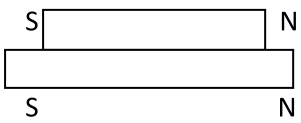
Where, B_H=horizontal component of earth magnetic field, I= moment of inertia of magnet about an axis of rotation through its centre of mass and equal to $I = m\frac{l^2+b^2}{12}$, where l and b are length and breadth of the magnet.

Uses of vibration magnetometer:

1. **Measurement of magnetic moment of a magnet**: Set the vibration magnetometer in the north –south direction and place the bar magnet in its stirrup. Measure the time period as $T = 2\pi\sqrt{\frac{I}{MB_H}}$
2. **Comparison of magnetic moment of two magnets of same size and same mass:**

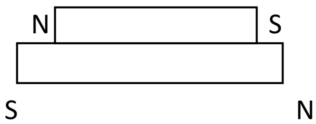
$$\frac{T_1}{T_2} = \frac{2\pi}{2\pi}\sqrt{\frac{\frac{I}{M_1B_H}}{\frac{I}{M_2B_H}}} = \sqrt{\frac{M_2}{M_1}}$$

3. **Comparison of magnetic moment of two unequal sizes and masses**: (Sum position and difference position)



(Sum position)

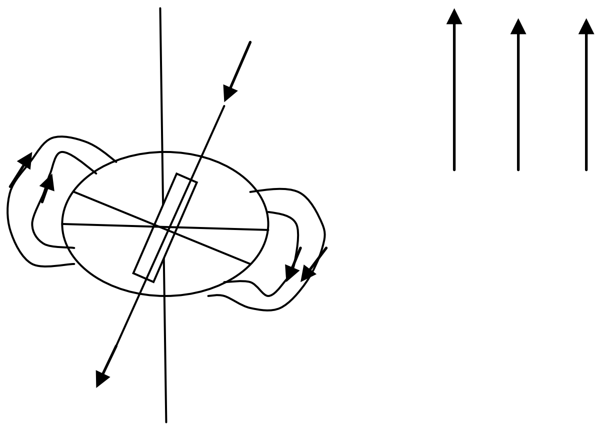
$$T_1 = 2\pi\sqrt{\frac{I_1+I_2}{(M_1+M_2)B_H}}$$



(Difference position)

$$T_2 = 2\pi\sqrt{\frac{I_1+I_2}{(M_1-M_2)B_H}}$$

Earth's magnetism:

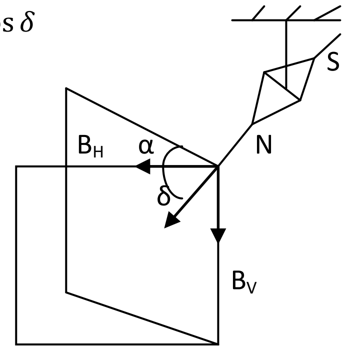


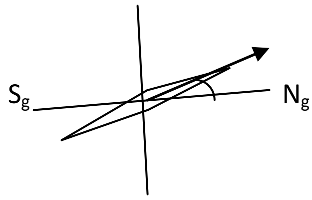
1. **Geographic axis**: The straight line passing through the geographical north and south poles of the earth is called its geographic axis. It is the axis of the rotation of the earth.
2. **Magnetic axis**: The straight line passing through the magnetic north and south poles of the earth is called its magnetic axis.
   The magnetic axis of the earth makes an angle of nearly 20^0 with geographic axis. At present, the magnetic south pole is located at a point in Northern Canada at a latitude of 70.5^0 N and a longitude of 96^0 W. The magnetic north pole is located diametrically opposite to S_m at latitude of 70.5^0 S and a longitude of 84^0 E. The magnetic poles are nearly 2000 km away from the geographic poles. The magnetic equator intersects the geographic equator at longitudes of 6^0 W and 174^0 E.
3. **Magnetic equator**: It is the great circle on the earth perpendicular to the magnetic axis.
4. Magnetic meridian: The vertical plane passing through the magnetic axis of a freely suspended small magnet is called magnetic meridian. The earth's magnetic field acts in the direction of magnetic meridian.
5. **Geographic meridian**: The vertical plane passing through the geographic north and south poles is called geographic meridian.

Elements of earth's magnetic field:

1. **Magnetic declination (α):** The angle between geographical meridian and the magnetic meridian at a place is called the magnetic declination at that place.
   Magnetic declination arises because the magnetic axis of the earth does not coincide with its geographic axis. In India, the value of α is small and it is 0^0 41' E for Delhi and 0^0 58' W for Mumbai. This means that the N-pole of a compass needle almost points in the direction of geographic north.
2. **Angle of dip or magnetic inclination (δ):** The angle made by the earth's total magnetic field B with the horizontal direction in the magnetic meridian is called angle of dip.
   The angle of dip is different at different places. At the magnetic equator, the dip needle rests horizontally so the angle of dip is zero at the magnetic equator. The dip needle rests vertically at the magnetic poles so that the angle of dip is 90^0 at the magnetic poles. At all other places, the dip angle lies between 0^0 and 90^0.

3. **Horizontal component of earth's magnetic field:** It is the component of earth's total magnetic $\vec{B}$ in the horizontal direction in the magnetic meridian.
   If δ be the angle of dip at any place, then
   $B_H = B\cos\delta$





$B_H = B\cos\delta\ and\ B_V = B\sin\delta$, therefore,

$B = \sqrt{B_H^2 + B_V^2}$ and $\tan\delta = \frac{B_V}{B_H}$

The lines passing through all such places where one of the three magnetic elements, has the same value.

1. **Isogonic lines**: The lines joining the places of equal declination are called Isogonic lines. The line of zero declination is called agonic line.
2. **Isoclinical lines:** The lines joining the places of equal dip or inclination are called Isoclinical lines. The line of zero dip is called aclinic line or magnetic equator. The points of 90^0 is called magnetic poles. The magnetic equator crosses geographic equator twice once in Atlantic and then in Pacific Ocean.
3. **Isodynamic lines**: The lines joining the places having the same value of the horizontal component of earth's magnetic field are called isodynamic lines. The horizontal component is zero at poles and maximum at the magnetic equator.

Neutral Point:

It is the point where the magnetic field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field. The resultant magnetic field at the neutral point is zero. If a compass needle is placed at such a point, it can stay in any position.

Suppose a small bar magnet is placed such that the north pole of the magnet is towards the magnetic south pole of the earth, then neutral points are obtained both sides on the axis of the magnet. If the distance of each neutral point from the middle point of the magnet be r and the magnitude of magnetic moment be M, then

$$B_H = \frac{\mu_0}{4\pi}\left(\frac{2M}{r^3}\right)$$

When North Pole of the bar magnet is towards magnetic north pole of the earth, the neutral points are obtained on perpendicular bisectors of the magnet.

$$B_H = \frac{\mu_0}{4\pi}\left(\frac{M}{r^3}\right)$$