Chapter-4: Moving Charges and Magnetism

1. Magnetic force on a moving charge

$$\boxed{\vec{F}_m = q\vec{v} \times \vec{B}}$$

Or

$$\boxed{F_m = qvB\sin\theta}$$

2. Path of charged particle in uniform magnetic field

- Path is straight line when

$$\boxed{\theta = 0^0 \ Or \ 180^0}$$

- Path is circle when

$$\boxed{\theta = 90^0}$$

- For any other angle, path is helix.

3. Circular path

$$\boxed{\vec{v} \cdot \vec{B} = 0}$$

- Centripetal force = Magnetic force

$$\boxed{m\frac{v^2}{r} = qvB}$$

- Radius of the circular path (r)

$$\boxed{r = \frac{mv}{qB}}$$

- Momentum (p)

$$\boxed{p = mv}$$

- Kinetic energy(K)

$$\boxed{p = \sqrt{2mK}}$$

- Time period,

$$\boxed{T = \frac{2\pi m}{Bq}}$$

- Angular velocity

$$\boxed{\omega = \frac{Bq}{m}}$$

- Frequency

$$\boxed{f = \frac{Bq}{2\pi m}}$$

4. Helical path

- Radius of helical path

$$\boxed{r = \frac{mv\sin\theta}{Bq}}$$

- Time period and

$$\boxed{T = \frac{2\pi m}{qB}}$$

- Frequency of helical path

$$\boxed{f = \frac{1}{T} = \frac{qB}{2\pi m}}$$

- Pitch of the helical path

$$\boxed{p = (v\cos\theta)T = \frac{2\pi mv\cos\theta}{Bq}}$$

5. Magnetic force on a current carrying conductor

- In Vector Form

$$\boxed{\vec{F}_m = I\vec{l} \times \vec{B}}$$

- In Magnitude Form

$$\boxed{F_m = IlB\sin\theta}$$

Here θ is the angle between $\vec{l}$ and $\vec{B}$ or between direction of current and magnetic field.

6. Lorentz Force

The total force on a charge Q moving in the presence of both electric and magnetic field is given by

$$\boxed{\vec{F} = \vec{F}_e + \vec{F}_m = Q\left(\vec{E} + \vec{v}X\vec{B}\right)}$$

7. Velocity selector

Path of a charged particle in uniform electric and magnetic field will remain unchanged if,

$$\boxed{\vec{F}_{net} = 0 \; or \; q\left(q\vec{E} + q\vec{v}x\vec{B}\right) = 0 \; or \; \vec{E} = -\vec{v}x\vec{B} \; or \; \vec{E} = \vec{B}x\vec{v}}$$

8. Cyclotron

It is a device used to accelerate charged particles (such as alpha particles, protons and deuterons or ions) to high energies to investigate nuclear structure

- The frequency of revolution of the particle

$$\boxed{f = \frac{qB}{2\pi m}}$$

- The Energy Gained by Charged particles

$$\boxed{K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{q^2B^2R^2}{m}\right)}$$

9. Magnetic dipole: and Magnetic dipole moment (unit Am)

- Current carrying loop is a magnetic dipole.
- Magnetic dipole moment of magnetic dipole

$$\boxed{M = NIA}$$

- Direction of $\vec{M}$ is the perpendicular to the plane of loop and given by right hand screw law.

10. Potential energy stored in dipole

$$\boxed{U = -\vec{M}.\vec{B} \ \ or\ U = -MB\cos\theta}$$

11. Work done in rotating dipole against torque

$$\boxed{W_{\theta_1\to\theta_2} = U_{\theta_2} - U_{\theta_1} = MB(\cos\theta_1 - \cos\theta_2)}$$

- If θ = 0^0, it is stable equilibrium position of the loop. In this condition, F = 0, $\tau = 0\ and\ U = -MB = Minimum$
- If θ = 180^0, it is unstable equilibrium position of the loop. Under thus condition,
  F = 0, $\tau = 0\ and\ U = +MB = Maximum$

12. Biot-Savart Law

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{I\vec{dl}x\vec{r}}{r^3}\right)\ or\ dB = \frac{\mu_0}{4\pi}\left(\frac{Idl\sin\theta}{r^2}\right)}$$

13. Magnetic field of a moving point charge

- The magnetic field vector $\vec{B}$ at point P at position vector $\vec{r}$ from the charge q moving with a velocity $\vec{v}$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\left[\frac{q(\vec{v}x\vec{r})}{r^3}\right] \; or \; B = \frac{\mu_0}{4\pi}\left(\frac{qvB\sin\theta}{r^2}\right)}$$

- Here θ is the angle between $\vec{v}$ and $\vec{r}$ and field is zero when θ = 0^0 or 180^0 and maximum at θ = 90^0, direction of $\vec{B}$ is along $(\vec{v}x\vec{r})$ if q is positive and opposite to $(\vec{v}x\vec{r})$ if q is negative.

14. Magnetic field induction at a point due to a straight wire carrying current I

- Magnetic field due at a point due to a straight wire of finite length (wire is held vertically)

$$\boxed{B = \frac{\mu_0 I}{4\pi x}\frac{\mu_0 I}{4\pi x}(\sin\alpha + \sin\beta)}$$

- Magnetic field at a point due to a straight wire of infinite length

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

- Magnetic field at a point due a straight wire with one end finite and other end infinite

$$\boxed{B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi r}\right) = \frac{\mu_0 I}{4\pi r}}$$

- Magnetic field created by long current carrying wire
  - r > R

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

  - r < R

$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$

15. Rules for finding direction of magnetic field in a straight conductor:

Right Hand Thumb Rule- Hold the straight conductor in your grip of right hand in such a way that your thumb points in the direction of current, the direction of curl fingers will point the direction of magnetic field.

16. Rules for finding direction of magnetic field in circular coil:

Right hand thumb rule (in case of axial position) or Cork Screw Rule (in case of centre of a circle)-The curl finger will point out to the direction of current and the thumb will give the direction of magnetic field.

17. Magnetic field induction at a point due to a circular wire carrying current I

- Magnetic field on the axis (either x, y or z-axis) of a circular loop (say, if coil placed in y-z plane so, $B_y = 0$, $B_z = 0$ and B_x

$$\boxed{B_x = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{\frac{3}{2}}}}$$

- Magnetic field at the centre (O) of ring (or circular loop)

$$\boxed{B_o = \frac{\mu_0 I}{2R}}$$

- If circular loop has N number of turns then magnetic field at its centre of radius R and carries current I

$$\boxed{B = \frac{\mu_0 N I}{2R}}$$

- Magnetic field when x >> R for N circular loops on axial position

$$\boxed{B_x = \frac{\mu_0 N I R^2}{2x^3} = \frac{\mu_0 N I}{4\pi}\left(\frac{2\pi R^2}{x^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{2NIA}{x^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{2M}{x^3}\right)}$$

- Magnetic field due to an arc of a circle at the centre(O)

$$\boxed{B = \left(\frac{\theta}{2\pi}\right)\frac{\mu_0 I}{2R}}$$

18. Ampere circuital law

It states that line integral of $\vec{B}$ around a closed path is equal to μ_0 times current enclosed by that path, provided that electric field inside the loop remains constant.

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{net}}$$

19. Magnetic field induction due to a solenoid carrying current I

- Magnetic field at the axis of a solenoid

$$\boxed{B = \frac{1}{2}\mu_0 n I(\cos\theta_1 - \cos\theta_2)}$$

- Magnetic field at the centre (on the axis) of a long solenoid

$$\boxed{B(centre) = \mu_0 n I}$$

- Magnetic field at ends of a long solenoid

$$\boxed{B(end) = \frac{1}{2}\mu_0 n}$$

20. Magnetic field induction due to a toroid carrying current I

- Magnetic field due to toroid

$$\boxed{B = \frac{\mu_0 NI}{2\pi r} = \mu_0 nI}$$

Where

$$\boxed{\boldsymbol{n = \frac{N}{2\pi r} = \frac{n}{l}}}$$

- Speed of light

$$\boxed{\boldsymbol{c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}}$$

21. Force between Two Long Parallel Wires

- Force on any one wire due to other parallel wire

$$\boxed{\boldsymbol{F = \frac{\mu_0}{2\pi}\left(\frac{I_1 I_2 L}{d}\right)}}$$

Where

- L= portion of length on the wire at which force to be determined
- d = separation between two long parallel wires
- $\frac{\mu_0}{2\pi} = 2 \times 10^{-7}$H/m

Note:

- When two parallel wires carrying current in same direction, the magnetic force acting between them is attractive force and
- When two parallel wires carrying current in opposite direction, the magnetic force acting between is repulsive force. [applying Fleming left hand rule]

22. Definition of Ampere

If two parallel long wires kept 1 m apart in vacuum, carry equal current in same direction and there is a force of attraction of 2×10^{-7} Nm^{-1} of each wire, the current in each wire is said to be 1 A.

The magnitude of magnetic field at distance x from long conductor is given as,

23. Moving coil Galvanometer:

$$\boxed{deflection, \varphi \propto current\ in\ the\ coil\ (I)}$$

The current to be measured is passed through the galvanometer. As the coil is in the magnetic field, it experiences a torque given by-

$$\boxed{\tau = MB\sin\theta = (NIA)B\sin\theta}$$

Where

N = Total number of turns of the coil,

I = Current passing through the coil,

A = Area of cross section of the coil,

B = Magnitude of radial magnetic field

The pole pieces are made cylindrical, the magnetic field always remains parallel to the plane of the coil.

That is angle between $\vec{M}\ and\ \vec{B}$ always remains 90^0.

This torque is balanced by spring in galvanometer, called spring torque or counter torque as in equilibrium,

$$\boxed{k\varphi = NIAB}$$

Here, k is the torsional constant of the spring. So,

$$\boxed{I = \left(\frac{k}{NAB}\right)\varphi}$$

Hence, the current $I \propto \varphi$; $\boldsymbol{Galvanometer\ constant} = \frac{\boldsymbol{k}}{\boldsymbol{NAB}}$

24. Sensitivity of Galvanometer:

A galvanometer is said to be sensitive if it shows large scale deflection even when a small current is passed through it or small voltage is applied across it.

There are two types of sensitivities:

- Current sensitivity

  Deflection per unit current is called current sensitivity

$$\boxed{current\ sensitivity = \frac{\varphi}{I} = \frac{NAB}{k}}$$

- Voltage sensitivity

  Deflection per unit P.D. is applied across the coil

$$\boxed{Voltage\ sensitivity = \frac{\varphi}{V} = \frac{\varphi}{IR} = \frac{NBA}{kR}}$$

$$\boxed{Current\ sensitivity = R(Voltage\ sensitivity)}$$

25. Sensitivity of a galvanometer can be increased by-

- Increasing the number of turns (N) in the coil or,
- Increasing the magnitude of magnetic field
- Increasing the Area A of the coil
- Decreasing the spring constant k

26. Conversion of galvanometer in to an ammeter:

A very low resistance is connected in parallel with a galvanometer

$$\boxed{\boldsymbol{I_g = I\frac{R_{sh}}{R_{sh} + R_g}}}$$

Hence an ammeter is a shunted or low resistance across a galvanometer.

Its effective resistance is,

$$\boxed{R_A = \frac{R_g R_{sh}}{R_g + R_{sh}} < R_{sh}}$$

Shunt is low resistance which is connected in parallel with galvanometer to protect it from strong currents.

27. Conversion of galvanometer into a voltmeter

A high resistance is connected in series with galvanometer to make a voltmeter.

$$\boxed{V = I(R_g + R_s)}$$

And total resistance of the circuit (resistance of voltmeter) is given by-

$$\boxed{R_V = R_g + R_s \gg R_g}$$

28. Expression of magnetic dipole moment of a revolving electron and thus define Bohr's Magneton

$$\boxed{\boldsymbol{M = \frac{evr}{2}}}$$

$$M_{min} = \frac{eh}{4\pi m} \text{ is called Bohr's magneton.}$$