Chapter-7: Alternating Current

1. AC Generator

$$\boxed{\varphi = BA\cos\theta = BA\cos\omega t}$$

$$\boxed{e = -N\frac{d\varphi}{dt}}$$

$$\boxed{e = E_0 \sin\omega t}$$

$$\boxed{E_0 = NBA\omega = 2\pi f NBA}$$

2. Average value of Voltage over half cycle of ac wave

$$\boxed{V_{av} = \frac{2V_0}{\pi} = 0.637V_0}$$

3. Average value of Current over half cycle of ac wave

$$\boxed{I_{av} = \frac{2I_0}{\pi} = 0.637I_0}$$

4. Phase and Phase Difference:

   In the circuit, $V = V_0 \sin(\omega t \pm \varphi_0)$ and $i = I_0 \sin(\omega t \pm \varphi_0)$

   Phase

$$\boxed{\omega t \pm \varphi_0}$$

   (φ_0 = initial phase angle)

   In the circuit, $V = V_0 \sin(\omega t \pm \theta_1)$ and $i = I_0 \sin(\omega t \pm \theta_2)$

   Phase Difference

$$\boxed{\varphi = (\pm\theta_1) - (\pm\theta_2)}$$

   (With respect to current phase)

5. Instantaneous Voltage Equation

$$\boxed{V = V_0 \sin(\omega t \pm \varphi) \;\; or \; V = V_0 \cos(\omega t \pm \varphi)}$$

6. Instantaneous Current Equation

$$\boxed{i = I_0 \sin(\omega t \pm \varphi) \;\; or \; i = I_0 \cos(\omega t \pm \varphi)}$$

   The phase angle ϕ is between voltage and current.

7. Relation between Time Period, Frequency and Angular Frequency

$$\boxed{T = \frac{2\pi}{\omega} \;\; or \; \omega = 2\pi f}$$

8. Root mean square (rms) value of current over one full cycle

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\ I_m$$

9. Root mean square (rms) value of voltage over one full cycle

$$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707 V_m$$

10. Power Factor of the Circuit

$$Power\ factor = \frac{Actual\ Power}{Apparent\ Power} = \cos\varphi = \frac{R}{Z}$$

11. Impedance of the LCR series ac circuit is Z.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

12. Average Power in the LCR series ac circuit

$$P_{av} = V_{rms} I_{rms} \cos\varphi$$

13. Purely Resistive ac circuit

If

$$V = V_0 \sin\omega t$$

Then

$$i = I_0 \sin\omega t$$

$$Z = R$$

$$R = \frac{V_{rms}}{I_{rms}} = \frac{V_0}{I_0}$$

$$V_R = I_{rms} R$$

$$P_{av} = I_{rms}{}^2 R = \frac{V_{rms}{}^2}{R} = \frac{I_0{}^2 R}{2}$$

14. Purely Inductive ac Circuit

If

$$V = V_0 \sin\omega t$$

Then

$$\boxed{i = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)}$$

$$\boxed{X_L = \omega L = 2\pi f L}$$

$$\boxed{Z = X_L}$$

$$\boxed{V_L = I_{rms} X_L}$$

$$\boxed{P_{av} = 0}$$

$$\boxed{Wattless\ Current = I_{rms} \sin\varphi}$$

15. Purely Capacitive Circuit

If

$$\boxed{V = V_m \sin\omega t}$$

Then

$$\boxed{i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)}$$

$$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$$

Or

$$\boxed{C = \frac{1}{2\pi f X_C}}$$

$$\boxed{V_C = I_{rms} X_C}$$

$$\boxed{Z = X_C}$$

16. R-L circuit

If

$$\boxed{V = V_m \sin\omega t}$$

Then

$$\boxed{i = I_m \sin(\omega t - \varphi)}$$

$$Z = \sqrt{R^2 + {X_L}^2}$$

$$V = \sqrt{{V_R}^2 + {V_L}^2}$$

$$\tan\varphi = \frac{X_L}{R} = \frac{\omega L}{R}$$

17. R-C circuit

If

$$V = V_m \sin\omega t$$

Then

$$i = I_m \sin(\omega t + \varphi)$$

$$z = \sqrt{R^2 + {X_C}^2}$$

$$\tan\varphi = \frac{X_C}{R} = \frac{1}{\omega CR}$$

18. In phasor representation in R-L circuit (inductive circuit),

$$\bar{Z} = R + jX_L$$

Or

$$\bar{V} = V_R + jV_L$$

LCR ac series circuit (also called Adaptor circuit)

RMS values of

$$V_R = I_{rms}R$$

$$V_L = I_{rms}X_L$$

$$V_C = I_{rms}X_C$$

Applied Voltage

$$\boxed{V_{applied} = I_{rms}Z = \sqrt{{V_R}^2 + (V_L - V_C)^2}}$$

For Inductive LCR series ac circuit,
If

$$\boxed{V = V_0 \sin \omega t}$$

Then

$$\boxed{i = I_0 \sin(\omega t - \varphi)}$$

For Capacitive LCR series ac circuit,
If

$$\boxed{V = V_0 \sin \omega t}$$

Then

$$\boxed{i = I_0 \sin(\omega t + \varphi)}$$

19. Resonance in LCR series ac circuit

Resonant frequency

$$\boxed{\omega_r = \frac{1}{\sqrt{LC}}}$$

Resonance Condition

$$\boxed{\omega = \omega_r}$$

$$\boxed{X_L = X_C}$$

$$\boxed{V_L = V_C}$$

20. Quality factor Q is defined as

$$\boxed{Q = \frac{Resonant\ frequency}{Bandwidth} = \frac{\omega_r}{\omega_2 - \omega_1}}$$

21. Quality factor at resonance is

$$\boxed{Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r\, L}{R}}$$

22. Quality factor can be in another form as,

$$\boxed{Q = \frac{1}{R}\left(\sqrt{\frac{L}{C}}\right)}$$

23. Transformer:

Transformation ratio

$$\boxed{\frac{e_s}{e_p} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{\eta I_p}{I_s}}$$

For an ideal transformer, there is no loss of power.

$$\boxed{\frac{e_s}{e_p} \vDash \frac{N_s}{N_p} = \frac{i_p}{i_s}}$$

Efficiency of a Transformer

$$\boxed{Efficiency(\eta) = \frac{Output\ Power}{Input\ Power \times Power\ Factor}}$$