CHAPTER-6: SYSTEM OF PARTICLE & ROTATIONAL MOTION

Part–1: Centre of Mass • Torque • Angular Momentum

1. Position of Centre of Mass (Two Particles)

For two masses m_1 and m_2,

$$\boxed{x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}}$$

Similarly,

$$\boxed{y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}}$$

2. Centre of Mass (n Particles)

$$\boxed{\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}}$$

3. Velocity of Centre of Mass

$$\boxed{\vec{V}_{CM} = \frac{\sum m_i \vec{v}_i}{\sum m_i}}$$

4. Acceleration of Centre of Mass

$$\boxed{\vec{a}_{CM} = \frac{\sum m_i \vec{a}_i}{\sum m_i}}$$

5. Linear Momentum of a System

$$\boxed{\vec{P} = \sum m_i \vec{v}_i}$$

6. Newton's Second Law for System

$$\boxed{\vec{F}_{ext} = \frac{d\vec{P}}{dt}}$$

7. Torque (Moment of Force)

Vector Form

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

Magnitude

$$\tau = rF\sin\theta$$

SI Unit: Newton metre (N-m)

8. Special Cases of Torque

Maximum Torque

$$\theta = 90^0$$

$$\tau = rF$$

Zero Torque

$$\theta = 0^\circ$$

9. Couple

Torque produced by a couple

$$\tau = Fd$$

where
d = Perpendicular distance between forces

10. Angular Momentum

Definition
Moment of Linear Momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

Magnitude

$$L = mvr\sin\theta$$

11. Relation between Torque and Angular Momentum

$$\tau = \frac{dL}{dt}$$

12. Conservation of Angular Momentum

If

$$\tau_{ext} = 0$$

Then

$$L = \text{Constant}$$

Part–2:Moment of Inertia • Rotational Dynamics • Rolling Motion

13. Angular Displacement

$$\theta = \frac{s}{r}$$

SI Unit: radian (rad)

14. Angular Velocity

$$\omega = \frac{d\theta}{dt}$$

Uniform Motion

$$\omega = \frac{2\pi}{T} = 2\pi f$$

15. Angular Acceleration

$$\alpha = \frac{d\omega}{dt}$$

16. Rotational Equations of Motion

First Equation

$$\omega = \omega_0 + \alpha t$$

Second Equation

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

Third Equation

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

17. Moment of Inertia

Definition

Resistance offered by a body to rotational motion.

$$I = \sum mr^2$$

SI Unit: kg m²

18. Radius of Gyration

$$I = Mk^2$$

Therefore

$$k = \sqrt{\frac{I}{M}}$$

19. Standard Moments of Inertia

Thin Ring

$$I = MR^2$$

Solid Disc

$$I = \frac{1}{2}MR^2$$

Solid Sphere

$$I = \frac{2}{5}MR^2$$

Hollow Sphere

$$I = \frac{2}{3}MR^2$$

Thin Rod (Through Centre)

$$I = \frac{1}{12}ML^2$$

Rod (Through End)

$$I = \frac{1}{3}ML^2$$

20. Parallel Axis Theorem

$$I = I_{CM} + Md^2$$

21. Perpendicular Axis Theorem

(For Plane Lamina)

$$I_z = I_x + I_y$$

22. Rotational Kinetic Energy

$$\boxed{KE = \frac{1}{2} I\omega^2}$$

23. Relation Between Torque and Angular Acceleration

$$\boxed{\tau = I\alpha}$$

24. Angular Momentum

$$\boxed{L = I\omega}$$

25. Rolling Motion

For Pure Rolling

$$\boxed{v = r\omega}$$

26.Total Kinetic Energy

$$\boxed{KE = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2}$$

Standard MOI Table (NEET Favourite)

Body	Moment of Inertia
Thin Ring	MR^2
Solid Disc	$\frac{1}{2}MR^2$
Solid Sphere	$\frac{2}{5}MR^2$
Hollow Sphere	$\frac{2}{3}MR^2$
Rod (Centre)	$\frac{1}{12}ML^2$
Rod (End)	$\frac{1}{3}ML^2$

Exam Tips

- Centre of Mass always tends to higher mass.
- Torque and Angular Momentum both are **Vector Quantities**.
- If External Torque zero then Angular Momentum is conserved.
- Moment of inertia of **Ring** is greater for same mass and same radius.
- Always Remember $v = r\omega$ in **Pure Rolling**
- Parallel Axis Theorem and Standard MOI questions are important.