Chapter-1: Electric Charges and Fields

1. Property of Charges:

- Quantisation of charge

$$q = Ne$$

- Additivity of charges

$$q_{net} = \sum_{i=1}^{n} q_i$$

- Conservation of charges

$$q_{net}(isolated) = Constant$$

2. Coulomb's Force

Force between two point charge q_1 and q_2 separated by distance r and placed in vacuum

- In vector Form

$$\vec{F} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1 q_2}{r^2}\hat{r}$$

- In magnitude form

$$F = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{|q_1||q_2|}{r^2}$$

Also,

$$F \propto \frac{1}{r^2}$$

And,

$$F \propto |q_1||q_2|$$

3. Electric Field Intensity

- Electric field intensity E at a point at distance r due to a point charge q kept in free space

In vector form

$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{r^2}\grave{r}$$

In magnitude form

$$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{|q|}{r^2}$$

- The net electric field intensity at a point whose position vector is r due to other charges whose positions are r_1, r_2, r_3, -------can be expressed as-

$$\vec{E} = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{n}\frac{q_i(\vec{r}-\vec{r_i})}{|\vec{r}-\vec{r_i}|^3}$$

4. Dipole

- Dipole Moment
  In vector form

$$\vec{p} = 2aq\hat{\imath}$$

  In magnitude form

$$p = 2aq$$

- Net Electric Field Intensity due to a dipole
  At **axial point** at distance r from dipole centre

$$E = \frac{1}{4\pi\epsilon_0}\frac{4aqr}{[r^2-a^2]^2}$$

  At **Axial point** when r >> a

$$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{2p}{r^3}\right)$$

  This shows that

$$E \propto \frac{1}{r^3}$$

- Net Electric field intensity due to a dipole
  At the **equatorial point** at distance r from dipole centre

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{2aq}{[r^2+a^2]^{\frac{3}{2}}}\right)$$

  At the **equatorial point** when r >> a

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{p}{r^3}\right)$$

- Torque in a dipole
  In vector form

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

  In magnitude form

$$\boxed{\tau = pE\sin\theta}$$

- Potential energy in rotating a dipole at an angle θ
  In vector form

$$\boxed{U = -\vec{p}.\vec{E}}$$

  In magnitude form, Potential in the dipole is

$$\boxed{U = -\mathrm{pE}\cos\theta}$$

5. Electric Flux

For uniform field

In vector form

$$\boxed{\varphi_E = \vec{E}.\vec{S}}$$

In magnitude form,

$$\boxed{\varphi = ES\ \cos\theta}$$

6. Charge Densities (continuous charge distribution)

- Linear Charge Density

$$\boxed{\lambda = \frac{q}{L}}$$

- Surface Charge Density

$$\boxed{\sigma = \frac{q}{A}}$$

- Volume Charge Density

$$\boxed{\rho = \frac{q}{V}}$$

7. Gauss's Law

$$\boxed{\varphi_E = \oint \vec{E}.\overrightarrow{dS} = \frac{q_{in}}{\varepsilon_0}}$$

The flux contributes all charges inside a Gaussian surface, not outside charge

And

$$\boxed{\varphi = \frac{q_{in}}{\varepsilon_0}}$$

8. Electric field intensity due to Infinite Line Charge (linear charge density λ distributed)

- Electric Field Intensity due to Line charge

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

This shows that

$$\boxed{E \propto \frac{1}{r}}$$

9. Electric Field Intensity due to two parallel infinite line charges having $(+\lambda \; and - \lambda)$

- Inside between two at distance r (separation is 2r)

$$\boxed{E = \frac{\lambda}{\pi\epsilon_0 r}}$$

{Towards line (2)}

- Outside (1) at distance r

$$\boxed{E = \frac{\lambda}{3\pi\epsilon_0 r}}$$

{Away from line charge (1)}

- Outside (2) at distance r

$$\boxed{E = \frac{\lambda}{3\pi\epsilon_0 r}}$$

{Towards line charge (2)}

10. Electric field intensity due to an infinite thin plane sheet (Surface charge density σ is distributed)

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

(This shows that E is independent of r.)

11. Electric field intensity due to a charged conductor –charged on both sides (near the conductor)

$$\boxed{E = \sigma/\varepsilon_0}$$

12. Electric field intensity in between and outside two long parallel plane sheet having surface charge densities ± σ

Outside parallel sheets,

$$\boxed{E = 0}$$

Between parallel sheet,

$$\boxed{E = \sigma/\varepsilon_0}$$

13. Electric field intensity due to hollow sphere or spherical shell

(r is the distance of a point inside the sphere from its centre, R is the radius of sphere)

Inside the sphere (r < R)

$$\boxed{E = 0}$$

Outside the sphere, r > R

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\left(\frac{|q|}{r^2}\right)}$$

On its surface r = R

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\left(\frac{|q|}{R^2}\right) = \frac{\sigma}{\epsilon_0}}$$

14. Electric Field due to two concentric spherical shells of radius, say r (inner sphere and 2r (outer sphere), charge distributed +q on inner sphere and – q on outer sphere surface

At distance $r_1 = 1.5$ r $(r < r_1 < 2r)$

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\left(\frac{4q}{9r^2}\right)}$$

At distance $r_2 = 2.5\, r \;\; (r_2 > 2r)$

$$\boxed{E = 0}$$