Chapter-2: Electrostatics Potential and Capacitance

1. Relation between work done and potential energy:

$$\boxed{W(i \to f) = U_f - U_i \;=\; \Delta U}$$

Against the electric force or by external force (external agent)

$$\boxed{W(i \to f) = U_i - U_f \;=\; -\Delta U}$$

By the electric force

[Since, Work done = - $\Delta U = -(U_f - U_i)$]

2. Potential energy of two point charge system (if charges q_1 and q_2 are separated by distance r)

$q_1 \bullet \longleftarrow r \longrightarrow \bullet q_2$

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}}$$

3. Potential energy of system of charges (due to multiple charges)

Charges kept at vertices of triangle or square or rectangle or at any geometrical shape.

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\sum_{\substack{i=1\\ i<j}}^{n}\frac{q_j q_i}{|\overrightarrow{r_{ji}}|}}$$

For example, in a triangle, if charges q_1, q_2 and q_3 are placed at vertex A, B and C respectively of a triangle ABC. The distance is r_{32} between charges q_3 and q_2, distance r_{31} between charges q_3 and q_1, distance r_{21} between charges q_2 and q_1, then

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_3 q_2}{r_{32}} + \frac{q_3 q_1}{r_{31}} + \frac{q_2 q_1}{r_{21}}\right]}$$

If potential energy (U) of system of charges is negative, work done will be positive and if potential energy is positive, work done will be negative. [By conservative force]
That is, if work done decreases (example, when charge is brought from any point or from infinity (initial point) to a point (final point) by electric force), potential energy increases and vice-versa.

4. Relation between electric potential and potential energy

$$\boxed{V = \frac{U}{q_0}}$$

5. If a test charge q_0 is brought from a point to another point against the electric force or by external force or by electric force.

- By Conservative Force

$$\boxed{V_i - V_f = \frac{U_i - U_f}{q_0} = W_{i\to f}}$$

- By External Agent or against Conservative Force

$$\boxed{V_f - V_i = \frac{U_f - U_i}{q_0} = W_{i\to f}}$$

- Similarly, if initial point is at ∞ and final point is say f so work done to bring a charge (say q_0) from infinity to a point (say f) by external force or against electric force is given by-

$$\boxed{V_f - V_\infty = \frac{U_f - U_\infty}{q_0} = W_{\infty\to f}}$$

[Here V_∞ and U_∞ are zero as seen that $U \propto \frac{1}{r}$ and $\propto \frac{1}{r}$]

- Work done to bring a charge say q_3 if two point charges q_1 (at point A) and q_2(at point B) kept in free space. A third point charge q_3 is moved from infinity to a point (say C , then formula of work done shall be applied as-

$$\boxed{W_{\infty\to C} = q_3(V_C - V_\infty) = q_3 V_C}$$

Where $V_\infty = 0$

- Given r_1 is the distance from point C to charge q_1 at point A, r_2 is the distance of point C to charge q_2 at point B as shown in the below figure of triangle ABC. Where V_C is

$$\boxed{V_C = V_{CA} + V_{CB} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]}$$

6. Relation between electric potential and electric field intensity

$$\boxed{dV = -\vec{E}.\overrightarrow{dr}}$$

Or,

$$\boxed{\int_{V_B}^{V_A} dV = -\int_{r_B}^{r_A} \vec{E}.\overrightarrow{dr}}$$

7. Electric potential difference say V_{AB} or $V_A – V_B$ in a uniform electric field

$$\boxed{V_B - V_A = \vec{E}.(\vec{r}_A - \vec{r}_B)}$$

8. Electric potential at a point

- At distance r due to a point charge q

q• ⟵ r ⟶ P

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r}\right)}$$

- Whose position vector r due to a point charge q kept at position vector r_0

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{|\vec{r} - \vec{r_0}|}\right)}$$

- Whose position vector r due to system of charges q_1, q_2, q_3..... kept at positions r_1, r_2, r_3 ....respectively

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{|\vec{r} - \vec{r_i}|}}$$

- Whose distance is same (r) from each charge

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\left(\frac{q_{net}}{r}\right)}$$

- At distance r due to continuous charge distribution

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r}}$$

09. Electric potential in a dipole at any point p, at distance r from the centre of dipole, making angle θ from the dipole axis

$$\boxed{V_p = \frac{1}{4\pi\epsilon_0}\left(\frac{p\cos\theta}{r^2}\right)}$$

- When $\theta = 0^0$, (axial position of dipole)

$$\boxed{V_p = \frac{1}{4\pi\epsilon_0}\left(\frac{p}{r^2}\right)}$$

- When $\theta = 90^0$, (at the equatorial point),

$$\boxed{V = 0}$$

10. Electric potential in case of hollow spherical conductor,

- Inside the hollow spherical conductor (r<R) where R is the radius of conductor and q is the charge distributed on the surface of conductor

$$\boxed{V_{inside} = V_{surface} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{R}\right)}$$

- Outside at any point at distance r from the centre of the spherical shell

$$\boxed{V_r = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r}\right)}$$

11. Electric potential in case of two metallic hollow concentric shells:

- If the inner shell A has radius R_1 and charge distributed is q_1 and has outer shell B has radius R_2 and charge distributed is q_2, then potential V_A and V_B are given as

$$\boxed{V_A = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{R_1} + \frac{q_2}{R_2}\right)}$$

$$\boxed{V_B = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{R_2} + \frac{q_2}{R_2}\right)}$$

- Thus, potential difference

$$\boxed{V_A - V_B = \frac{1}{4\pi\epsilon_0} q\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

12. Capacitance C: SI Unit: Farad (F)

- Of a Capacitor of charge q and V the potential difference

$$\boxed{C = \frac{q}{V}}$$

- Of a metallic spherical conductor of radius R and charge q on its surface

$$\boxed{C = 4\pi\epsilon_0 R}$$

- Of a two concentric metallic hollow spherical conductor of inner and outer shell having radii R_1 and R_2 and charges on inner and outer surface q and –q respectively

$$\boxed{C = 4\pi\epsilon_0 \frac{R_1 R_2}{R_2 - R_1}}$$

- Of a two co-axial metallic hollow cylindrical conductor placed in free space of inner and outer shell having radii R_1 and R_2 and charge on inner outer surface of sphere is q and charge on outer surface of shell B is –q

$$\boxed{C = \frac{2\pi\epsilon_0 L}{ln\frac{R_2}{R_1}}}$$

13. Parallel Plate Capacitor

- Capacitance C of a parallel plate capacitor having plate area A with charge q and –q on each plate placed in medium K (K is a dielectric constant) and separation between the plates d

$$\boxed{C = \frac{\epsilon_0 K A}{d}}$$

- Capacitance C of n similar plates of equal separation (d) and same plate area (A) connected alternatively

$$\boxed{C = (n-1)\frac{\epsilon_0 A}{d}}$$

- The capacitance C of a parallel plate capacitance of separation d and plate area A is filled partially between the gap of plates with dielectric constant K and thickness t with same area

$$\boxed{C = \frac{\epsilon_0 A}{d - t + \frac{t}{K}}}$$

Special cases:

- If more than one dielectric slabs of different thickness and different constants are placed between the parallel plates capacitor, then

$$\boxed{C = \frac{\epsilon_0 A}{(d - t_1 - t_2 - \cdots..) + (\frac{t_1}{K_1} + \frac{t_2}{K_2} + \cdots.)}}$$

- If a dielectric slab is completely filled then t = d, then

$$\boxed{C = \frac{\epsilon_0 K A}{d}}$$

- If a conductor is completely filled in the gap, then t = d and $K = \infty$, then

$$\boxed{C = \infty}$$

- If the conductor is partially filled with thickness t and same area, then

$$\boxed{C = \frac{\epsilon_0 A}{d - t}}$$

- The capacitance will be infinity if one of the plates is earthed..

14. Energy stored in capacitor is given by

$$\boxed{U = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{1}{2}\frac{q^2}{C}}$$

15. Energy density

- The energy per unit volume is called the energy density (u).

$$\boxed{u = \frac{U}{Volume}}$$

- Energy density of air capacitor

$$\boxed{u = \frac{1}{2}\epsilon_0 E^2}$$

- Energy density in a dielectric medium (dielectric constant K)

$$\boxed{u = \frac{1}{2}(\epsilon_0 K E^2)}$$

16. Electric field intensity on the –q plate due to positive charge in case of two parallel plates

$$\boxed{E = \frac{q}{2A\epsilon_0}}$$

17. Force of attraction with which both parallel plates (charge q and –q and area A with separation d) attract each other

$$\boxed{F = \frac{q^2}{2A\epsilon_0}}$$

18. Electric field intensity (E) in the dielectric (dielectric constant K) inserted between parallel plates

$$\boxed{E = \frac{\sigma}{\epsilon} = E_0 - E_i}$$

19. In free space between the plates, External electric field (E_0)

$$\boxed{E_0 = \frac{\sigma}{\epsilon_0}}$$

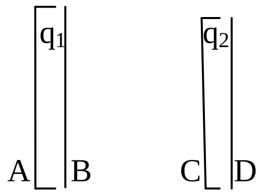
20. Dielectric Constant K

$$\boxed{K = \frac{E_0}{E} \ or, K = \frac{E_0}{E_0 - E_i}}$$

21. Relation between induced charge, charge on plates and dielectric constant K

$$\boxed{q_i = q\left(1 - \frac{1}{K}\right)}$$

22. If the charge on plate 1 is q_1 and on plate 2 is q_2 then charge at each point (inner & outer surface of plate-1 and plate-2 respectively are



- Charge on point A and D

$$\boxed{q_A = q_D = \frac{q_1 + q_2}{2}}$$

Where q_1 is

$$\boxed{q_1 = Charge\ at\ point\ A + Charge\ at\ point\ B}$$

And q_2 is

$$\boxed{q_2 = \text{Charge at point C} + \text{Charge at point D}}$$

- Charge at point B

$$\boxed{q_B = \frac{q_1 - q_2}{2}}$$

- Charge at point C

$$\boxed{q_C = -\left(\frac{q_1 - q_2}{2}\right)}$$

23. Charge redistribution:

When two conductors of charge q_1 and q_2 on its surfaces and having capacitances as C_1 and C_2 with radii as R_1 and R_2, are connected by a conducting wire, then charges on conductors after joining the wire are in the ratio of their capacities as given-

$$q_1' = C_1 V' \ \ and\ q_2' = C_2 V'$$

Where

$$\boxed{V_{com} = Common\ potential = \frac{q_1 + q_2}{C_1 + C_2}}$$

Since,

$$\boxed{C \propto R\ So, q_1' = (q_1 + q_2)\left(\frac{R_1}{R_1 + R_2}\right)}$$

24. Loss of energy during redistribution of charge

$$\boxed{\Delta U = U_i - U_f = \frac{C_1 C_2}{2(C_1 + C_2)}((V_1 - V_2)^2)}$$

Where,

$$\boxed{U_i = \frac{1}{2}\left(\frac{q_1^2}{C_1}\right) + \frac{1}{2}\left(\frac{q_2^2}{C_2}\right) \; and \; U_f = \frac{1}{2}\left[\frac{(q_1+q_2)^2}{C_1+C_2}\right]}$$

25. Capacitors Combination

If C_1, C_2 capacitors are connected in

- Series (same charge in each capacitor)

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \ldots .}$$

- Parallel (same potential difference across each capacitor)

$$\boxed{C = C_1 + C_2 + \cdots \ldots \ldots}$$

26. If three capacitors of same capacity (say, C)

- If they are connected in STAR

  Then, their DELTA connection will be C/3

- If they are connected in DELTA,

  Then their star connection will be 3C.

27. Relation between Electrical susceptibility and Polarisation

Polarisation is defined as dipole moment per unit volume and its magnitude is referred to as polarisation density.
Electrical susceptibility is given by

$$\boxed{\chi = \frac{\vec{p}}{\epsilon_0 \vec{E}}}$$

28. Relation between dielectric constant (K) and electrical susceptibility (χ)

$$\boxed{K = 1 + \chi}$$

29. Polarisation density (P)

$$\boxed{P = \frac{q_i t}{At} = \frac{dipole\ moment\ in\ the\ slab}{volume} = \frac{q_i}{A} = \sigma_i}$$

Where A is the area of dielectric slab, t the thickness of slab, q_i the induced charge in the dielectric slab.

30. Effect of dielectric on various parameters

Effect of dielectric

Initially No dielectric and Supply is Connected

Let q_0, V_0, C_0, E_0 and U_0 be the charge, potential difference, capacitance, electric field Intensity and potential energy stored respectively before dielectric slab is inserted. Then,

$$\boxed{V_0 = \frac{q_0}{C_0} \text{ and } V_0 = E_0 d \text{ and } U_0 = \frac{1}{2} C_0 V_0^2}$$

Finally

(i) When supply is disconnected and dielectric slab is inserted:

- Charge (remains same)

$$\boxed{q = q_0}$$

The charge on capacitor plate remains q_0 as battery has been disconnected before the insertion of dielectric slab.

- Electric field (decreased by K times

$$\boxed{E = \frac{E_0}{K}}$$

- Potential difference (decreased by K times)

$$\boxed{V = \frac{V_0}{K}}$$

- Capacitance (increased by K times)

$$\boxed{C = C_0 K}$$

- Potential energy stored (decreased by K times)

$$\boxed{U = \frac{1}{2} C V^2 = \frac{U_0}{K}}$$

Finally

(ii) When battery remains connected across the capacitor

- Potential difference (remains same)

$$\boxed{V = V_0}$$

- Electric field intensity (remains same)

$$\boxed{E = E_0}$$

- Capacitance (increased by K times)

$$\boxed{C = C_0 K}$$

- Charge (increased by K times)

$$\boxed{q = K q_0}$$

- Potential energy stored:

$$\boxed{U = K U_0}$$

31. Charging of capacitor in C-R circuit

$$\boxed{q = q_0\left(1 - e^{\frac{-t}{\tau}}\right)}$$

$$where\ q_0 = CV\ and\ \tau = RC = Time\ constant\ in\ C - R\ circuit$$

Charging current i is given by $i = i_0 e^{\frac{-t}{\tau}}$

Where $i_0 = \frac{V}{R}\ is\ the\ current\ at\ time\ t = 0$

32. Discharging of capacitor in C-R circuit

$$\boxed{q = q_0\left(e^{\frac{-t}{\tau}}\right)}$$

At time t = τ, $q = q_0 e^{-1} = 0.368\ q_0$

Note:

- A capacitor offers zero resistance in a circuit when it is uncharged (t = 0) that it can be assumed as short circuited and
- It offers infinite resistance when it is fully charged (t = ∞) as it can be assumed as open circuited.