Class 12 Physics Notes Chapter-2: Electrostatic Potential and Capacitance

2.0. Introduction

In the previous chapter, we studied **electric force** and **electric field**, which describe how charges exert forces on one another.
However, in many practical situations, it is more convenient to describe an electric field in terms of **energy** rather than force. This leads to the concepts of **electrostatic potential** and **potential difference**.
Just as a body possesses **gravitational potential energy** due to its position in a gravitational field, a charge possesses **electrostatic potential energy** due to its position in an electric field.

2.1. ELECTROSTATIC POTENTIAL:

Definition:

Electric Potential at a point is the work done per unit positive test charge by an external agent in bringing the test charge from infinity to that point, without acceleration.

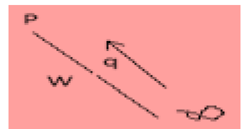


Figure 2.1-1

$$V = \frac{W_{ext}}{q_o}$$

Where, $q_0 \to 0$ as it could not disturb the existing field

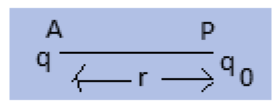
Also,

Electric potential is the electrostatic potential energy per unit positive test charge.

It is scalar quantity. Its SI unit is volt (V).

$$\boxed{W_{i\to f} = Q\left(V_f - V_i\right)}$$

$$\boxed{V_P = Lim_{q_0\to 0}\frac{U}{q_0}}$$



2.2. RELATION BETWEEN POTENTIAL DIFFERENCE, WORK DONE AND POTENTIAL ENERGY:

1. by the conservative force: work done to move a point charge q from initial point i to final point j:

$$\boxed{W_{i\to f} = q\left(V_i - V_f\right) = U_i - U_f = -\Delta U}$$

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2. against the conservative force or by the external force: work done to move a point charge q from initial point i to final point j:

$$\boxed{W_{i\to f} = q\left(V_f - V_i\right) = \left(U_f - U_i\right) = \Delta U}$$

3.If nothing is given to find the work done by the conservative force or by the external force, it is simply asked to find or calculate the work done, then apply the formula mentioned above in point 3).

Example-1:
Calculate the work done to move a charge of 1 μC from infinity to a point which has the potential of 12 V.
Solution:
Since, $W_{\infty \to A} = q(V_A - V_\infty) = 1 \times 10^{-6}(12 - 0) = 12 \times 10^{-6} = 1.2 \times 10^{-5}$
Answer: $\boxed{1.2 \times 10^{-5} V}$

2.3. Electric Potential and Electric Field due to a point Charge at distance r:

Graph-

Graph between E and V versus r

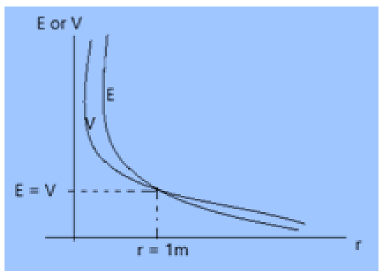

Figure 2.3-1

2.4. POTENTIAL DUE TO A POINT CHARGE:

1. Electric potential at a point (sat at point P) **due to a positive point charge** q located at a point (say at A) is given as:

$$\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q}{r}\right)}$$

Where r = distance AP. whereas, **due to a negative point charge**, potential at distance r from point P is:

$$\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{-q}{r}\right)}$$

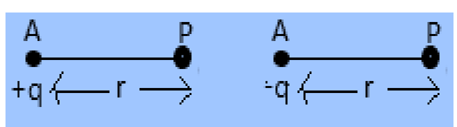

Figure 2.4-1

2. As distance increases, potential decreases due to positive point charge, and potential increases due to negative point charge. In the figure below, AB = r_1 and AC = r_2; since $r_2 > r_1$ therefore, due to + q charge, $V_C < V_B$ and due to –q charge, $V_C > V_B$.

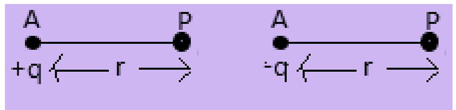

Figure 2.4-1

3. Graph between V and r:

For a point charge, the magnitude of electric potential varies inversely with distance r.
due to positive point charge and negative point charge, is given below:

$$V \propto \left(\frac{+q}{r}\right)$$

and

$$V \propto \left(\frac{-q}{r}\right)$$

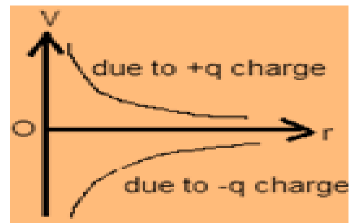

Figure 2.4-3

4. For an isolated finite charge distribution, the reference potential is conventionally taken as: $V_\infty = 0$.

The corresponding potential energy is also taken as zero at infinity.

The Electric potential is a scalar quantity. It has no direction. Its SI unit is volt.

$$\boxed{1V = 1J/C}$$

2.5. POTENTIAL DUE TO A SYSTEM OF CHARGES:

1. Electric potential at a point due to several charges q_1, q_2, q_3 ………..which are at distances r_1, r_2, r_3, ..respectively is given as

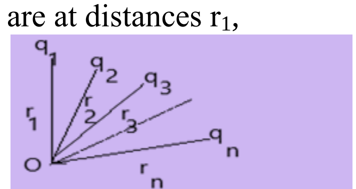

Figure 2.5-1: Electrical Potential at point O due to several charges

$$\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + - - - - - - - + \frac{q_n}{r_n}\right)}$$

$$\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\sum_{i=1}^{n}\frac{q_i}{r_i}}$$

2. Electric potential at a point (say at P):

If several charges $(q_1, q_2, q_3, \dots)$ are at the same distance r from point P, then

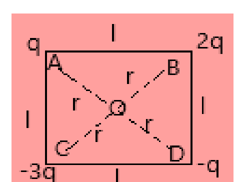

Figure 2.5-2

$$\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_{net}}{r}\right)}$$

Example-1

Find the Electric Potential at the centre O of a square ABCD of each side l. Charges are shown at each vertex in Figure 2.5-2

Solution:

Here $q_{net} = -q; r = l/\sqrt{2}$

Therefore, net potential at O

$$V_0 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_{net}}{r}\right)$$

Or

$$V_{net} = \frac{1}{4\pi\epsilon_0}\left(\frac{-q\sqrt{2}}{l}\right)$$

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3. Electric potential at a point (say at point P) at position vector $\vec{r}$ due to charges $q_1, q_2, q_3, \dots$.. which are placed at position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$, ...respectively, given as:

$$\boxed{V = \sum_{i=1}^{n} \frac{q_i}{|\vec{r} - \vec{r}_i|}}$$

Example-2:

The four charges, +q, -q, + q –q are located at each vertex of a square ABCD respectively. Calculate the electric potential at a point O of a square of each side L.

Answer:

Since, electric potential at a point at distance r due to a positive charge q is given as: $V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q}{r}\right)$.

So, $V_O = V_{OA} + V_{OB} + V_{OC} + V_{OD} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{\sqrt{2}q}{a} + \frac{\sqrt{2}(-q)}{a} + \frac{\sqrt{2}q}{a} + \frac{\sqrt{2}q}{a}\right) = 0$

2.6. POTENTIAL DUE TO AN ELECTRIC DIPOLE:

Suppose a dipole (+q and –q) is separated by a distance 2l and we have to find the electric potential at distance r from the centre of dipole and inclined an angle θ from the dipole axis, then it is given as:

$$V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p\cos\theta}{r^2}\right)$$

Derivation-1:

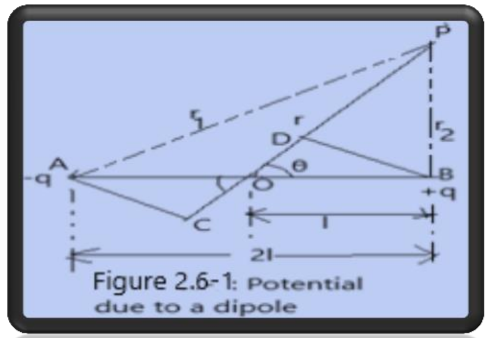


Figure 2.6-1: Potential due to a dipole

As shown in the figure 2.6-1,
Let AP = r_1, PB = r_2; $AP \approx PC\ and\ PD \approx PB$
$r_1 = PO + OC = r + l\cos\theta;$
$r_2 = PO - O\,D = r - l\cos\theta,$
Hence, Electric Potential at point P due to dipole is
$V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{-q}{r_1} + \frac{q}{r_2}\right)$

Or,

$$V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{-q}{r + l\cos\theta} + \frac{q}{r - l\cos\theta}\right)$$

Or,

$$V = \left(\frac{q}{4\pi\epsilon_0}\right)\left(\frac{2l\cos\theta}{r^2 - l^2 cos^2\theta}\right)$$

Since, dipole moment p = 2lq and for r >> l, $l^2 cos^2\theta$ is neglected. So, the expression of electric potential due to dipole at distance r from the centre of dipole, makes an angle θ is,

$$\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p\cos\theta}{r^2}\right) - - - - - - -(1)}$$

From the above equation (1),

1. Axial Position of a dipole:

The electric potential at Axial position of dipole is: $\theta = 0^0\ Or, 180^0$

$$\boxed{V_{axial} = \pm\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{p}{r^2}\right)}$$

2. Equatorial Position of a dipole: $\theta = 90^0$, $\cos 90^0 = 0$

$$\boxed{V_{equatotial} = 0}$$

2.7. POTENTIAL ENERGY OF A SYSTEM OF TWO CHARGES WHEN NO EXTERNAL FIELD:

Suppose two charges $q_1$and q_2 are initially at infinity and determine the work done by an external agency to bring the charges to the locations of position vectors $\vec{r}_1\ and\ \vec{r}_2$ relative to some origin.

Derivation-2:

Suppose the first charge q_1 is brought from infinity to a point of position $\vec{r}_1$. There is no external field against which work to be done. So, work done in bringing q_1 from infinity to $\vec{r}_1$is zero. So potential at point P (position vector $\vec{r}_2$) at distance r_{12} from point charge q_1 (position vector $\vec{r}_2$) is:

$$V_1 = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1}{r_{12}}\right)$$

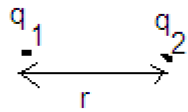
Now, work done to bring charge q_2 from infinity to point P whose position vector $\vec{r}_2$ is:

$$W = q_2 V_1 = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1 q_2}{r}\right) \; where \; r = r_{12}$$

Since, electric force is a conservative force, so, this work is stored as potential energy. Hence,

$$U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r}\right) \quad \ldots (\boldsymbol{Derived})$$

So, potential energy is directly proportional to the product of two charges and inversely proportional to the separation between two charges.



$$U = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1 q_2}{r}\right)$$

1. When a charge q is brought near to another positive charge Q, its potential energy increases to overcome the repulsive force as shown in the above equation and its kinetic energy decreases as the total energy is conserved.
2. The charge is brought slowly with constant velocity, so there is no change in its kinetic energy.
3. Potential Energy of the system due to various charges $q_1, q_2, q_3, ..$ at positions $r_1, r_2, r_3, ..$is given as:

$$\boxed{U = \sum_{i=1\, i<j}^{n} \frac{q_j q_i}{r_j i}}$$

Example-3

Three charges of $1\mu C \; are \; placed$ at the vertex A, B, C of an equilateral triangle of side 10 cm. Find the total potential energy of the system.

Solution:

Since,

$$U = 9\times 10^9\left(\frac{q_3 q_2}{r_{32}} + \frac{q_3 q_1}{r_{31}} + \frac{q_2 q_1}{r_{21}}\right)$$

$$= 9\times 10^9\left(\frac{1\times 10^{-6}\times 1\times 10^{-6}}{10\times 10^{-2}} + \frac{1\times 10^{-6}\times 1\times 10^{-6}}{10\times 10^{-2}} + \frac{1\times 10^{-6}\times 1\times 10^{-6}}{10\times 10^{-2}}\right)$$

$$= 9\times 3\times 10^9\times 10^{-12}\times 10^1 = 0.12\, J$$

2.8. Potential energy due to a single point charge in an external electric field:

Work done to bring a point charge q from infinity to a point of potential V against the conservative force is

$$W_{i\to f} = q(V_f - V_i) = U_f - U_i$$

Since,

$$U_i = V_i = 0$$

Therefore, from above equation,

$$W_{i\to f} = q(V_f) = U_f$$

Or

$$U = U_f = qV$$

2.9. Potential energy of a system of two point charges in an external electric field:

To find the potential energy of charges q_1 and q_2 located at positions $\vec{r}_1$ and $\vec{r}_2$ respectively in an external field.

Derivation-3:

Work done to bring charge q_1 from infinity to a point A at position vector $\vec{r}_1$in the external electric field is:

$$W_1 = q_1 V_{(\vec{r}_1)} - - - - - (i)$$

Work done to bring the charge q_2 from infinity to a point B at position vector $\vec{r}_2$ in the external electric field

$$W_2 = q_2 V_{(\vec{r}_2)} - - - - - - (ii)$$

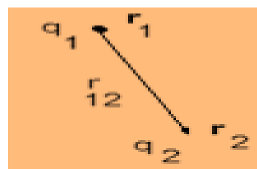


Figure 2.9-1

And, work done on q_2 due to the charge q_1:

$$W_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}}\right) - - - - - (iii)$$

Where, r_{12} = distance between charge q_1 and q_2.

Now, potential energy of the system = Total work done in assembling the configuration. So,

$$\boxed{U = W_1 + W_2 + W_3 = q_1 V_{(\vec{r}_1)} + q_2 V_{(\vec{r}_2)} + \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1 q_2}{r_{12}}\right)}$$

Example-4

(i) Find the potential energy of a system of two charges 5 μC and -2 μC placed at $(-5\ cm, 0, 0)$ and $(5\ cm, 0, 0)$respectively.

(ii) If the same charges is now placed in an external electric field $E = A(1/r^2)$; where $A = 5 \times 10^5 NC^{-1}m^2$, then what will be the potential energy of the system?

Solution:

(i)Electrostatic potential energy,

$$U = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1 q_2}{r}\right)$$

Here, r = $r_2 - r_1 = 5 — (-5) = 10\ cm,\ q_1 = 5\mu C = 5 \times 10^{-6} C\ and\ q_2 = -2\mu C$

$= -3 \times 10^{-6}$C, therefore,

$$U = 9 \times 10^9 \times \frac{5 \times 10^{-6} \times (-2) \times 10^{-6}}{18 \times 10^{-2}} = -0.5\,J$$

Therefore, work done = -U = + 0.5 J

If both charges are in the external electric field, then potential energy,

$$U = q_1 V_{\vec{r}_1} + q_2 V_{\vec{r}_2} + \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_2 q_1}{r}\right)$$

Here, $V_{\vec{r}_1} = E \times r_1 = A\left(\frac{1}{{r_1}^2}\right)(r_1) = 5 \times 10^5 \times \frac{1}{r_1} = 5 \times 10^5 \times \frac{1}{5\times10^{-2}} = 10^7 V$

$$V_{\vec{r}_2} = E \times r_2 = A\left(\frac{1}{{r_2}^2}\right)(r_2) = 5 \times 10^5 \times \frac{1}{r_2} = 5 \times 10^5 \times \frac{1}{5 \times 10^{-2}} = 10^7 V$$

Therefore,

$$U = 5 \times 10^{-6} \times 10^7 + (-2) \times 10^{-6} \times 10^7 + 9 \times 10^9 \times \left(\frac{(-2 \times 10^{-6})(5 \times 10^{-6})}{10 \times 10^{-2}}\right)$$
$$= 50 - 20 - 0.5 = 29 - 0.5 = 28.5\,J$$

2.10. POTENTIAL ENERGY OF A DIPOLE IN AN EXTERNAL ELECTRIC FIELD:

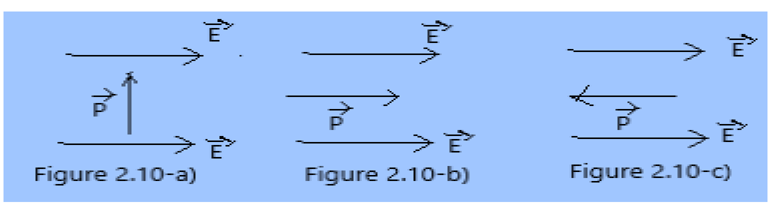
The potential energy due a dipole of dipole moment $\vec{p}$ placed in an external electric field $\vec{E}$ and its dipole axis making angle θ (angle between dipole moment and electric field) is given as:

$$\boxed{U = -pE\cos\theta}$$

(a) From the above formula, mentioned in equation, it can be explained the followings: Potential energy will be zero if a dipole is placed perpendicular to the electric field, $\theta = 90^0$.

(b) Potential energy in **stable equilibrium position** (dipole moment vector and electric field vector parallel to each other, $\theta = 0^0$) is minimum and its value is –PE.

(c) Potential energy in **unstable equilibrium position** (dipole moment vector and electric field vector anti- parallel to each other, $\theta = 180^0$) is maximum and its value is +PE.



(d) Work done to rotate a dipole from angle θ_1 to angle $\theta_2$by an external agent (if not mentioned, apply this formula) as:

$$\boxed{W = pE(\cos\theta_1 - \cos\theta_2)}$$

(e) Work done to rotate a dipole from angle θ_1 to angle $\theta_2$by conservative force is:

$$W = pE(\cos\theta_2 - \cos\theta_1)$$

Potential energy due to electric dipole in vector form in the external uniform electric field can be expressed as:

$$\boxed{U = -\vec{p}\cdot\vec{E}}$$

2.11. RELATION BETWEEN ELECTRIC FIELD AND ELECTRIC POTENTIAL:

The electric field $\vec{E}$ and potential gradient dV/dr is related as:

$$\boxed{dV = -\vec{E}\cdot d\vec{r}}$$

It can be **derived** as:

The small work done by the conservative force to displace a body dr is

$$dW = \vec{F}\cdot d\vec{r} - - - - - (i)$$

Since,

$$\vec{F} = q\vec{E} - - - - - (ii)$$

Also, small work done to move a charge q from infinity to a point of small potential difference

$$dW = q[dV_i - dV_f] = -qdV_f = -qdV - - - -(iii)$$

From the equation (i), (ii) and (iii), we get

$$q\vec{E}\cdot d\vec{r} = -qdV$$

Or,

$$\boxed{dV = -\vec{E}\cdot d\vec{r} - - - - - (iv)}$$

Where,

$$\boxed{d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}}$$

Or,

$$\boxed{\int_{V_2}^{V_1} dV = -\int_{r_2}^{r_1} \vec{E}\cdot d\vec{r}\,, where\ d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}}$$

Where,

$$\boxed{\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k}}$$

From the expression in equation (iv), the negative sign indicates that electric potential decreases in the direction of the electric field.

Note: If potential at point 1 is V_1 and at point 2, it is V_2 and distance between these two points is d, then,

$$\boxed{V_1 - V_2 = Ed}$$

2.12. EQUIPOTENTIAL SURFACE AND ITS PROPERTIES:

A surface on which potential is equal at every point is known as equipotential surface.
For an electric dipole, the equatorial plane passing through the midpoint and perpendicular to the dipole axis is an equipotential surface of zero potential.
In a two-dimensional cross-section, this equipotential surface appears as a straight line.

2.12.1. Properties of equipotential surface:

(a) Work done to move a charge on equipotential surface ΔV is zero because potential difference between any two points is zero as: $W = q\Delta V$

(b) Electrical field is perpendicular to the equipotential surface. As $dV = 0$ and $dV = -\vec{E} \cdot d\vec{r}$. Hence, $\vec{E} \cdot d\vec{r} = 0$. That is, $\vec{E}$ is perpendicular to $d\vec{r}$.

(c) Two equipotential surfaces never cross each other. If they do so, then at the point of intersection, there will be two potential at the same point and this is not possible.

(d) The spacing between equipotential surfaces is smaller in regions of stronger electric field and larger in regions of weaker electric field.

$$E \approx \frac{\Delta V}{\Delta r}$$

(e)Electric potential decreases along the direction of electric field, because, $dV = -\vec{E} \cdot d\vec{r}$.
(f)Equipotential surfaces are planar for the uniform electric field.
(g)For a single isolated charge, equipotential surfaces are spherical in shape and spacing increases.

2.13. To Derive Electric Potential Energy due to a Dipole: Derivation-4

As shown in the figure, a small work done to rotate a dipole against the torque,

$$dW = -\tau d\theta - - - - (i)$$

Since,

$$\tau = pE \sin\theta - - - - (ii)$$

Hence,

$$dW = -pE \sin\theta \, d\theta - - - (iii)$$

The work done by a conservative force is,

$$dW = -dU - - (iv)$$

Integrating equation (iv) both sides, we have,

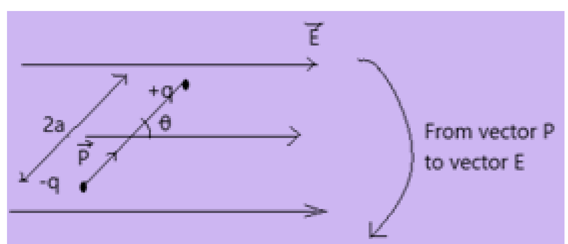


Figure 2.13-1

$$U = \int_{90^0}^{\theta} pE \sin\theta \, d\theta = -PE\cos\theta - - - (v)$$

Note:

At, 90^0, the electric potential energy is zero.

Hence,

$$\boxed{U = -pE\cos\theta}$$

2.14. Properties of Conductors:

(a) In an electrostatic equilibrium, the electric field inside the material of a conductor is zero because free charges rearrange themselves on the surface until the internal electric field becomes zero. .

Or,

In electrostatic equilibrium, the electric field inside the conducting material is zero because free charges redistribute themselves on the conductor's surface until the net electric field inside the conductor becomes zero.

(b) Electric potential is same throughout inside a conductor and its value is equal to the potential at the surface.

(c) Electric field is perpendicular on the surface of a conductor irrespective of its shape and size.

(d)The electric field just outside the surface of a conductor is

$$E_{out} = \frac{\sigma}{\epsilon_0}$$

And, inside

$$E_{in} = 0$$

Where, $\sigma = \frac{q}{A}$ = surface charge density.

2.15. Electrostatic Shielding

Definition

Electrostatic shielding is the phenomenon in which the electric field inside a closed conducting enclosure becomes zero, protecting its interior from external electric fields.

Principle

Since E= 0 inside a conductor in electrostatic equilibrium. In electrostatic equilibrium, a closed conducting enclosure shields its interior from external electrostatic fields, provided no charge is placed inside the cavity.

Applications

- Faraday cage
- Protection of sensitive electronic instruments
- Shielded cables
- Metal body of cars protecting occupants during lightning strikes

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Faraday Cage

A **Faraday cage** is a hollow conducting enclosure that prevents external electric fields from penetrating its interior.

Working Principle

Charges induced by an external electric field rearrange themselves on the outer surface of the conductor, cancelling the field inside.

Electrostatic Shielding in Daily Life

- Aircraft and automobiles during thunderstorms
- Coaxial cables used in communication systems
- Shielding of laboratory equipment
- Protection of electronic circuits from external interference

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2.16. CAPACITORS AND CAPACITANCE:

Definition of a capacitor

A capacitor is a system of two conductors separated by an insulating medium (dielectric). When charged, the conductors carry equal and opposite charges.

A capacitor stores the charge as well as potential energy in the electrostatic field. The capacity of a capacitor is called its capacitance.

A capacitance is defined as the ratio of charge stored (q) to the potential difference (V) across it.

$$q \propto V\ Or, q = CV$$

$$\boxed{C = \frac{q}{V} = Constant}$$

Here, proportionality constant C is known as capacitance.

- The SI unit of capacitance is Farad and 1 F = 1 C/V
- For a given capacitor under fixed physical conditions, capacitance is independent of q and V.
- The capacitance depends upon:
  (i) Area (A) of plates, $C \propto A$,
  (ii) Separation (d) between the plates, $C \propto \frac{1}{d}$
  (iii)Permittivity $(\epsilon_0 r)$of the medium, $C \propto \epsilon_r\ Or, C \propto K$, where K = dielectric constant and equals to relative permittivity
- For n parallel plates, equivalent capacitance will be $(n-1)C$.
- The graph between q and V is a straight line and its slope = C if q on y-axis and V on x-axis as shown below.

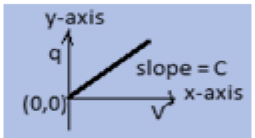


Figure 2.16-1

- If charge on positive plate of the capacitor is q_1 and on negative plate, it is q_2, then its capacitance will be:

$$\boxed{C = \frac{q_1 - q_2}{2V}}$$

Actually, capacitance on positive plate is taken as the charge on capacitor.

2.17. Capacitance of an Isolated Spherical Conductor

Consider an isolated conducting sphere of radius R. Potential at its surface is

$$V = \frac{q}{4\pi\epsilon_0 R}$$

Using

$$C = \frac{q}{V}$$

We obtain

$$C = 4\pi\epsilon_0 R$$

2.18. PARALLEL PLATE CAPACITOR:

The capacitance of a parallel plates of plate area A, separated by distance d and filled with air (air capacitor with dielectric constant K = 1) is given by:

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

The capacitance of a parallel plates of plate area A, separated by distance d and filled with dielectric slab of dielectric constant K and thickness t, (partially inserted) is given by

$$C = \frac{\epsilon_0 A}{(d-t) + \frac{t}{K}}$$

$\boldsymbol{Derivation - 5}$:

Suppose, A capacitor C of plate area A, separated by distance d and a dielectric slab of dielectric constant K is partially filled between the gap of a parallel plate capacitor, having thickness t.

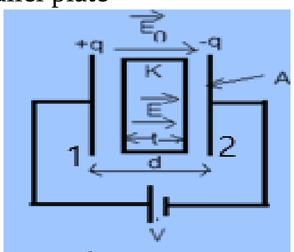


Figure 2.16-2

As shown in the arrangement, the potential difference V is given as,

$$V = V_1 - V_2 = Et + E_0(d-t) - - - (i)$$

Since, dielectric constant,

$$K = \frac{E_0}{E} - - - (ii)$$

Where,

E_0 = Electric field in the air and E = Net electric field inside the dielectric.

$From\ equation\ (i) and\ (2)$, we get,

$$V = \frac{E_0 t}{K} + E_0(d-t) - - - (iii)$$

By definition,

$$V = \frac{q}{C} - - - - (iv)$$

From equation (iii) and (iv), we have,

$$\frac{q}{C} = E_0\left[(d - t) + \frac{t}{K}\right] - - - - (v)$$

Now,
Electric field in the air,

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0} - - - (vi)$$

Where,

$\sigma = Surface\ charge\ density\ , \epsilon_A = permittivity\ of\ free\ space$

So, from equation (v) and (vi), we get

$$\frac{q}{C} = \frac{q}{A\epsilon_0}\left[(d - t) + \frac{t}{K}\right] - - - - (vii)$$

Hence, from equation (vii), we get the capacitance of a parallel plate capacitor filled with dielectric of constant K is,

$$\boxed{C = \frac{\epsilon_0 A}{(d - t) + \frac{t}{K}}}$$

From the above equation of Parallel Plates Capacitor,
(JEE/Competitive Exam Enrichment — Beyond CBSE Core)

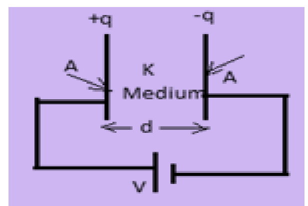


Figure 2.16-3

- If dielectric slab is fully inserted between the plates, i. e. d = t, then its capacitance will be as:

$$\boxed{C = \frac{\epsilon_0 K A}{d}}$$

- If a conductor is completely inserted between the plates, i. e. d = t, and K =∞ $\left(K = \frac{E_0}{E}\ and\ E = 0\ inside\ a\ conductor\right)$then its capacitance will be as:

$$\boxed{C = \infty}$$

- If a conductor is partially inserted between the plates, i. e. $d \neq t$, and K =∞, then

$$\boxed{C = \frac{\epsilon_0 A}{(d - t)}}$$

- If two dielectric slabs of dielectric constant $K_1$and K_2 with their respective thickness t_1 and t_2 are partially inserted, then its capacitance will be

$$\boxed{C = \frac{\epsilon_0 A}{[d - (t_1 + t_2)] + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2}\right)}}$$

The arrangements of parallel plate capacitors for air, dielectric and a conductor are shown below:

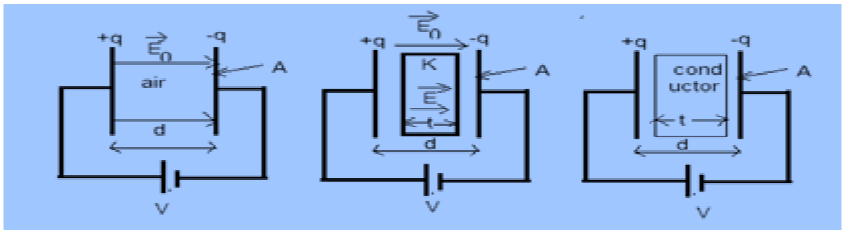


Figure 2.16-4

Solved Example 1
A capacitor stores 6μC of charge at a potential difference of 12V. Find its capacitance.

Solution
Using

$$C = \frac{q}{V} = \frac{6 \times 10^{-6}}{12} = 0.5\ \mu F$$

Solved Example 2
A parallel plate capacitor has an area of 0.02m^2 and a plate separation of 1mm. Calculate its capacitance.

Solution
Using

$$C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 0.02}{1 \times 10^{-3}} = 1.77 \times 10^{-10} F$$

Exam Tips
- Capacitance depends only on the geometry and dielectric medium.
- Increasing the plate area increases capacitance.
- Increasing the plate separation decreases capacitance.
- For a given capacitor geometry, inserting a dielectric of relative permittivity K > 1 increases its capacitance by a factor K.
- The SI unit of capacitance is the **farad (F)**.

Common Mistakes
 Confusing capacitance with charge stored.
 Forgetting that capacitance is independent of Q and V.
 Using $C = \epsilon_0 Ad$ instead of the correct formula.
 Ignoring unit conversions from mm to m or μF to F.

2.18. EFFECT OF DIELECTRIC ON CAPACITANCE:
When a single capacitor of same plate area A, separated by distance d and has charge equal and opposite on plates, is first connected with a battery and then after some time, **battery is**

disconnected and a dielectric slab of dielectric constant K is completely inserted, then its new parameters will be as

Initial values are: initial capacitance, C_0, initial potential difference V_0, initial electric field E_0, initial charge q_0, initial potential energy U_0 and initial energy density u_0):

Final values are:

(**Shown in Fig-1 and Fig-2 below**)

- capacitance, $C = C_0K$ (**increases by K times**)
- potential difference across plates will be, $V = \frac{V_0}{K}$ (**decreased and becomes 1/K times**)
- electric field,$E = \frac{E_0}{K}$ (**decreased and becomes 1/K times)**
- electric charge, $q = q_0$(**remains same**)
- potential energy, $U = \frac{U_0}{K}$ (**decreased and becomes 1/K times**)
- energy density, $u = u_0/K$ (**decreased and becomes 1/K times**)

When a single capacitor is first connected with a battery and then after some time, a dielectric slab of dielectric constant K is completely inserted, **when battery remains connected**, then its new parameters will be as **(Initial values are: initial capacitance, C_0, initial potential difference V_0, initial electric field E_0, initial charge q_0, initial potential energy U_0 and initial energy density u_0).**

(Shown in Fig-1 and Fig-3 below)

- capacitance, $C = C_0K$ (**increases by K times**)
- potential difference across plates will be $V = V_0$
- electric field, $E = E_0$
- electric charge, $q = q_0K$
- potential energy, $U = U_0K$
- energy density,$u = u_0K$

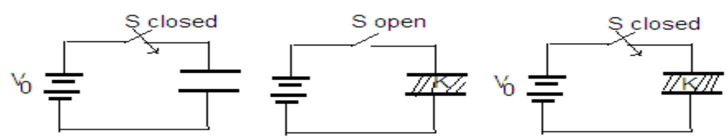


Fig-1: Initially Closed

Fig-2: Disconned and slab is fully inserted

Fig-3: Remains connected with battery and dielectric is fully inserted

Note: Initially, $C_0 = \frac{\epsilon_0 A}{d}$; $V_0 = \frac{q_0}{C_0}$; $E_0 = \frac{q_0}{A\epsilon_0}$;Finally: $V = \frac{q}{C}$, $E = \frac{q}{A\epsilon}$, $\epsilon = \epsilon_0 K$; $U = \frac{q^2}{2C} = \frac{CV^2}{2}$; $u = \frac{\epsilon E^2}{2}$

JEE Insight (Beyond NCERT)

1. Why does a dielectric increase capacitance?

A dielectric reduces the **effective electric field** between the plates. Therefore, for the same charge, the potential difference decreases, and since $q = CV$, the capacitance increases.

---

2. Common JEE Concept

Even if the **electric field inside a dielectric decreases**, the **charge stored on a battery-connected capacitor increases** because the battery maintains a constant potential difference.

3. Battery Connected vs Battery Disconnected

Students should always identify the condition before solving numerical problems.

Quantity	Battery Connected	Battery Disconnected
Charge (Q)	Constantly supplied by battery (can change)	Constant
Potential Difference (V)	Constant	Changes
Capacitance (C)	Increases with dielectric	Increases with dielectric
Electric Field (E)	Electric field remains constant for fixed plate separation, since E= V/d and V and d remain constant.	Decreases
Stored Energy (U)	Increases	Decreases

This comparison is a **favourite topic in JEE Main**.

2.19. COMBINATION OF CAPACITORS-SERIES:

Let two unequal capacitors of capacitances C_1 and C_2 are connected in series and the combination is connected with a dc battery of potential difference V as shown below Fig-2.19-1.to know its equivalent capacitance and potential difference across each capacitor, the following steps must be followed:

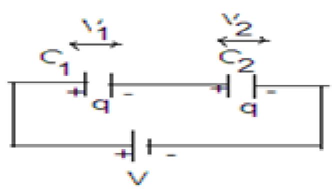


Figure 2.19-1 : Series Connection

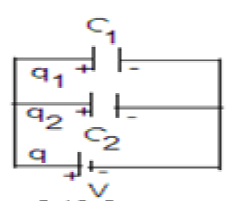


Figure 2.19-2Parallel Connection

Equivalent Capacitance (C_{eq}) in Series Combination of Two capacitors:

The charge stored by each capacitor will be equal. The voltage is divided.

$$C = \frac{q}{V}$$

And q = constant so,

$$V \propto \frac{1}{C}$$

As shown in the Figure 2.19-1:

$$V = V_1 + V_2 \Rightarrow \frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} - - - - (i)$$

Hence, if two capacitors are connected in series, then its equivalent capacitance will be:

$$C_{eq} = \frac{Product\ of\ Two\ capacitors}{Sum\ of\ Two\ capacitors} = \frac{C_1 C_2}{C_1 + C_2} --- (ii)$$

If more than two capacitors are connected in series and the combination is connected with a battery of potential difference V, then its equivalent capacitance will be as:

$$\boxed{\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}}$$

Potential Difference across each capacitor:

$$V_1 = \frac{q}{C_1}$$

And,

$$V_2 = \frac{q}{C_2}$$

Since,

$$q = C_{eq} V$$

Therefore,

$$V_1 = V\left(\frac{C_2}{C_1 + C_2}\right)$$

And,

$$V_2 = V\left(\frac{C_1}{C_1 + C_2}\right)$$

In series combination, a graph is plotted between V and C, which is rectangular hyperbola shape with q = constant and shown in Figure 2.19-3 below. Here V on y-axis and C on x-axis (Figure 2.19-3) in in Figure 2.19-4, q on y-axis and C on x-axis.

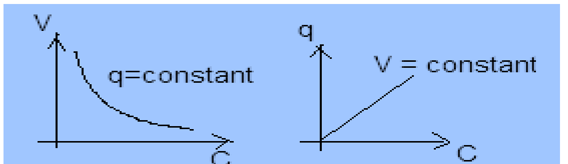


Figure 2.19-3 for Series Combination

Figure 2.19-4 for Parallel Combination

(Graph of V versus C for constant charge q: rectangular hyperbola.)

2.20. Equivalent Capacitance (C_{eq}) in Parallel Combination of Two capacitors:

When two capacitors of capacitance C_1 and C_2 are connected in parallel and the combination is connected with a battery of potential difference V across it as shown in the Fig-2 above. Then its equivalent capacitance will be found as:

The potential difference across each capacitor will be same. The charge is divided.

Since, $C = \frac{q}{V}$:and V = constant so, $q \propto C$ and the graph plotted between q and C will be a straight line as shown in Figure 2.19-4 above also.

As shown in the Fig-2.19-2.:

$$q = q_1 + q_2 \Rightarrow C_{eq}V = C_1V + C_2V \Rightarrow C_{eq} = C_1 + C_2 - - - (i)$$

Hence, if two capacitors are connected in parallel, then its equivalent capacitance will be

$$C_{eq} = sum\ of\ two\ capacitances = C_1 + C_2 - - - -(ii)$$

If more than two capacitors are connected in parallel and the combination is connected with a battery of potential difference V, then its equivalent capacitance will be as:

$$\boxed{C_{eqv} = \sum_{i=1}^{n} C_i}$$

Charge stored by each capacitor:

$$q_1 = C_1V\ and\ q_2 = C_2V\ \Rightarrow q_1 = q\left(\frac{C_1}{C_1 + C_2}\right)\ and\ q_2 = q\left(\frac{C_2}{C_1 + C_2}\right)$$

Some other methods to find Equivalent Capacitance

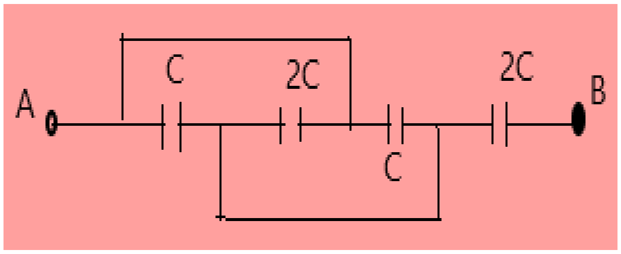
- Same Potential or Common Terminal method
- Wheatstone bridge method
- Symmetrical circuit

Same Potential Method

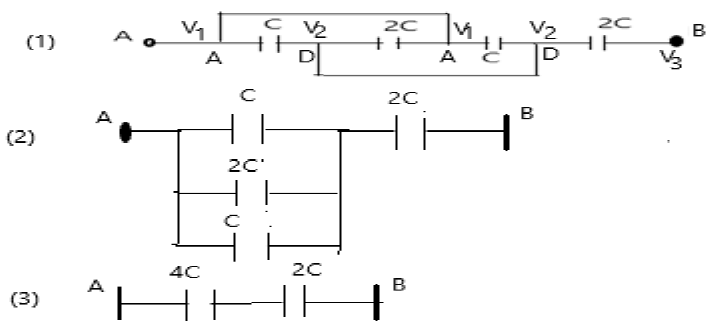
Potential of all terminals is same. Reduce the circuit accordingly.

Example-1

Find the equivalent capacitance between A and B as given below.



Solution:



In step-I, the circuit is reduced as shown.
In step-2, the circuit is again reduced and simplified.

$$C' = C + 2C + C = 4C$$

In step-(3),

$$C_{eq} = \frac{4C \times 2C}{4C + 2C} = \frac{4C}{3}$$

Answer: 4C/3

Wheatstone Bridge Method [Competitive Exam Enrichment (JEE)]

In this, for balanced bridge, one of diagonals branch capacitance will be opened.

Example-2

Find the equivalent capacitance between A and B as shown in the figure below.

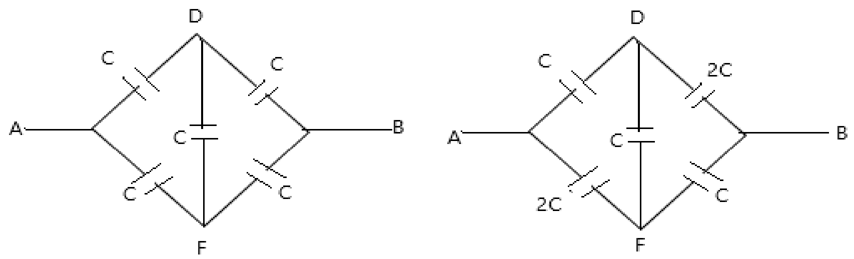


Fig-(i) Fig-(ii)

Solution:
In Fig-(i), the bridge is balanced as
$C\ (brnach\ AD) \times C(branch\ BF) = C(branch\ DB) \times C(branch\ AF)$

Therefore,
$C(branch\ DF)$ will be opened. Now, equivalent capacitance between A and B will be

$$C_{AB} = \frac{C}{2} + \frac{C}{2} = C$$

Answer: C
In Fig-(ii), the bridge is unbalanced as

$$C\ (brnach\ AD) \times C(branch\ BF) \neq 2C(branch\ DB) \times 2C(branch\ AF)$$

When C (AD) = C (BF) and C (DB) = C (AF), then applying the below formula for equivalent capacitance,

$$C_{eq} = \frac{C_1C_3 + C_2C_3 + 2C_1C_2}{C_1 + C_2 + 2C_3}$$

Where,
$C_1 =$ C (AD), $C_4 =$ C (BF) and $C_2 =$ C (DB), $C_3 =$ C (AF); $C_5 = C(DF)$
So,
Let $C_1 = C$; $C_2 = 2C$; $C_3(diagonal\ branch\ DF) = C$
Hence,

$$C_{AB} = C_{eq} = \frac{C \times C + 2C \times C + 2C \times 2C}{C + 2C + 2C} = \frac{7C^2}{5C} = \frac{7C}{5}$$

Answer: 7C/5

Symmetrical Circuit [Competitive Exam Enrichment (JEE)]

In this type, about the horizontal or vertical symmetry,:

The two symmetric points are at the same potential; therefore, no potential difference exists between them, so the branch connecting them carries no charge and can be removed.

Example-3

Find the equivalent capacitance between A and B in the circuit below.

The question is in Fig-(i). Its solution is given in Fig-(ii)

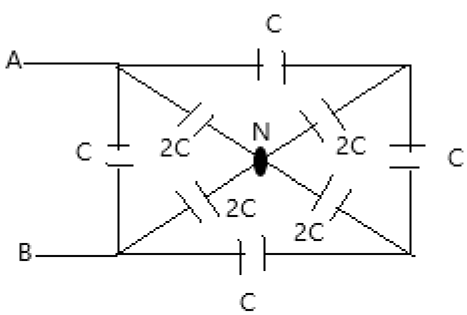


Fig-(iii)

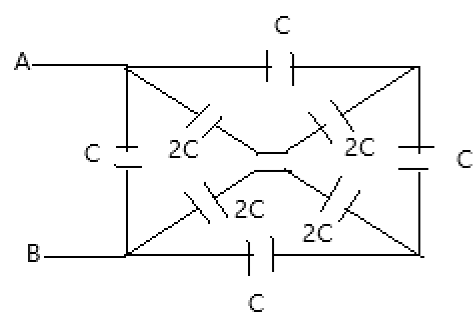


Fig-(iv)

Fig-(ii) is the question. Its solution is in Fig-(iv).

Solution:

In figure-(iv), @C and 2C are in series and C is in parallel of these two. Hence,

$$C_1 = 2C$$

Similarly,

$$C_2 = 2C$$

Now, 2C, C and 2C are in series. Therefore

$$C_3 = C/2$$

Now, C_3 *and* C are in parallel, then

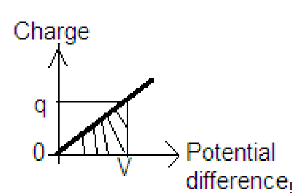
$$C_{AB} = C + C_3 = C + \frac{C}{2} = \frac{3C}{2}$$

Answer: 3C/2

2.21. ENERGY STORED IN A CAPACITOR (U) and Energy Density (u):

Energy stored in a parallel plate capacitor is:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{1}{2}\frac{q^2}{C}$$



The area under the graph between q and V gives the potential energy.

Energy density (u) 9s defined as the energy stored in the parallel plate capacitor per unit volume. That is,

$$u = \frac{Energy\ Stored}{Volume} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\left(\frac{\epsilon_0 A}{d}\right)(Ed)}{2Ad} = \frac{1}{2}\epsilon_0 E^2$$

Energy density in electrostatic field is,

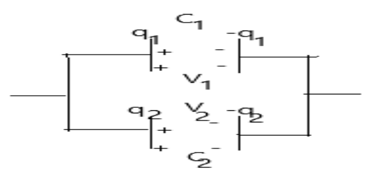
$$\boxed{u_E = \frac{1}{2}\epsilon_0 E^2}$$

(i) SI unit of energy density is J/m^3.

2.22. Loss of Electrostatic Energy in the Capacitor-

Suppose a capacitor of capacitance C_1 is charged to a potential difference of V_1 by a battery. Another capacitor of capacitance C_2 also charged by another battery after disconnection from each battery, they connected in parallel with C_1, then the capacitor C_1 will supply the charge to C_2 until the potential is equalized. This potential is said to be

Common Potential (V).



Let initially, a capacitor C_1 has charge q_1, potential V_1 and another capacitor C_2 has potential V_2. Finally, both are disconnected from battery and connected with wire in parallel, then charge is transferred from one capacitor to another until potentials are equalized. The common potential will be

$$\boxed{V_{com} = \frac{Net\ Charge}{Net\ Capacitance} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}} - - - (i)$$

The loss in electrostatic energy,

$$\boxed{U_i - U_f}$$

Hence,

$$U_i - U_f = \frac{1}{2}C_1{V_1}^2 - \frac{1}{2}(C_1 + C_2){V_{com}}^2 - - - - (ii)$$

Putting the value of V_{com} from equation (i) in equation (ii), we get,

$$U_i - U_f = \frac{1}{2}C_1{V_1}^2 - \frac{1}{2}(C_1 + C_2){V_{com}}^2 = \frac{1}{2}C_1{V_1}^2 - \frac{1}{2}(C_1 + C_2)\left(\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2\right) - - - (iii)$$

By solving the above equation (iii), we have,

$$\boxed{loss\ in\ Energy = U_i - U_f = \frac{(C_1C_2)(V_1 - V_2)^2}{2(C_1 + C_2)}}$$

Example-1

A 100 μF capacitor is charged by a 100 V supply. It is then disconnected from the supply and is connected to another uncharged 100 μF capacitor. How much electrostatic energy is lost in this process?

Solution:

We know that loss in electrostatic energy is given by,

$$U_i - U_f = \frac{C_1 C_2 (V_1 - V_2)^2}{2(C_1 + C_2)}$$

Given, $C_1 = 100\mu F = 1 \times 10^{-4} = C_2, V_1 = 100\,V, V_2 = 0$, hence,

$$U_i - U_f = \frac{1 \times 10^{-4} \times 1 \times 10^{-4} \times (100 - 0)^2}{2(1 \times 10^{-4} + 1 \times 10^{-4})} = \frac{10^{-4}}{4 \times 10^{-4}} = 0.25\,J - Ans.$$

2.23. Polarisation of Dielectrics, Polarisation Density and Advanced Concepts

Learning Objectives

After studying this part, students will be able to:

- Explain dielectric polarization.
- Distinguish between polar and non-polar dielectrics.
- Define polarization density.
- Understand electric susceptibility and dielectric constant.
- Relate microscopic behavior of molecules to macroscopic properties of dielectrics.

1. What is a Dielectric?

A **dielectric** is an insulating material that does not allow free flow of electric charge but becomes polarized when placed in an external electric field.

Examples:

- Air
- Vacuum
- Glass
- Mica
- Plastic
- Paper
- Ceramic

2. Polar and Non-Polar Dielectrics

(A) Polar Dielectrics

Polar dielectrics possess **permanent electric dipole moments** even in the absence of an external electric field.

However, due to random molecular orientation, the net dipole moment is zero.

Examples

- Water (H_2O)

- Hydrogen Chloride (HCl)
- Ammonia (NH_3)

(B) Non-Polar Dielectrics

These molecules do **not** possess a permanent dipole moment.
When an external electric field is applied, the positive and negative charge centres shift slightly, inducing a dipole moment.

Examples

- Nitrogen (N_2)
- Oxygen (O_2)
- Carbon Dioxide (CO_2)

---

3. Polarisation of a Dielectric

When a dielectric is placed in an external electric field:

- Polar molecules tend to align with the field.
- Non-polar molecules develop induced dipole moments.
- The dielectric acquires a net dipole moment.

This process is called **polarisation**.
As a result:

- The electric field inside the dielectric decreases.
- The capacitance of a capacitor increases.

4. Polarisation Density (Polarisation Vector)

Definition

Polarization vector $\vec{P}$ is defined as the electric dipole moment per unit volume of the dielectric.
Mathematically,

$$\boxed{Polarisation\ Density = \frac{Total\ Dipole\ Moment}{Volume}}$$

Or

$$\boxed{P = \frac{p_{net}}{V}}$$

Where:

- P = Polarisation density
- p_{net} = Net electric dipole moment
- V = Volume of the dielectric

SI Unit: C/m^2

Nature

- Vector quantity
- For a linear isotropic dielectric, the polarization vector $\vec{P}$ is parallel to the applied electric field $\vec{E}$.

---

5. Electric Susceptibility

The ease with which a dielectric becomes polarized is measured by its **electric susceptibility**.
It is represented by χ_e
For a linear dielectric,

$$\boxed{P = \epsilon_0 E \chi_e}$$

Where

- E = Applied electric field
- ϵ_0 = Permittivity of free space

6. Relative Permittivity (Dielectric Constant)

The dielectric constant is related to susceptibility by

$$K = 1 + \chi_e$$

Where

- $K = \epsilon_r$

Since $\chi_e > 0$, $K > 1$ for all ordinary dielectric materials.

---

7. Types of Polarisation

(A) Electronic Polarisation

It occurs due to the displacement of the electron cloud relative to the nucleus.
Present in **all dielectric materials**.

(B) Ionic Polarisation

It occurs in ionic crystals because positive and negative ions shift slightly in opposite directions.
Examples:

- Sodium Chloride (NaCl)
- Potassium Chloride (KCl)

(C) Orientation Polarisation

It occurs only in polar molecules.
The permanent dipoles rotate and align partially with the applied electric field.
Examples:

- Water
- Ammonia

---

8. Effect of Polarisation

Polarisation causes:

- Reduction of the effective electric field inside the dielectric.
- Increase in capacitance.
- Increase in charge storage capability.
- Better insulation between capacitor plates.

---

9. Applications of Dielectrics

- Capacitors
- Printed circuit boards (PCBs)
- High-voltage insulators
- Underground power cables
- Radio-frequency circuits
- Communication systems