Chapter-4: Moving Charges and Magnetism

1. Magnetic force on a moving charge

$$\boxed{\vec{F}_m = q\vec{v} \times \vec{B}}$$

Or

$$\boxed{F_m = qvB \sin\theta}$$

2. Path of charged particle in uniform magnetic field

- Path is straight line when

$$\boxed{\theta = 0^0 \ Or \ 180^0}$$

- Path is circle when

$$\boxed{\theta = 90^0}$$

- For any other angle, path is helix.

3. Circular path

$$\boxed{\vec{v} \cdot \vec{B} = 0}$$

- Centripetal force = Magnetic force

$$\boxed{m\frac{v^2}{r} = qvB}$$

- Radius of the circular path (r)

$$\boxed{r = \frac{mv}{qB}}$$

- Momentum (p)

$$\boxed{p = mv}$$

- Kinetic energy(K)

$$\boxed{p = \sqrt{2mK}}$$

- Time period,

$$\boxed{T = \frac{2\pi m}{Bq}}$$

- Angular velocity

$$\boxed{\omega = \frac{Bq}{m}}$$

- Frequency

$$\boxed{f = \frac{Bq}{2\pi m}}$$

4. Helical path

- Radius of helical path

$$\boxed{r = \frac{mv\sin\theta}{Bq}}$$

- Time period and

$$\boxed{T = \frac{2\pi m}{qB}}$$

- Frequency of helical path

$$\boxed{f = \frac{1}{T} = \frac{qB}{2\pi m}}$$

- Pitch of the helical path

$$\boxed{p = (v\cos\theta)T = \frac{2\pi mv\cos\theta}{Bq}}$$

5. Magnetic force on a current carrying conductor

- In Vector Form

$$\boxed{\vec{F}_m = I\vec{l}\times\vec{B}}$$

- In Magnitude Form

$$\boxed{F_m = IlB\sin\theta}$$

Here θ is the angle between $\vec{l}\ and\ \vec{B}$ or between direction of current and magnetic field.

6. Lorentz Force

The total force on a charge Q moving in the presence of both electric and magnetic field is given by

$$\boxed{\vec{F} = \vec{F}_e + \vec{F}_m = Q\left(\vec{E} + \vec{v}X\vec{B}\right)}$$

7. Velocity selector

Path of a charged particle in uniform electric and magnetic field will remain unchanged if,

$$\boxed{\vec{F}_{net} = 0 \; or \; q\left(q\vec{E} + q\vec{v}x\vec{B}\right) = 0 \; or \; \vec{E} = -\vec{v}x\vec{B} \; or \; \vec{E} = \vec{B}x\vec{v}}$$

8. Cyclotron

- The frequency of revolution of the particle

$$\boxed{f = \frac{qB}{2\pi m}}$$

- The Energy Gained by Charged particles

$$\boxed{K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{q^2B^2R^2}{m}\right)}$$

9. Magnetic dipole: and Magnetic dipole moment (unit Am)

- Current carrying loop is a magnetic dipole.
- Magnetic dipole moment of magnetic dipole

$$\boxed{M = NIA}$$

- Direction of $\vec{M}$ is the perpendicular to the plane of loop and given by right hand screw law.

10. Potential energy stored in dipole

$$\boxed{U = -\vec{M}.\vec{B} \;\; or \; U = -MB\cos\theta}$$

11. Work done in rotating dipole against torque

$$\boxed{W_{\theta_1 \to \theta_2} = U_{\theta_2} - U_{\theta_1} = MB(\cos\theta_1 - \cos\theta_2)}$$

12. Biot-Savart Law

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{I\overrightarrow{dl}x\vec{r}}{r^3}\right) \; or \; dB = \frac{\mu_0}{4\pi}\left(\frac{Idl\sin\theta}{r^2}\right)}$$

13. Magnetic field of a moving point charge

The magnetic field vector $\vec{B}$ at point P at position vector $\vec{r}$ from the charge q moving with a velocity $\vec{v}$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\left[\frac{q(\vec{v}x\vec{r})}{r^3}\right] \; or \; B = \frac{\mu_0}{4\pi}\left(\frac{qv\sin\theta}{r^2}\right)}$$

14. Magnetic field induction at a point due to a straight wire carrying current I

- Magnetic field due at a point due to a straight wire of finite length (wire is held vertically)

$$\boxed{B = \frac{\mu_0 I}{4\pi x}\frac{\mu_0 I}{4\pi x}(\sin\alpha + \sin\beta)}$$

- Magnetic field at a point due to a straight wire of infinite length

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

- Magnetic field at a point due a straight wire with one end finite and other end infinite

$$\boxed{B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi r}\right) = \frac{\mu_0 I}{4\pi r}}$$

- Magnetic field created by long current carrying wire
  - ❖ r > R

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

  - ❖ r < R

$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$

15. Magnetic field induction at a point due to a circular wire carrying current I

- Magnetic field on the axis (either x, y or z-axis) of a circular loop (say, if coil placed in y-z plane so, $B_y = 0$, $B_z = 0$ and B_x

$$\boxed{B_x = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{\frac{3}{2}}}}$$

- Magnetic field at the centre (O) of ring (or circular loop)

$$\boxed{B_o = \frac{\mu_0 I}{2R}}$$

- If circular loop has N number of turns then magnetic field at its centre of radius R and carries current I

$$\boxed{B = \frac{\mu_0 N I}{2R}}$$

- Magnetic field when x >> R for N circular loops on axial position

$$\boxed{B_x = \frac{\mu_0 N I R^2}{2x^3} = \frac{\mu_0 N I}{4\pi}\left(\frac{2\pi R^2}{x^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{2NIA}{x^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{2M}{x^3}\right)}$$

- Magnetic field due to an arc of a circle at the centre(O)

$$\boxed{B = \left(\frac{\theta}{2\pi}\right)\frac{\mu_0 I}{2R}}$$

16. Ampere circuital law

It states that line integral of $\vec{B}$ around a closed path is equal to μ_0 times current enclosed by that path, provided that electric field inside the loop remains constant.

$$\boxed{\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \boldsymbol{\mu_0 I_{net}}}$$

17. Magnetic field induction due to a solenoid carrying current I

- Magnetic field at the axis of a solenoid

$$\boxed{\boldsymbol{B = \frac{1}{2}\mu_0 n I(\cos\theta_1 - \cos\theta_2)}}$$

- Magnetic field at the centre (on the axis) of a long solenoid

$$\boxed{\boldsymbol{B(centre) = \mu_0 n I}}$$

- Magnetic field at ends of a long solenoid

$$\boxed{B(end) = \frac{1}{2}\mu_0 n}$$

18. Magnetic field induction due to a toroid carrying current I

- Magnetic field due to toroid

$$\boxed{B = \frac{\mu_0 N I}{2\pi r} = \mu_0 n I}$$

Where

$$\boxed{\boldsymbol{n = \frac{N}{2\pi r} = \frac{n}{l}}}$$

- Speed of light

$$\boxed{\boldsymbol{c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}}$$

18. Force between Two Long Parallel Wires

- Force on any one wire due to other parallel wire

$$\boxed{\boldsymbol{F = \frac{\mu_0}{2\pi}\left(\frac{I_1 I_2 L}{d}\right)}}$$

- $\frac{\mu_0}{2\pi} = 2 \times 10^{-7}$H/m

19. Moving coil Galvanometer:

$$\boxed{deflection, \varphi \propto current\ in\ the\ coil\ (I)}$$

torque-

$$\boxed{\tau = MB\sin\theta = (NIA)B\sin\theta}$$

This torque is balanced by spring in galvanometer, called spring torque or counter torque as in equilibrium,

$$\boxed{k\varphi = NIAB}$$

Here, k is the torsional constant of the spring. So,

$$\boxed{I = \left(\frac{k}{NAB}\right)\varphi}$$

Hence, the current $I \propto \varphi$; $\boldsymbol{Galvanometer\ constant} = \frac{\boldsymbol{k}}{\boldsymbol{NAB}}$

20. Sensitivity of Galvanometer:

- Current sensitivity

  Deflection per unit current is called current sensitivity

$$\boxed{current\ sensitivity = \frac{\varphi}{I} = \frac{NAB}{k}}$$

- Voltage sensitivity

  Deflection per unit P.D. is applied across the coil

$$\boxed{Voltage\ sensitivity = \frac{\varphi}{V} = \frac{\varphi}{IR} = \frac{NBA}{kR}}$$

$$\boxed{Current\ sensitivity = R(Voltage\ sensitivity)}$$

21. Conversion of galvanometer in to an ammeter:

A very low resistance is connected in parallel with a galvanometer

$$\boxed{\boldsymbol{I_g = I\frac{R_{sh}}{R_{sh} + R_g}}}$$

Hence an ammeter is a shunted or low resistance across a galvanometer.
Its effective resistance is,

$$\boxed{R_A = \frac{R_g R_{sh}}{R_g + R_{sh}} < R_{sh}}$$

22. Conversion of galvanometer into a voltmeter

A high resistance is connected in series with galvanometer to make a voltmeter.

$$\boxed{V = I(R_g + R_s)}$$

And total resistance of the circuit (resistance of voltmeter) is given by-

$$\boxed{R_V = R_g + R_s \gg R_g}$$

23. Bohr's Magneton

$$\boxed{\boldsymbol{M = \frac{evr}{2}}}$$

$$M_{min} = \frac{eh}{4\pi m} \text{ is called Bohr's magneton.}$$