Chapter-3: Current Electricity

1. Relation among charge (q) to flow, current (I) and time t

$$\boxed{I = \frac{q}{t} = \frac{ne}{t}}$$

For positive q, current is in forward direction and for negative q, it is in backward direction.

2. Relation between very small charge dq to flow for small time dt

$$\boxed{I = \frac{dq}{dt}}$$

3. Current density (J) is the current (I) distributed uniformly over the cross section area (A)

$$\boxed{J = \frac{I}{A}}$$

Or

$$\boxed{I = \vec{J} \cdot \vec{A}}$$

4. Ohm's law

$$\boxed{I \propto V \; or \; V \propto I \Rightarrow V = RI}$$

The value of R depends upon length, shape and nature of the material. Resistance(R) is directly proportional to length (l) of conductor and is inversely proportional to the area (A) of the cross section of the conductor if the temperature remains constant, that is,

$$\boxed{R \propto l, R \propto \frac{1}{A} \; So, R = \rho \frac{l}{A}}$$

Where ρ is the resistivity or specific resistance of the conductor and ρ depends upon nature of the material and temperature of the conductor.

5. Effect of temperature on resistance:

Resistance of a metallic conductor at temperature t ^{0}C is given by

$$\boxed{R_t = R_0(1 + \alpha t + \beta t^2)}$$

If R_1 be the resistance at temperature t_1 and R_2 be the resistance at temperature t_2, then

$$R_2 = R_1(1 + \alpha \Delta t)$$

Where α is the temperature coefficient of resistance and $\Delta t = t_2 - t_1$.

If the temperature is not sufficiently large, as in practical cases, then the equation is

$$\boxed{R_t = R_0(1+\alpha t) \ or \ \alpha = \frac{R_t - R_0}{R_0 t} = \frac{Change\ in\ resistance}{original\ resistance\ x\ rise\ in\ temperature}}$$

6. Temperature dependence of resistivity

Resistivity of a metal conductor is given by

$$\boxed{\rho_T = \rho_0[1 + \alpha(T - T_0)]}$$

Where, ρ is the resistivity at temperature T (in Kelvin), and resistivity at temperature T_0 isρ_0.
So, temperature coefficient of electrical resistivity is defined as fractional change in electrical resistivity per unit change in temperature.

7. Conductance (G) and conductivity (σ):

Conductance (G) is given by reciprocal of resistance and will be written as

$$\boxed{G = \frac{1}{R}}$$

And conductivity (σ) is given by reciprocal of resistivity and will be written as

$$\boxed{\sigma = \frac{1}{\rho}}$$

8. Drift velocity and relaxation time

drift velocity

$$\boxed{\vec{v}_d = \frac{-e\vec{E}\tau}{m}}$$

Here, negative sign shows that $\vec{v}_d$ is opposite the direction of $\vec{E}$.
Magnitude of drift velocity = Drift speed

$$\boxed{|\vec{v}_d| = v_d = \frac{eE}{m}\tau}$$

Relaxation time is the average time of electrons between successive collisions.

9. Relation between average drift velocity, relaxation time and current

$$\boxed{I = neAv_d}$$

Here, n number of electrons per unit volume or electron density or number density and N = number of electrons in a metal conductor of length l and cross section A, so volume = Al

Ohm's law from the expression of drift velocity

$$\boxed{R = \frac{ml}{ne^2 A\tau} = constant}$$

10. Relation between resistivity (ρ), number density (n) and relaxation time (τ)

$$\boxed{\rho = \frac{m}{ne^2\tau}}$$

a) When temperature increases, relaxation time decreases and resistivity of conductor (metal) increases so, conductivity decreases. (Relaxation time depends upon temperature of conductor)
b) When temperature increases, number of free electrons increases, so, resistivity of semiconductor decreases. (Number of free electrons depend upon nature of material)

11. Mobility is the magnitude of drift velocity per unit electric field applied

$$\boxed{\mu = \frac{q\tau}{m}}$$

a) Mobility due to charge carrier electrons in a conductor is given by

$$\boxed{\mu_e = e\frac{\tau_e}{m_e}}$$

b) Mobility due to charge carrier holes in a conductor is given by

$$\boxed{\mu_h = e\frac{\tau_h}{m_h}}$$

c) The total current in a conducting material is the sum of current due to the positive current carriers and negative current carriers.

12. Equivalent Resistance in series

$$\boxed{R = R_1 + R_2 + R_3 - - - -}$$

The current is same in all resistors.

Since, V = RI, for same current

$$\boxed{V \propto R}$$

Hence,
For two resistors R_1 and R_2 connected in series and the combination is connected with a battery of potential difference across it,
Then

$$\boxed{V_1 = V\left(\frac{R_1}{R_1 + R_2}\right)}$$

And,

$$\boxed{V_2 = V - V_1}$$

Or

$$\boxed{V_2 = V\left(\frac{R_2}{R_2 + R_1}\right)}$$

Where, V_1 is potential difference across R_1 and V_2 is potential difference across R_2.

13. Equivalent Resistance in Parallel

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} - - - - -}$$

The voltage is same in all resistors.

Since, V = RI, for same potential difference

$$\boxed{I \propto \frac{1}{R}}$$

Hence,
For two resistors R_1 and R_2 connected in parallel and the combination is connected with a battery of potential difference V across it,
Then

$$\boxed{I_1 = I\left(\frac{R_2}{R_1 + R_2}\right)}$$

And,

$$\boxed{I_2 = I - I_1}$$

Or

$$\boxed{I_2 = I\left(\frac{R_1}{R_2 + R_1}\right)}$$

Where, I_1 is current in R_1 and I_2 is current in R_2
Suppose n resistances of same ohmic resistances are connected in series and same n resistances are connected in parallel, then R_{eqvt} in both cases are given by:

$$\boxed{R_{eqvt} = nR}$$

$$\boxed{\frac{1}{R_{eqvt}} = \frac{n}{R} \Rightarrow R_{eqvt} = \frac{R}{n}}$$

14. Power Supplied by a Battery

When the current enters the positive plate of a battery
Then

$$\boxed{P = +EI}$$

15. Power Delivered / Consumed by a Battery

When current enters the negative plate of a battery
Then

$$\boxed{P = -EI}$$

Where, E = emf of a cell / battery

16. Electrical Power (P) and Heat generated across a resistor

$$\boxed{\int dW = \int V\,dq \Rightarrow W = VI \int dt \Rightarrow W = VIt = P = VI}$$

\Electrical power (P) dissipated or generated across a resistor is given by

$$\boxed{P = VI = I^2R = \frac{V^2}{R}}$$

Heat generated across a resistor is given by

$$\boxed{H = VIt = I^2Rt = \frac{V^2}{R}t}$$

17. Principle of bulbs

Resistance of a bulb is given by

$$R = \frac{V^2}{P} \Rightarrow R \propto \frac{1}{P} \Rightarrow P \propto \frac{1}{R} keeping\ the\ voltage\ fixed\ across\ the\ bulb$$

In series (suppose two bulbs are connected in series having the same rated voltage but rated power of both bulbs are P_1 and P_2 respectively), total power is given by (as $\frac{1}{P} \propto R$)

$$\boxed{\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} \Rightarrow P = \frac{P_1P_2}{P_1 + P_2}}$$

In parallel (suppose two bulbs are connected in parallel having the same rated voltage but rated power of both bulbs are P1 and P2 respectively), total power is given by (as $P \propto \frac{1}{R}$)

$$\boxed{P = P_1 + P_2}$$

In $R = \frac{V^2}{P}$ V and P are rated values (written) of that bulb.

In parallel, a bulb having more rated power, glows more brightly ($H \propto \frac{V^2}{R}$) as R becomes less while in series, a bulb having less rated power, glows more brightly ($H \propto I^2R$).

In series, since I is same, hence, $P \propto R \text{ or } \frac{P_1}{P_2} = \frac{R_1}{R_2}$

18. EMF (E) of cell and internal resistance (r)

EMF (E) of cell is defined as work done by the cell in moving unit positive charge through the whole circuit (including the cell) is called electromotive force (emf) of the cell.

$$\boxed{E = \frac{W}{q}}$$

When the cell / battery is charged and connected with a load resistance R then

Discharging Equation of a cell

In Closed Circuit

When a load resistance R is connected across the cell,
Then

$$\boxed{V < E}$$

And,

$$\boxed{V = E - Ir}$$

For V = IR;

$$\boxed{I = \frac{E}{r + R}}$$

For $I = \frac{V}{R}$

$$\boxed{V = \frac{E}{r + R}}$$

Note:

Draw graphs between

(i). V and I

(ii). I and R

(iii). V and R

If a cell is shorted, then

$$\boxed{V = 0}$$

And,

$$\boxed{I = \frac{E}{r}}$$

Note:

Draw graphs between

(i). I and r for same E

(ii). E and I for same r

- **In Open Circuit**

$$\boxed{I = 0}$$

Then

$$\boxed{V = E}$$

This shows that

$$\boxed{V \leq E}$$

Charging Equation of a cell

In Closed Circuit

When battery is discharged (it draws the current)

$$\boxed{V = E + Ir}$$

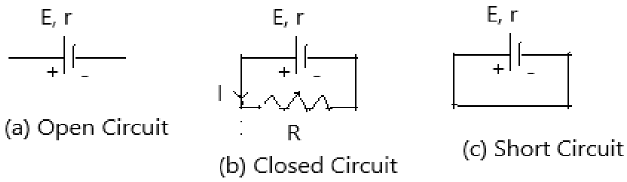
If resistance R is connected in series with a battery which supplies the current and charges the cell then

$$\boxed{I = \frac{V' - E}{r + R}}$$

Where, V' = Large battery voltage, E is the EMF of a cell.

Note:

Draw a graph between I and R for V', E and r



19. Internal resistance (r) of cell:

The potential difference across a real source in a circuit is not equal to the emf of a cell. The reason is that charge moving through the Electrolyte of the cell encounters resistance and this resistance is called internal resistance (r) of the cell.

Remember:

When current leaves the terminal, the voltage is taken as positive and when current enters the terminal, the voltage is taken as negative.

Voltage rise can be taken from negative to positive in cell and voltage drop can be taken from positive to negative across resistance.
Voltage rise as from negative to positive is taken plus sign before it and voltage drop as from positive to negative to minus sign before it.

20. Combination of cells

a) **Series Connection**:
Equivalent emf (same polarity)

$$\boxed{E_{eq} = E_1 + E_2 + E_3 + ------}$$

Equivalent Internal Resistance

$$\boxed{r_{eq} = r_1 + r_2 + r_3 + ---}$$

If external resistance R is connected in a closed loop,

$$\boxed{I = \frac{E_1 + E_2 + E_3 + ---}{(r_1 + r_2 + r_3 + ---) + R}}$$

So, it can be said that two cells connected in series,

$$\boxed{Net\ EMF = E_1 + E_2}$$

$$Net\ Resistance = Total\ internal\ resistance + Total\ External\ Resistance$$

$$\boxed{I = \frac{Net\ EMF}{Net\ Resistance}}$$

Note:
(i). N cells of same emf (E) and same internal resistance (r) are connected in series, then

$$\boxed{I = \frac{nE}{nr + R}}$$

(ii). In series connection, there are n cells of each emf E. If polarity of m cells is reversed, then equivalent emf

$$\boxed{E_{eq} = (n - 2m)E}$$

Whereas, total resistance is

$$\boxed{R_{eq} = R + nr}$$

$$\boxed{I = \frac{(n - 2m)E}{nr + R}}$$

b) Parallel Combination

- If n cells of emf E_1, E_2, E_3, ---- (polarity of each same) with internal resistance r_1, r_2, r_3---are connected in parallel (with no external resistance R across the combination), then

$$\boxed{\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + ----}$$

- And, Equivalent internal resistance

$$\boxed{\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + ----}$$

- If n cells each emf E (polarity of each same), internal resistance r, are connected in parallel, then
  Equivalent emf

$$\boxed{E_{eq} = E}$$

  Equivalent resistance

$$\boxed{E_{eq} = nr}$$

- If an external resistance R is connected across the parallel combination of n cells with each emf E and internal resistance r, then
  Current

$$\boxed{I = \frac{E}{R + \frac{r}{n}}}$$

Note:

If two cells or three cells with integral multiple of E connected in parallel and no internal resistance given, then take the average of all as,

$$\boxed{E_{eq} = Average\ value\ of\ all\ emfs}$$

There are n identical cells in a row and number of rows are m. EMF of each cell is E and internal resistance is r.

Net EMF= nE and Total internal resistance = $\frac{nr}{m}$

Total external resistance = R, current through the external resistance is given by

$$\boxed{I = \frac{nE}{R + \frac{nr}{m}}}$$

23. Maximum Power Transfer Theorem

Current I will be maximum when

$$\boxed{\sqrt{mR} = \sqrt{nr}}$$

Or,

$$\boxed{R = \frac{nr}{m}}$$

Hence, power transferred to the load is maximum when load resistance = total internal resistance

21. Kirchhoff's Law:

Junction Rule:

Kirchhoff's Junction Rule is based on conservation of charge.

Kirchhoff's First law is also called Kirchhoff's Current Law (KCL). It is applied at a junction.

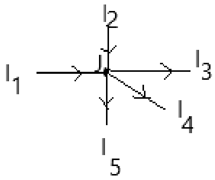
According to this law:

The algebraic sum of current at any junction is equal to zero.

$$\boxed{\sum_{junction} I = 0}$$

That is, net current entering the junction = net current leaving the junction

Here, I_1 and I_2 are current entering the junction J and I_3, I_4 and I_5 are current leaving the junction J.



So,

$$\boxed{I_1 + I_2 = I_3 + I_4 + I_5}$$

Kirchhoff's Loop Rule

Voltage Law (KVL): Kirchhoff's Loop Rule is based on the law of conservation of energy.

It is applied in a closed loop circuit.

The net voltage in a closed loop is equal to zero.

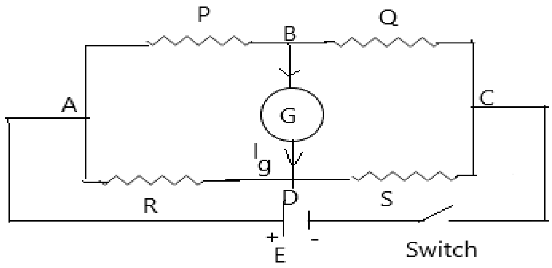
$$\boxed{\sum_{closed\ loop} V = 0 \ or \sum E_{net} = \sum IR}$$

The voltage equations in the closed loop can be wriiten along the direction of Loop Current Assumed.

If the Loop Current and Branch Current are in the same direction, then for resistor, Voltage Drop and is written $-RI$. The cell voltage from negative plate to positive plate is voltage rise and is taken $+E$, and the cell voltage from positive plate to negative plate is voltage drop and is taken $-E$, where E is the value of cell voltage.

22. Wheatstone bridge:

By this bridge, the unknown resistance S can be found out in terms of known resistance.



Principle: The bridge is said to be balanced when deflection in the galvanometer is zero. That is $I_g = 0$ and hence, $V_B = V_D$

$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

Following two points are important regarding a Wheatstone bridge,

- In Wheatstone bridge, if cell and galvanometer arms are inter-changeable. In this case, no effect of balancing of the bridge.
- If bridge is not balanced, current will flow from D to B if, $PS > QR$