CHAPTER-3: MOTION IN A PLANE

Vectors

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1. Scalars and Vectors

Scalar Quantity

Only Magnitude

Examples

- Mass
- Time
- Distance
- Speed
- Temperature

Vector Quantity

Magnitude + Direction

Examples

- Displacement
- Velocity
- Acceleration
- Force
- Momentum

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2. Position Vector

A particle located at point **P(x, y)** has position vector,

$$\boxed{\vec{r} = x\hat{\imath} + y\hat{\jmath}}$$

where

x = x-coordinate

y = y-coordinate

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3. Unit Vector

Magnitude of unit vector = 1

$$\boxed{|\hat{\imath}| = |\hat{\jmath}| = |\hat{k}| = 1}$$

If $\vec{r}$ is given, then

$$\boxed{\check{r} = \frac{\vec{r}}{|\vec{r}|}}$$

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4. Magnitude of a Vector

For

A⃗=Axi^+Ayj^

$$\boxed{\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}}$$

Magnitude

$$|\vec{A}| = \sqrt{{A_x}^2 + {A_y}^2}$$

5. Direction of Vector

If θ is angle with x-axis

$$\tan\theta = \frac{A_y}{A_x}$$

6. Addition of Two Vectors

If

$\vec{R} = \vec{A} + \vec{B}$

Then

$$R_x = A_x + B_x$$

$$R_y = A_y + B_y$$

Magnitude

$$|\vec{R}| = \sqrt{{R_x}^2 + {R_y}^2}$$

We can also write by law of Parallelogram method when two vectors are acting at a point,

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

7. Subtraction of Vectors

$\vec{R} = \vec{A} - \vec{B}$

Components

$$R_x = A_x - B_x$$

$$R_y = A_y - B_y$$

And its magnitude can also be written as

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

8. Resolution of Vector

Horizontal Component

$$A_x = A\cos\theta$$

Vertical Component

$$A_y = A\sin\theta$$

10. Dot Product (Scalar Product)

$$\boxed{\vec{A} \cdot \vec{B} = AB\cos\theta}$$

Special Cases
θ = 0°

$$\vec{A} \cdot \vec{B} = AB$$

θ = 90°

$$\vec{A} \cdot \vec{B} = 0$$

Note: If two vectors are perpendicular to each other, then their dot product is zero.

11. Cross Product (Vector Product)

$$\boxed{\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}}$$

$$\boxed{|\vec{A} \times \vec{B}| = AB\sin\theta}$$

Magnitude of $|\hat{n}| = 1$

Note: If two vectors are parallel with each other then their cross product is zero.

Direction
Right Hand Rule

12. Kinematics in Vector Form

Position Vector

$$\boxed{\vec{r} = \vec{r}_0 + \vec{v}t}$$

Velocity

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt}}$$

Acceleration

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt}}$$

13. First Equation of Motion (Vector Form)

$$\boxed{\vec{v} = \vec{u} + \vec{a}t}$$

In component form,
In x-direction,

$$\boxed{v_x = u_x + a_x t}$$

In y-direction,

$$\boxed{v_y = u_y + a_y t}$$

14. Second Equation of Motion (Vector Form)

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^1$$

In component form,
In x-direction,

$$s_x = u_x t + \frac{1}{2}a_x t^2$$

In y-direction,

$$s_y = u_y t + \frac{1}{2}a_y t^2$$

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15. Third Equation of Motion

$$\boldsymbol{v^2 = u^2 + 2\vec{a}\cdot\vec{s}}$$

In component form,
In x-direction,

$$\boldsymbol{v^2 = u^2 + 2a_x s_x}$$

In y-direction,

$$\boldsymbol{v^2 = u^2 + 2a_y s_y}$$

□ NEET Quick Revision Formulae

✔ Position Vector

$$\boldsymbol{\vec{r} = \vec{r}_0 + \vec{v}t}$$

✔ Magnitude

$$A^2 = {A_x}^2 + {A_y}^2$$

✔ Components
Horizontal Component

$$A_x = A\cos\theta$$

Vertical Component

$$A_y = A\sin\theta$$

✔ Dot Product

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

✔ Cross Product

$$\boldsymbol{|\vec{A}\times\vec{B}| = AB\sin\theta}$$

✔ Motion Equation

$$\boldsymbol{\vec{v} = \vec{u} + \vec{a}t}$$

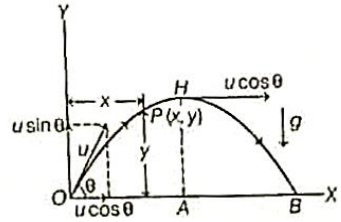
$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^1$$

$$\boldsymbol{v^2 = u^2 + 2\vec{a}\cdot\vec{s}}$$

Projectile Motion & Circular Motion

1. Horizontal and Vertical Components

For a projectile projected with velocity u at angle θ , ($\theta \neq 0^{0}, 90^{0} or 180^{0}$), the path is parabola.



Horizontal Component

$$u_x = u\cos\theta$$

Vertical Component

$$u_y = u\sin\theta$$

2. Horizontal Velocity

$$\boldsymbol{v_x = u\cos\theta}$$

(Constant)

3. Vertical Velocity

$$v_y = u_y\sin\theta - gt$$

At the maximum height,

$$v_y = 0$$

Note: Horizontal component $u_x = u\cos\theta$ is same everywhere on the path of projectile motion.

4. Resultant Velocity

When particle is thrown with initial velocity; $\vec{u} = u_x\hat{\imath} + u_y\hat{\jmath}$

$$u = \sqrt{{u_x}^2 + {u_y}^2}$$

5. Angle of projection

$$\tan\theta = \frac{u_y}{u_x}$$

6. Time of Flight

$$T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$$

7. Maximum Height

$$H = \frac{u^2 sin^2\theta}{2g} = \frac{{u_y}^2}{2g}$$

8. Horizontal Range

$$\boxed{R = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x u_y}{g}}$$

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9. Equation of Trajectory

$$\boxed{y = x \tan\theta - \frac{1}{2}\frac{gx^2}{u^2 cos^2\theta}}$$

In terms of Range ® and angle of projection θ,

$$\boxed{y = x \tan\theta \left(1 - \frac{x}{R}\right)}$$

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10. Greatest Range

For

$$\boxed{\theta = 45^0}$$

Maximum Range

$$\boxed{R_{max} = \frac{u^2}{g}}$$

Range will be maximum for complementary angles, $\theta \ and\ 90^0 - \theta$,

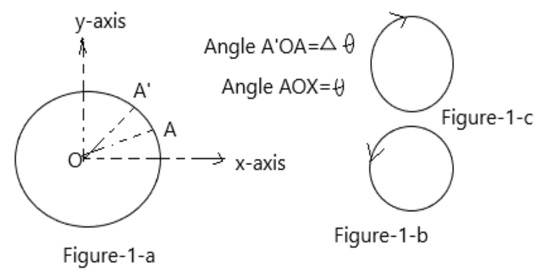
$$\boxed{R_\theta = R_{90^0-\theta}}$$

Circular Motion:

In circular motion, the particle has two velocities, (i) Angular velocity and Linear velocity. Acceleration is also is of two types, (i) Angular acceleration and (ii) Linear acceleration

11. Angular Velocity

$$\boxed{\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}}$$



Where θ is angular displacement.
SI Unit
rad/s

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12. Linear Velocity

$$\boxed{v = \frac{ds}{dt} = r\omega}$$

Here, $ds = rd\theta$

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13. Centripetal Acceleration or Radial Acceleration

$$\boxed{a_c = \frac{v^2}{r} = r\omega^2}$$

14. Centripetal Force

$$\boxed{F_C = \frac{mv^2}{r} = mr\omega^2}$$

15. Period of Revolution

$$\boxed{T = \frac{2\pi}{\omega}}$$

16. Frequency

$$\boxed{f = \frac{I}{T}}$$

17. Relation between Angular Velocity and Frequency

$$\boxed{\omega = 2\pi f}$$

18. Centrifugal Force

Pseudo Force

Magnitude

$$\boxed{F = m\frac{v^2}{r}}$$

Direction

Away from Centre

19. Banking of Roads (Without Friction)

$$\boxed{\tan\theta = \frac{v^2}{rg}}$$

20. Angular Acceleration:

$$\boxed{\alpha = \frac{d\omega}{dt}}$$

If angular acceleration and angular velocity are in same direction, Speed increases and if both are in opposite direction, speed decreases.

21. Linear Acceleration:

It is of two types, (i) Tangential acceleration and (ii) Centripetal acceleration.

Tangential Acceleration:

$$\boxed{a_t = \frac{dv}{dt} = r\alpha}$$

22. Resultant Linear Acceleration in Circular Motion:

$$\boxed{\boldsymbol{a} = \sqrt{\boldsymbol{a_c}^2 + \boldsymbol{a_t}^2}}$$

$$\boxed{\tan\theta = \frac{a_c}{a_t}}$$

23. In Uniform Circular Motion,

$$\boxed{Linear\ Speed = constant}$$

$$\boxed{Angular\ speed = constant}$$

$$\boxed{Tangential\ Acceleration\ (a_{t)} = 0}$$

$$\boxed{Angular\ Acceleration\ (\alpha) = 0}$$

Exam Tips

Students are often confused in **Dot Product** and **Cross Product**.

Remember:

- Dot Product → **Scalar**
- Cross Product → **Vector**
- Maximum Range is at **45°**.
- Equal Range for complementary angles θ and (90^0−θ).
- Velocity is always tangent in Circular Motion.
- Centripetal Acceleration and Force always **act towards Centre.**