Part – 1: Magnetic Force, Magnetic Field and Lorentz Force

Learning Objectives

After studying this part, students will be able to:

- Understand the concept of a magnetic field.
- Distinguish between electric and magnetic fields.
- Define magnetic flux density.
- Explain the force experienced by a moving charge in a magnetic field.
- Derive the expression for the Lorentz force.
- Apply the right-hand rule to determine the direction of magnetic force.
- Solve basic numerical problems related to magnetic force.

4.1 Introduction

Electricity and magnetism are closely related phenomena. A stationary electric charge produces only an **electric field**, whereas a **moving electric charge** produces both an **electric field** and a **magnetic field**.

The interaction between moving charges and magnetic fields forms the basis of:

- Electric motors
- Loudspeakers
- Particle accelerators
- Cathode-ray tubes
- Mass spectrometers
- Galvanometers

So, magnetic field can be produced by

- A magnet
- Moving charge
- Current carrying conductor

Oersted demonstrated that when current flows in a wire, magnetic field is produced. He noticed that there is a deflection in the compass towards left when placed below the straight conductor upon current flows (south to North).

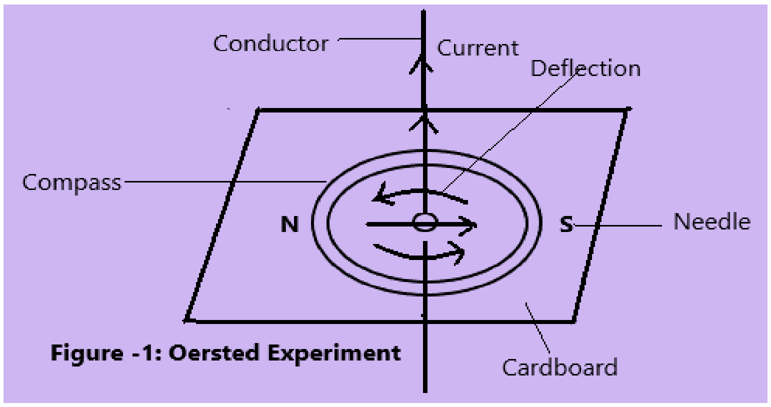

Figure -1: Oersted Experiment

4.2 Magnetic Field

Definition

A **magnetic field** is the region around a magnet or a current-carrying conductor where a magnetic force can be experienced by a moving charge, a current-carrying conductor, or a magnetic material.
It is represented by the symbol

$$\boxed{\vec{B}}$$

and is also called the **magnetic flux density**.

SI Unit

$$\boxed{tesla\ (T)}$$

CGS unit:

$$\boxed{Gauss\ (G)}$$

$$\boxed{1T = 10^4 G}$$

Dimensions

$$\boxed{[ML^0T^{-2}A^{-1}]}$$

4.3 Magnetic Field Lines

Magnetic field lines are imaginary curves used to represent the direction and strength of a magnetic field.

Properties

- Outside a magnet, field lines emerge from the **North Pole** and enter the **South Pole**.
- Inside the magnet, they travel from **South to North**, forming closed loops.
- Two magnetic field lines never intersect.
- The closer the field lines, the stronger the magnetic field.

Uniform Magnetic Field Lines

It is represented by crossed (X) field or dot (.) field or Lines parallel to each other.

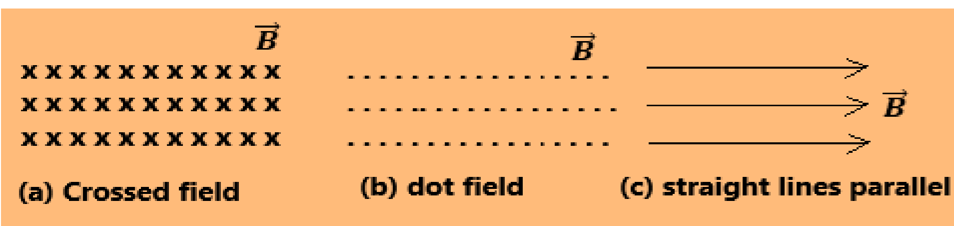

Figure-1-1 (a), (b), (c): Uniform Magnetic Field Lines

4.4 Magnetic Force on a Moving Charge

A stationary charge does **not** experience any magnetic force.
A charge moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$
experiences a magnetic force.

The magnitude of this force is:

$$F = qvB\sin\theta$$

Where:

- q = charge
- v = speed of the particle
- B = magnetic field strength
- θ = angle between $\vec{v}$ and $\vec{B}$

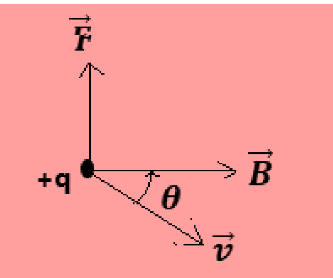

Figure-2: Force on moving charge

4.5 Lorentz Force

When a charged particle is under the influence of both an electric field and a magnetic field simultaneously, the total force acting on it is called the **Lorentz Force**.

The vector equation is:

$$\boxed{\vec{F} = q\vec{E} + q\vec{v}\times\vec{B} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)}$$

Where

- $\vec{E}$ = Electric field
- $\vec{B}$ = Magnetic field

Special Cases

(i) Only Electric Field Present

$$\boxed{\vec{F} = q\vec{E}}$$

(ii) Only Magnetic Field Present

$$\boxed{\vec{F} = q\vec{v}\times\vec{B}}$$

(iii) Velocity Parallel to Magnetic Field

If

$$\theta = 0^0$$

Then

$$\boxed{F = 0}$$

No magnetic force acts on the particle.

(iv) Velocity Perpendicular to Magnetic Field

If

$$\theta = 90^0$$

Then

$$\boxed{F = qvB}$$

The magnetic force is maximum.

4.6 Direction of Magnetic Force

The direction of the magnetic force is determined by the **Right Hand Crossed Product Rule**.

Direction of magnetic force on a positive charge is determined by the right-hand rule for the cross product $\vec{v}\times\vec{B}$.
For a **negative charge (electron)**, the force is opposite to the thumb.

4.7 Characteristics of Magnetic Force

- Always acts perpendicular to both the velocity and the magnetic field.
- Changes only the **direction** of velocity, not its magnitude.
- Does **no work** on the charged particle.
- Can produce straight line, circular or helical motion.

4.8 Why Does the Magnetic Force Do No Work?

Since the magnetic force is always perpendicular to the displacement,

$$W = \vec{F}\cdot\vec{s} = Fs\cos\theta$$

Here $\theta = 90^0$
Therefore,

$$W = Fs\cos 90^0 = 0$$

Hence,

- Speed remains constant.
- Kinetic energy remains constant.
- Only the direction of motion changes.

Solved Example 1

A proton moves with a speed of 2×10^6 m s^{-1} perpendicular to a magnetic field of 0.5 T. Find the magnetic force acting on it.

Solution

Using

$$F = qvB\sin\theta$$

Given, v = 2 x 10^6 m/s, $\theta = 90^0$, q = charge on proton = 1.6 x 10^{-19} C,
Therefore,

$$F = 1.6\times10^{-19}\times2\times10^6\times0.5 = 1.6\times10^{-13}N$$

Answer: $\boxed{1.6\times10^{-13}\ N}$

Solved Example 2

An electron enters a magnetic field parallel to the field lines. Find the magnetic force.

Solution

Since

$$\theta = 0^0$$

Answer: $\boxed{0\ N}$

Exam Tips

- A stationary charge experiences **no magnetic force**.
- Magnetic force is maximum when the velocity is perpendicular to the magnetic field.
- A magnetic field changes the **direction** of motion but not the **speed**.
- The force on an electron is opposite in direction to that on a proton.

Common Mistakes

- Using F= qE instead of F= qvB for magnetic force.
- Forgetting the sin θ factor.
- Applying the right-hand rule directly to electrons without reversing the direction.
- Assuming a magnetic field changes the kinetic energy of a charged particle.

JEE Insight (Beyond NCERT)

1. Circular Motion in a Uniform Magnetic Field

When a charged particle enters a uniform magnetic field **perpendicular** to its velocity, the magnetic force acts as the **centripetal force**, resulting in uniform circular motion.

2. Velocity Selector (Preview)

If electric and magnetic fields act simultaneously and satisfy

$$qE = qvb$$

the particle passes undeflected. This principle is widely used in **mass spectrometers** and is important for **JEE**.

Numerical on Magnetic Force Due to moving charge

Solved Example-1

A charge is moving with acceleration $\vec{a} = (x\hat{\imath} - 2\hat{\jmath} + \hat{k})$m/s^2 in the uniform magnetic field $\vec{B} = (\hat{\imath} + \hat{\jmath} - \hat{k}) \times 10^{-2} T$. Find the value of x.

Solution:

Since,

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Or,

$$m\vec{a} = q(\vec{v} \times \vec{B})$$

This implies that $\vec{a} \perp \vec{v}$ and $\vec{a} \perp \vec{B}$.

For, $\vec{a} \perp \vec{B}$, we get that

$$\vec{a} \cdot \vec{B} = 0$$

Therefore,

$$(x\hat{\imath} - 2\hat{\jmath} + \hat{k}) \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) \times 10^{-2} = 0$$

Or,

$$x - 2 - 1 = 0 \Rightarrow x = x = 3\,ms^{-2}$$

Answer: $\boxed{3m/s2}$

Part – 2: Motion of a Charged Particle in a Uniform Magnetic Field

Learning Objectives

After studying this part, students will be able to:

- Explain the motion of a charged particle in a magnetic field.
- Derive the radius of the circular path.
- Derive the time period, frequency, and angular frequency.
- Explain helical motion.
- Understand the concept of a velocity selector.
- Solve Board, NEET, and JEE numerical problems.

4.9 Motion of a Charged Particle in a Uniform Magnetic Field

When a charged particle enters a **uniform magnetic field**, the nature of its motion depends on the angle between its velocity $\vec{v}$ and the magnetic field $\vec{B}$.

There are three important cases:

1. Velocity **perpendicular** to the magnetic field
2. Velocity **parallel** to the magnetic field
3. Velocity at an **arbitrary angle**

Case I: Velocity Perpendicular to the Magnetic Field

Suppose a particle of charge q and mass m enters a uniform magnetic field with velocity v such that $\theta = 90^0 \Rightarrow \sin 90^0 = 1$

The magnetic force is F = $|q|vB$

Since the force is always perpendicular to the velocity, it acts as the **centripetal force**.

Hence,

$$|q|vB = m\frac{v^2}{r}$$

Therefore, the radius of the circular path is

$$\boxed{r = \frac{mv_{\perp}}{|q|B}}$$

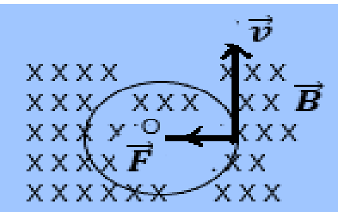

Figure-3: Circular motion of a moving charge

Important Observations

The radius:

- is directly proportional to mass,
- is directly proportional to speed,
- is inversely proportional to charge,
- is inversely proportional to magnetic field strength.

Thus,

- heavier particles move in larger circles,
- stronger magnetic fields produce smaller circular paths.

4.10 Time Period of Circular Motion

The circumference of the circular path is 2πr.
Time period,

$$T = \frac{2\pi r}{v}$$

Substituting $r = mv/qB$,

$$\boxed{T = \frac{2\pi m}{|q|B}}$$

Important Result
The time period is **independent of the particle's speed and radius**.

4.11 Frequency of Revolution

Frequency is the reciprocal of time period.

$$\boxed{f = \frac{|q|B}{2\pi m}}$$

4.12 Angular Frequency (Cyclotron Frequency): **(JEE/NEET Enrichment — Beyond CBSE 2026–27 Core)**

The angular frequency is $\omega = 2\pi f$.

$$\boxed{\omega_C = \frac{|q|B}{m}}$$

And,

$$\boxed{f_C = \frac{|q|B}{2\pi m}}$$

This is also known as the **cyclotron frequency**.

Summary of Circular Motion

Quantity	Expression
Radius	$r = mv_{\perp}/\lvert q\rvert B$
Time Period	$T = 2\pi m/\lvert q\rvert B$
Frequency	$f = \lvert q\rvert B/2\pi m$
Angular Frequency	$\omega = \lvert q\rvert B/m$

Case II: Velocity Parallel to Magnetic Field

When θ=0^0, the magnetic force becomes

$$F = qvB\sin\theta = qvB \times \sin 0^0 = 0$$

Therefore,

- no magnetic force acts,
- the particle continues with **uniform straight-line motion**.

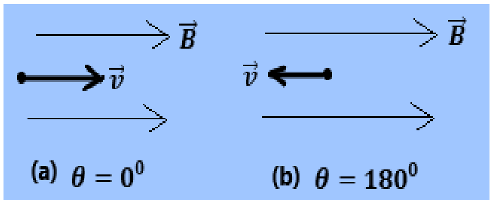

Figure-4 (a) and (b): Straight line motion of a charge particle

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Case III: Velocity at an Arbitrary Angle

Suppose the particle enters the magnetic field at an angle θ.
Resolve the velocity into two components:

- Parallel component:

$$v \parallel v\cos\theta$$

- Perpendicular component:

$$v \perp = v\sin\theta$$

The parallel component remains unchanged, while the perpendicular component produces circular motion.
Therefore, the particle follows a **helical (spiral) path**.

4.13 Radius of the Helix

Only the perpendicular component contributes to circular motion.
Hence,

$$\boxed{r = \frac{mv\sin\theta}{qB}}$$

4.14 Pitch of the Helix

The **pitch** is the distance travelled by the particle along the magnetic field during one complete revolution.

$$\boxed{Pitch = v\|T}$$

Therefore,

$$\boxed{Pitch = \frac{v\cos\theta\,(2\pi m)}{qB}}$$

4.15 Velocity Selector (Crossed Electric and Magnetic Fields)-

(JEE/NEET Enrichment — Beyond CBSE 2026–27 Core)

A **velocity selector** uses mutually perpendicular electric and magnetic fields to allow only particles of a particular speed to pass undeflected.
For mutually perpendicular electric and magnetic fields, a charged particle passes undeflected when the electric and magnetic forces balance:
For zero net force,

$$qE = qvB$$

Hence,

$$\boxed{v = \frac{E}{V}}$$

Only particles with this speed travel in a straight line.
The particle must have its velocity perpendicular to the magnetic field for this simple condition.

Solved Example 1

A proton enters a magnetic field of 0.5 T with speed 4×10^6 m/s perpendicular to the field.
Find the radius of its path.

Solution
Using

$$r = mv/qB$$

Where, m=1.67×10^{-27} kg, q=1.6×10^{-19} C
Therefore,

$$r = \frac{1.67\times10^{-27}\times4\times10^{6}}{1.6\times10^{-19}\times0.5} = 8.35\times10^{-2}\ m$$

Answer: $\boxed{8.35\ Cm}$

Solved Example 2

A charged particle enters crossed electric and magnetic fields.
Given:
E=2×10^{4} V/m, B=0.5T
Find the speed of the particle for zero deflection.

Solution

$$v = \frac{E}{B}$$

Or

$$v = \frac{2\times10^{4}}{0.5} = 4\times10^{4} m/s$$

Answer: $\boxed{4\times10^{4}\ m/s}$

Exam Tips

- Magnetic force changes only the **direction**, not the **speed**, of a charged particle.
- The time period in a uniform magnetic field does **not** depend on the particle's speed.
- In helical motion, only the **perpendicular component** determines the radius.
- Remember the velocity selector condition: v=E/B.

Common Mistakes

 Using the total velocity instead of the perpendicular component while calculating the radius of a helix.
 Forgetting that the parallel component remains unchanged.
 Assuming that kinetic energy changes in a magnetic field.
 Confusing frequency with angular frequency.

JEE Insight (Beyond NCERT)

1. Radius Comparison

If two particles have the same charge but different masses and speeds, $r \propto mv$
Thus, heavier or faster particles move in larger circular paths.

2. Energy Remains Constant

Because the magnetic force is always perpendicular to the velocity,

- kinetic energy remains constant,
- speed remains constant,
- only the direction of motion changes.

This concept is frequently tested in **JEE Main** and **NEET** conceptual questions.

Part – 3: Force on a Current-Carrying Conductor & Torque on a Current Loop

Learning Objectives

After studying this part, students will be able to:

- Explain the origin of magnetic force on a current-carrying conductor.
- Derive the expression for force on a straight conductor.
- Apply Fleming's Left-Hand Rule.
- Understand the historical definition of one ampere.
- Derive the torque acting on a current loop.
- Define magnetic dipole moment.
- Solve Board, NEET, and JEE numerical problems.

4.16 Force on a Current-Carrying Conductor

When a conductor carrying electric current is placed in a magnetic field, it experiences a force because the moving charge carriers (electrons) are acted upon by the magnetic field.
This principle is the basis of:

- Electric motors
- Loudspeakers
- Moving-coil galvanometers
- Ammeters
- Voltmeters

4.17 Derivation of Force on a Straight Conductor

Consider a straight conductor of:

- Length = L
- Current = I
- Magnetic field = B
- Angle between conductor and magnetic field = θ\theta

If the conductor contains n free electrons per unit volume and has cross-sectional area A,
Current,

$$I = neAv_d$$

Where, v_d is the drift velocity.
The total magnetic force on all moving electrons leads to the force on the conductor:

$$\boxed{F = ILB\sin\theta}$$

Special Cases

(i) Conductor Parallel to the Magnetic Field

$$\boxed{\theta = 0^0}$$

(ii) Conductor Perpendicular to the Magnetic Field

$\theta = 90^0$ and

$$\boxed{F = BIL}$$

Maximum force acts on the conductor.

Vector Form

$$\boxed{\vec{F} = I\vec{L} \times \vec{B}}$$

The direction is perpendicular to both the current and the magnetic field.
(The final vector relation is written in terms of conventional current I)

4.18 Fleming's Left-Hand Rule

This rule gives the direction of the force acting on a current-carrying conductor in a magnetic field.

Stretch the thumb, forefinger, and middle finger of the left hand mutually perpendicular.

- **Forefinger** → Magnetic Field ($\vec{B}$)
- **Middle Finger** → Current ($I\vec{L}$)
- **Thumb** → Force or Motion ($\vec{F}$)

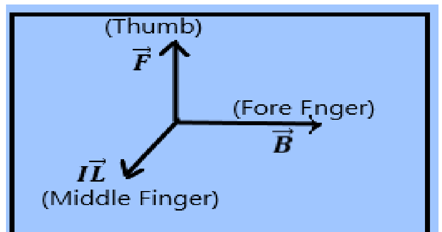


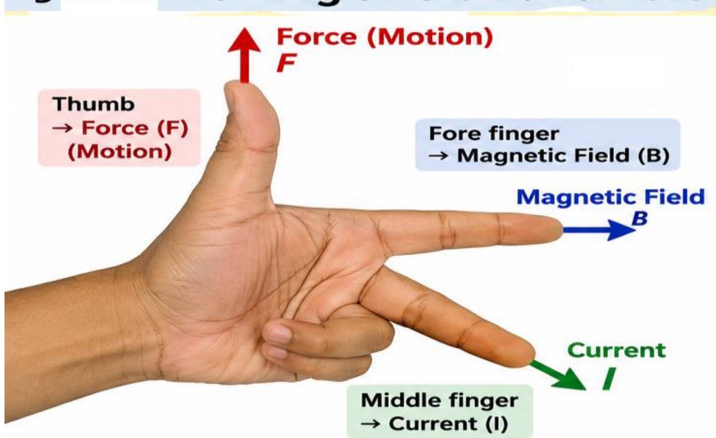
Figure-5: Direction of Force by Fleming's Left Hand Rule

Application

- Electric motors
- Loudspeakers
- Moving-coil instruments

Remember:
F – B – I
Thumb – Forefinger – Middle Finger

Figure 5.1 Fleming's Left Hand Rule



4.19 Historical Definition of One Ampere (Historical Definition of Ampere (Pre-2019 SI)

The constant current which, if maintained in two straight, parallel conductors of infinite length placed 1 m apart in vacuum, produces a force of 2×10^{-7} N per metre between them.

Note: This historical definition is useful for understanding the SI system, though the ampere is now defined using the elementary charge.

Current SI definition: The ampere is defined by fixing the numerical value of the elementary charge ee to exactly $1.602176634\times10^{-19}$ C.

4.20 Torque on a Rectangular Current Loop

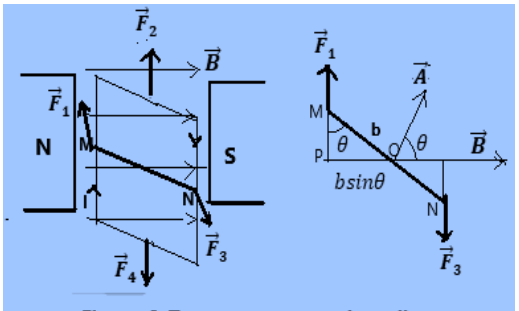


Figure-6: Torque on rectangular coil

Derivation-1

Considering a rectangular coil having:

- Length = l
- Breadth = b
- Area = A= lb
- Number of turns = N
- Current = I
- Magnetic field = B

When placed in a uniform magnetic field, opposite sides of the coil experience equal and opposite forces, forming a couple. The side MN = b, The upper side and lower side forces $\vec{F}_2$ and $\vec{F}_4$ are acting with equal magnitude and in opposite directions, cancel each other.

The torque is

$$\tau = Force\ \times Perpendicular\ distance\ PO$$

Since, $\vec{F}_1$ and $\vec{F}_2$ are equal and let it be $\vec{F}$. Therefore,

$$\tau = F \times b \sin\theta ----(1)$$

Since,

$$F = IlB --- (2)$$

Then.

$$\tau = IlB(b\sin\theta) --- (3)$$

Therefore,

$$\tau = I(l \times b)B\sin\theta$$

Or,

$$\tau = IAB\sin\theta --- (4)$$

Let the coil has N turns, then

$$\boxed{\tau = NIAB\sin\theta}$$

Where, θ\theta is the angle between the **normal to the plane of the coil** and the magnetic field.

Special Cases

Maximum Torque

When $\theta=90^0$,

$$\boxed{\tau = NIAB}$$

Zero Torque

When $\theta = 0^0$

$$\boxed{\tau = 0}$$

4.21 Magnetic Dipole Moment

A current-carrying loop behaves like a magnetic dipole.
It is the product of current and cross-sectional area normal to the plane of the coil.
The magnetic dipole moment is defined as

$$\boxed{\vec{M} = NI\vec{A}}$$

Where:
- N = Number of turns
- I = Current
- A = Area vector

SI Unit

$\boxed{Am^2}$

Torque in Terms of Magnetic Dipole Moment

Using

$$M = NIA$$
$$\boxed{\vec{\tau} = \vec{M} \times \vec{B}}$$

Magnitude:

$$\boxed{\tau = MB \sin\theta}$$

4.22 Applications

The torque on a current loop is the operating principle of:
- Moving-coil galvanometer
- Electric motor
- Analog ammeter
- Analog voltmeter
- Loudspeaker voice coil

Solved Example 1

A conductor of length 0.40 m carries a current of 5 A and is placed perpendicular to a magnetic field of 0.60 T. Find the magnetic force.
If the conductor makes angle 30^0 with magnetic field, what is the magnetic force?

Solution

Using

$$F = ILB\sin\theta$$

Given, $\theta = 90^0$ and $\sin 90^0 = 1$
So,

$$F = ILB$$

Substituting the values from the question, we get

$$F = 5 \times 0.4 \times 0.6 = 1.2\ N$$

Again,
Given, $\theta = 30^0 \Rightarrow \sin 30^0 = 1/2$
Hence,

$$F = \frac{1.2}{2} = 0.6\ N$$

Answer:
1.2 N and 0.6 N

Solved Example 2

A rectangular coil has:

- Number of turns = 50
- Current = 2 A
- Area = 0.02 m^2
- Magnetic field = 0.5 T

The plane of the coil is parallel to the magnetic field (so the normal is perpendicular to the field). Find the torque.

Solution

Since,

$$\tau = NIAB \sin \theta$$

Therefore,

$$\tau = 50 \times 2 \times 0.20 \times 0.5 = 10\ Nm$$

Answer: $\boxed{10\ Nm}$

Exam Tips

- Use **Fleming's Left-Hand Rule** for motors and force on a conductor.
- Use the **Right-Hand Rule** for the force on a moving positive charge.
- Torque is maximum when the **normal to the coil** is perpendicular to the magnetic field.
- Always identify whether the question gives the angle with the **plane** of the coil or with its **normal**.

Common Mistakes

□ Confusing Fleming's Left-Hand Rule with Fleming's Right-Hand Rule.
□ Forgetting the sin θ factor in force and torque expressions.
□ Using the angle with the plane instead of the normal without conversion.
□ Omitting the number of turns N in torque calculations.

JEE Insight (Beyond NCERT)

1. Angle Convention

If the angle ϕ is given between the **plane of the coil** and the magnetic field, then

$$\tau = NIAB \cos \varphi$$

because ϕ and the angle made by the **normal** with the magnetic field are complementary.

2. Equilibrium of a Current Loop

- **Stable Equilibrium:** Magnetic dipole moment $\vec{M}$ is parallel to $\vec{B}$.
- **Unstable Equilibrium:** $\vec{M}$ is antiparallel to $\vec{B}$.

These concepts are frequently tested in **JEE Main** and **NEET** conceptual questions.

Try yourself:

Q.1.

The magnetic force on a charge particle 2 μC is acting as: $\vec{F} = (\hat{\imath} - 2\hat{\jmath} + \hat{k}) \times 10^{-3} N$.The particle is moving in uniform magnetic field $\vec{B} = (\hat{\imath} + \hat{\jmath} - \hat{k}) \times 10^{-2} T$. Find the velocity of charge particle.

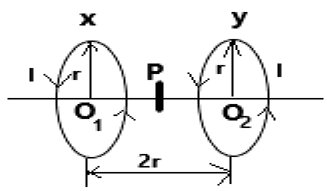
Q.2.

What is the force on an electron moving with velocity 2×10^{-4} m/s in uniform magnetic field of 0.02 T, when making

- angle 60^0 and
- parallel to the magnetic field?

Q.3.

Two coils x and y are placed coaxially and carry the same current 2 A in the same direction. They are separated by 2 m and radius of 1 m also. Determine the net magnetic field at mid point of the line joining two centres. Given r = 1 m



Part – 4: Biot–Savart Law & Magnetic Field Due to Current-Carrying Conductors

Learning Objectives

After studying this part, students will be able to:

- State and explain the Biot–Savart Law.
- Determine the direction of the magnetic field using the Right-Hand Thumb Rule.
- Derive the magnetic field at the centre of a circular current loop.
- Understand the magnetic field on the axis of a circular loop.
- Solve Board, NEET, and JEE numerical problems.

4.23 Biot–Savart Law

The **Biot–Savart Law** gives the magnetic field produced by a small current element at any point in space.

Consider a small current element I $\vec{dl}$.

The magnetic field $\vec{dB}$ at a point located at a distance r is:

- directly proportional to the current I,
- directly proportional to the length element dl,
- directly proportional to sin θ,
- inversely proportional to the square of the distance.

Hence,

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

Introducing the proportionality constant,

$$\boxed{dB = \frac{\mu_0 Idl\sin\theta}{4\pi r^2}}$$

Where

- μ_0 = permeability of free space

$$\boxed{\mu_0 = 4\pi \times 10^{-7} Hm^{-1} = 4\pi \times 10^{-7} TmA^{-1}}$$

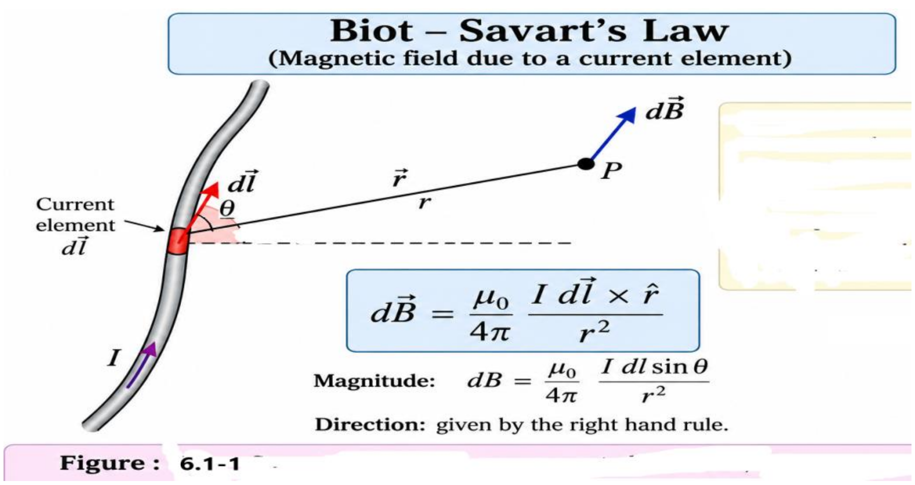


Figure : 6.1-1

Vector Form

$$\boxed{d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2} = \frac{\mu_0}{4\pi}\left(\frac{I d\vec{l} \times \vec{r}}{r^3}\right)}$$

Where, r^ is the unit vector from the current element to the observation point.

Characteristics of the Biot–Savart Law

- Magnetic field is directly proportional to current.
- It decreases with the square of the distance from the current element.
- The direction is perpendicular to both the current element and the radius vector.
- It is analogous to Coulomb's law in electrostatics.

4.24 Right-Hand Thumb Rule

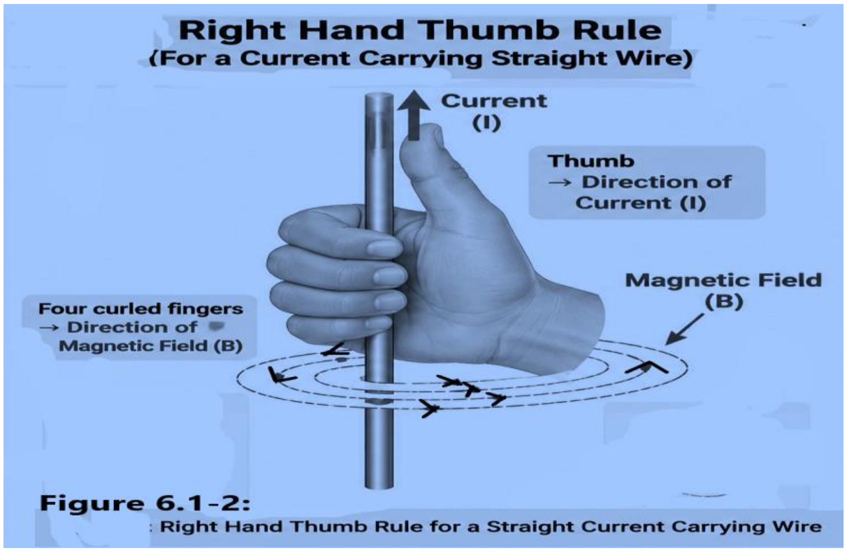
The direction of the magnetic field around a straight current-carrying conductor is determined by the **Right-Hand Thumb Rule**.

Rule

Hold the conductor in your right hand.

- Thumb → Direction of current.
- Curled fingers → Direction of magnetic field lines.

The field lines form **concentric circles** around the conductor.



4.25 Magnetic Field Due to a Long Straight Current-Carrying Conductor

Using the Biot–Savart Law and integrating over the entire conductor,
the magnetic field at a distance rr from a long straight conductor is

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

This formula can also be obtained using Ampere-circuital law.

- Magnetic field at a point making angle α and β from a fixed wire at distance r perpendicular to the current carrying straight wire

$$\boxed{B = \frac{\mu_0 I(\sin\alpha + \sin\beta)}{4\pi r}}$$

If one end fixed and othe end infinite of a straight wire, then magnetic field at perpendicular distance r from a wire

$$\boxed{B = \left(\frac{1}{2}\right)\frac{\mu_0 I(\sin\alpha + \sin\beta)}{4\pi r}}$$

Important Observations

The magnetic field:

- is directly proportional to the current,
- is inversely proportional to the perpendicular distance,
- forms concentric circular field lines around the conductor.

Graphical Relation

B∝I and B∝1/r

Hence:

- Doubling the current doubles the magnetic field.
- Doubling the distance halves the magnetic field.

Derivation-2:

4.26 Magnetic Field at the Centre of a Circular Current Loop

To derive the expression of magnetic field at the centre of a circular current carrying wire-

Considering a circular loop of: Radius = R and Current = I.

Let a circular coil is placed in x-y plane and current is clock-wise.

Using Biot-Savart's law,

$$d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{Id\vec{l} \times \hat{r}}{r^2}\right) - - - (1)$$

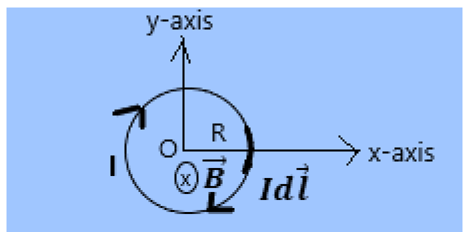


Figure-6.1: Magnetic field at the centre

Or, in magnitude form,

$$dB = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{Idl\sin\theta}{r^2}\right) - - - (2)$$

Since, angle between $Id\vec{l}$ and $\hat{r}$ is 90^0, $\sin 90^0 = 1$.

Let radius of the circle placed in x-y plane be R.

Therefore,

$$dB = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{Idl}{R^2}\right) - -(2)$$

Integrating equation (2) both sides with limit 0 to $l = 2\pi R$

$$B_z = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I}{R^2}\right)\int_0^{2\pi R} dl = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I}{R^2}\right)(2\pi R) = \frac{\mu_0 I}{2R}$$

Or,

$$\boxed{B = \frac{\mu_0 I}{2R}}$$

For a coil of **N turns**,

$$\boxed{B = \frac{\mu_0 NI}{2R}}$$

Graph:

A graph is plotted between magnetic field B and radius of circle R.

Observations

The magnetic field:

- increases with current,
- increases with the number of turns,
- decreases with increasing radius.

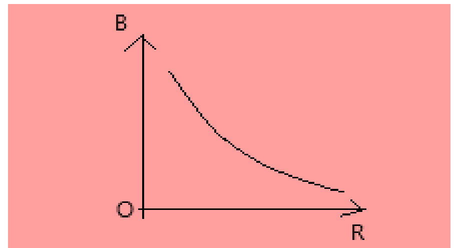


Figure-6.2: Graph between B and R

Magnetic Field at the centre due to an arc:

The magnetic field at the centre due to an arc is given by

$$\boxed{B_{centre} = \frac{\mu_0 I}{2r}\left(\frac{\theta}{360^0}\right)}$$

Where,

- r is the radius of the circle
- θ is the angle made by an arc at the centre O.

Note: $\vec{B}$ at O due to arm OA = OB = 0

$$B_{OA} = B_{OB} = 0$$

B due to arc at O is

$$B_{arc\ AB} = \frac{\mu_0 I}{2r}\left(\frac{\theta}{360^0}\right)$$

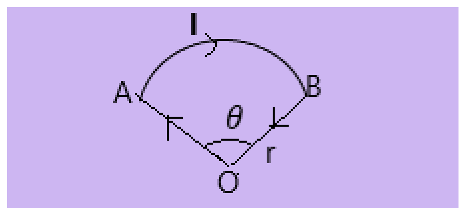


Figure-6.3: Magnetic field at Centre O due to an arc

4.27 Magnetic Field on the Axis of a Circular Loop

Derivation-3:

From the figure-7,

Let the radius of circular coil placed in y-z plane, be R.

At axial position at distance x from the centre of the circle, we need to find the magnetic field.

Using Biot-Savart's law,

$$d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{Id\vec{l} \times \hat{r}}{r^2}\right) - -(1)$$

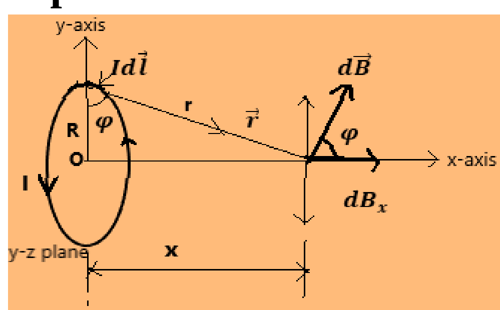


Figure-7: Magnetic Field at Axial Position

From equation (1), $d\vec{B} \perp Id\vec{l}$ and $d\vec{B} \perp \vec{r}$.

Now, $Id\vec{l} \perp \hat{r}$,

Therefore, $\theta = 90^0 \Rightarrow \sin 90^0 = 1$

Hence,

$$dB = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{Idl}{r^2}\right) ----(2)$$

Its vertical component upward and downwards will cancel each other.
Let $d\vec{B}$ makes an angle φ from the axial line.
The horizontal component:

$$dB_x = dB\cos\varphi --- (3)$$

Also,

$$\cos\varphi = \frac{R}{r} ----(4)$$

And,

$$r = \sqrt{R^2 + x^2} ----(5)$$

From equation (2), (3) and (4), we have

$$dB_x = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{Idl}{r^2}\right) \times \frac{R}{r} --- (6)$$

Integrating equation (6) with limits 0 to l, we get

$$B_x = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{IR}{r^3}\right)\int_0^l dl$$

Or,

$$B_x = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{IR}{r^3}\right)(l) --- (7)$$

Putting $l = 2\pi R$ in equation (7) and using equation (5), we obtain

$$B_x = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{IR}{r^3}\right)2\pi R = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}$$

Hence,

$$\boxed{B = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}}$$

For N-turns,

$$\boxed{B = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}}}$$

Hence, notice that, $B_y = 0$ and $B_Z = 0$

Graph between B and x:

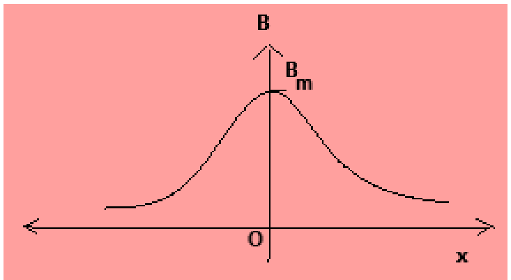


Figure-7.1: Variation of B with x

Special Cases

At the Centre

If $x = 0$, then

$$\boxed{B = \frac{\mu_0 NI}{2R}}$$

Far Away from the Loop

If x≫R, then
The magnetic field decreases rapidly and varies approximately as

$$B \propto \frac{1}{x^3}$$

Solved Example 1

A long straight conductor carries a current of 10 A. Find the magnetic field at a point 5 cm away.

Solution

Using

$$B = \frac{\mu_0 I}{2\pi r}$$

Therefore,

$$B = 2 \times 10^{-7} \times \frac{10}{5 \times 10^{-2}} = 4 \times 10^{-5} T$$

Answer:

$\boxed{4 \times 10^{-5} T}$

Solved Example 2

A circular coil of radius 20 cm carries a current of 4 A. Find the magnetic field at its centre.

Solution

Using

$$B = \frac{\mu_0 I}{2r}$$

Here, r = R = 20 cm = 0.20 m
Therefore,

$$B = 2\pi \times 10^{-7} \times \frac{4}{0.2} = 4\pi \times 10^{-6} T = 1.26 \times 10^{-5}\ T$$

Answer: $\boxed{B = 1.26 \times 10^{-5} T}$

In the above solved example-2, determine the magnetic field (magnitude and direction)
(i) At distance 20 cm and
(ii) At distance $x \gg R$ (radius of the circle) and $x = 200\ cm$
from the centre of a circular current carrying wire placed in y-z plane and direction of current is anti-clockwise.

Solution:
Using

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}} - - - (1)$$

(i) Here, $x = 20\ cm = 0.20\ m$, that is, $x = R$
Therefore, from equation (

$$B = \frac{\mu_0 I}{4\sqrt{2}R}$$

Therefore,

$$B = \frac{\pi \times 4 \times 10^{-7}}{\sqrt{2} \times 0.2} = \frac{40\pi}{2\sqrt{2}} \times 10^{-7} = 10\sqrt{2}\pi\ \times 10^{-7}T = 4.44 \times 10^{-6}T$$

Direction is straight line at a point of 20 cm away from the point ($+x$ direction)
(ii)In this case, R is neglected from the denominator of equation-1.
Therefore,

$$B = \frac{\mu_0 I R^2}{2x^3}$$

Or

$$B = \frac{2\pi \times 10^{-7} \times 4 \times (0.2)^2}{2(2)^3} = \frac{8\pi \times 10^{-7}}{0.2}\left(\frac{0.2}{2}\right)^3 = 40\pi \times 10^{-10} = 4\pi \times 10^{-9}\ T$$

Direction is away from point of 200 cm on x axis.
Answer:
(i):$10\sqrt{2}\pi\ T$ and (ii): $4\pi \times 10^{-9}\ T$

Exam Tips

- Always use the **Right-Hand Thumb Rule** to determine the direction of the magnetic field around a conductor.
- For a long straight wire, remember that the magnetic field varies as 1/r, not $1/r^2$.
- At the centre of a circular loop, all current elements contribute in the same direction, resulting in the maximum magnetic field.

Common Mistakes

☐ Using the electric field formula instead of the magnetic field formula.
☐ Forgetting to convert centimetres into metres before substitution.
☐ Ignoring the number of turns N for a multi-turn coil.
☐ Confusing the centre of the loop with a point on its axis.

JEE Insight (Beyond NCERT)

1. Comparison of Distance Dependence

Source	Magnetic Field Dependence
Long straight conductor	$B \propto 1/r$
Circular loop (far on axis)	$B \propto 1/x^3$

This comparison is frequently asked in conceptual questions.

2. Superposition Principle

If multiple current-carrying conductors are present, the resultant magnetic field is the **vector sum** of the magnetic fields due to each conductor individually. This principle is widely used in JEE Main and Advanced problems involving multiple wires and loops.

Try yourself

Q.1

Find the magnetic field at the centre O in each figure (a), (b) and (c). The radius of circular wire is a and current is I.

Figure-a

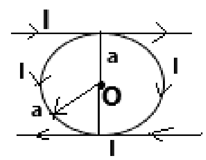


Figure-b

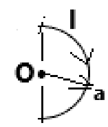


Figure-c

Q.2

Find the net magnetic force on

(a) square coil of each side a and carries current I due to infinite wire at distance a and carries also current I. (Figure-a)

(b) 15 A wire and net magnetic field at 10 A wire.

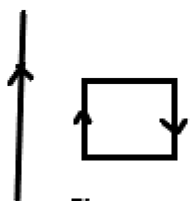


Figure-a

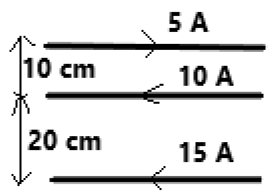
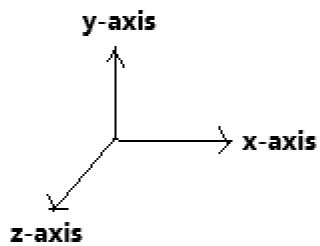


Figure-b



Part – 5: Ampere's Circuital Law, Solenoid, Toroid & Force Between Parallel Conductors

Learning Objectives

After studying this part, students will be able to:

- State and apply Ampere's Circuital Law.
- Derive the magnetic field due to a long straight wire
- Derive the magnetic field due to a long solenoid.
- Explain the magnetic field due to a toroid.
- Derive the force between two parallel current-carrying conductors.
- Distinguish between a solenoid and a toroid.
- Solve Board, NEET, and JEE numerical problems.

4.28 Ampere's Circuital Law

Ampere's Circuital Law relates the magnetic field around a closed path to the total current enclosed by that path.

Statement

The line integral of the magnetic field around any closed path is equal to **μ_0 times the net current enclosed** by that path.

Mathematically,

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}}$$

This law is especially useful for systems with **high symmetry**, such as:

- Long straight conductors
- Solenoids
- Toroids

4.29 Magnetic Field Due to a Long Straight Wire

Derivation-4:

To find magnetic field at a point at distance r perpendicular from the wire:

Let a long straight wire carries current I.

Taking Amperean loop as a circle of radius r as shown in the figure.

Using Ampere Circuital law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

Or,

$$Bl = \mu_0 I_{enclosed} \; --- \; (1)$$

(as $\vec{B}$ and $d\vec{l}$ are parallel to each other and makes angle $\theta = 0^0$ and $\cos 0^0 = 1$)

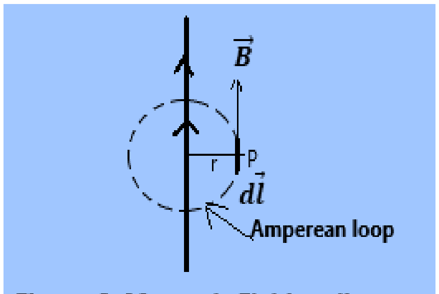


Figure-8: Magnetic Field at distance r from a long stright wire

The length of the Amperean circular loop is $l = 2\pi r$. It encloses current I.

Therefore, from equation (1),

$$B(2\pi r) = \mu_0 I$$

Or

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Magnetic Field due to a long straight wire of cross-section of radius R:

Derivation-5:

Derive the magnetic field inside and outside the straight long wire of radius R-

By Using Ampere Circuital law,

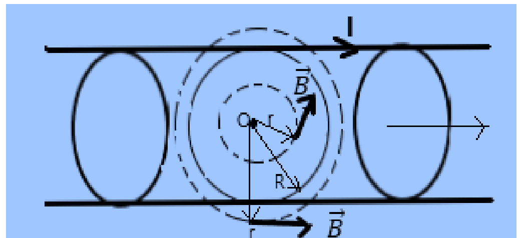


Figure-9: Magnetic field inside and outside a long wire

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

Or,

$$Bl\cos\theta = \mu_0 I_{encl} - -(1)$$

The magnetic field $\vec{B}$ and $d\vec{l}$ are along same line.

Therefore, $\theta = 0^0 \Rightarrow \cos 0^0 = 1$

From equation (1),

$$Bl = \mu_0 I_{encl} - - - (2)$$

(a)Inside the wire at distance $r < R$,

Since, area of inner Amperean circular loop is

$$A_1 = \pi r^r$$

Area of circle (cross-section) of radius R is

$$A = \pi R^2$$

Assuming uniform current density throughout the wire, therefore, The current is directly proportional to area. So,

$$\frac{I_1}{I} = \frac{r^2}{R^2} - - - (3)$$

Again, from equation (2),

$$B_{inside}(2\pi r) = \mu_0 I_1 - - - (4)$$

Substituting I_1 from equation (3) in equation (4), we obtain

$$B_{inside} = \left(\frac{\mu_0}{2\pi r}\right)\left(\frac{Ir^2}{R^2}\right)$$

Or

$$\boxed{B_{inside} = \frac{\mu_0 Ir}{2\pi R^2}}$$

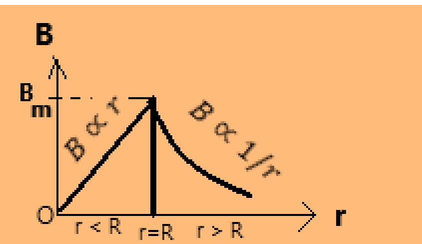


Figure 9-1: Graph between B and r

(b)Outside the radius R $(r > R)$

The Amperean loop encloses the current I.

Therefore, from equation (2), we obtain

$$B(2\pi r) = \mu_0 I$$

Or,

$$\boxed{B_{outside} = \frac{\mu_0 I}{2\pi r}}$$

4.30 Magnetic Field Due to a Long Solenoid

A **solenoid** is a long cylindrical coil made by winding many turns of insulated wire closely together.

Characteristics

- Produces a strong and nearly uniform magnetic field inside.
- The magnetic field outside is very weak.
- Acts like a bar magnet with distinct north and south poles.

Using Ampere's Circuital Law, the magnetic field inside a long solenoid is

$$\boxed{B = \mu_0 n I}$$

Where

- n = N/L = number of turns per unit length,
- I = current.

Derivation-6:

Derive the magnetic field at the centre of a long solenoid of length L:-

Applying Ampere's law to the Amperean loop b-a-d-c

$\oint \vec{B}.\vec{dl} = \mu_0 I = Bl\cos\theta ----(1)$

(As $\vec{B}$ is uniform and parallel to $\vec{dl}$.

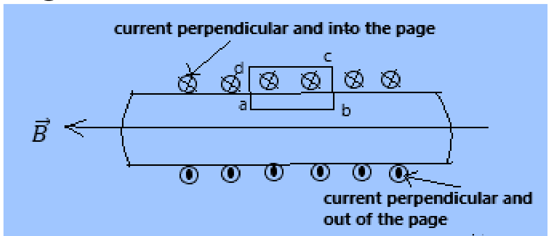


Figure-10: Magnetic field due to a solenoid

Here, ab = dl (small elemental length)

For side c-d, B = 0 as side cd is outside solenoid.

For side b-c and ad, $\theta = 90^0 \Rightarrow \cos 90^0 = 0$

Now, magnetic field due to side a-b ($\theta = 0^0 \Rightarrow \cos 0^0 = 1$) is

$$BL = \mu_0 I --- (2)$$

If the solenoid has n turns per unit length, then number of turns in length L of the solenoid =nL, so,

$BL = \mu_0 nLI \Rightarrow B = \mu_0 nI$, Hence, magnetic field inside a solenoid is given by-

$B = \mu_0 nI ----- (3)$

Properties of the Magnetic Field Inside a Solenoid

- Uniform throughout the central region.
- Parallel to the axis of the solenoid.
- Independent of the radius (for a long solenoid).

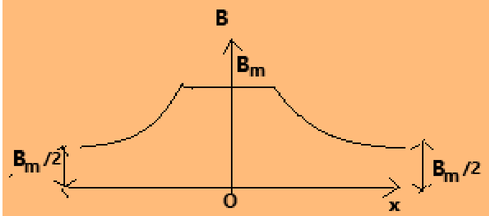


Figure-10-1: Graph between B and x

Field Outside the Solenoid

For an ideal long solenoid,

$$\boxed{B_{outside} \approx 0}$$

4.31 Applications of a Solenoid

- Electromagnets
- Electric bells
- Relays
- Magnetic switches
- MRI machines
- Inductors

Graph between B and x in solenoid

4.32 Magnetic Field Due to a Toroid (JEE/Competitive Exam Enrichment — Not required for CBSE 2026–27 Core)

A **toroid** is formed by bending a solenoid into a closed circular ring.

Characteristics

- Magnetic field is confined inside the toroid.
- The field outside is nearly zero.
- There are no free magnetic poles.

Using Ampere's Circuital Law,

$$\boxed{B = \frac{\mu_0 N I}{2\pi r}}$$

Where

- N = total number of turns,
- r = mean radius of the toroid.

Important Features

- Strong magnetic field inside.
- Negligible magnetic field outside.
- Widely used in transformers and inductors to minimize magnetic leakage.

4.33 Solenoid vs Toroid

Solenoid	Toroid
Straight cylindrical coil	Circular ring-shaped coil
Has north and south poles	No free magnetic poles
Weak field outside	Field outside is nearly zero
Used as electromagnets	Used in transformers and inductors

4.34 Force between Two Parallel Current-Carrying Conductors

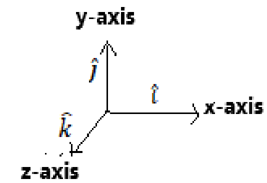
Derivation-7:

Consider two long parallel conductors (wires) separated by a distance d as shown in Figure-11(a)
Let the currents be I_1 and I_2 .
The magnetic field produced by conductor 2 at conductor 1 is

$$\vec{B}_{12} = \frac{\mu_0 I_2}{2\pi d}(\hat{k}) - -(1)$$

The magnetic field produced by conductor 1 at conductor 2 is

$\vec{B}_{21} = \frac{\mu_0 I_1}{2\pi d}(-\hat{k}) - -(2)$



Let l be the length of conductor -1 or conductor-2 at which Magnetic force is to be determined.
The magnetic Force at conductor-1 by conductor-2 is

$$\vec{F}_{12} = I_1 l(\hat{j}) \times \vec{B}_{12} - -(3)$$

Substituting the $\vec{B}_{12}$ from equation (1) in equation (3), we have

$$\vec{F}_{12} = I_1 l(\hat{j}) \times \frac{\mu_0 I_2}{2\pi d}(\hat{k}) = \left(\frac{\mu_0}{2\pi}\right)\left(\frac{I_1 I_2 l}{d}\right)(\hat{\imath}) - - - (4)$$

Similarly,
The magnetic Force at conductor-2 by conductor-1 is

$$\vec{F}_{21} = I_2 l(j) \times \vec{B}_{21} - - - (5)$$

Substituting the $\vec{B}_{21}$ from equation (2) in equation (5), we have

$$\vec{F}_{21} = I_2 l(j) \times \frac{\mu_0 I_1}{2\pi d}(-\hat{k}) = \left(\frac{\mu_0}{2\pi}\right)\left(\frac{I_1 I_2 l}{d}\right)(-\hat{\imath}) - - - (6)$$

From equations (4) and (6), we get magnitude of $\vec{F}_{12}$ and $\vec{F}_{21}$is equal and direction opposite to each other.
Let

$$\vec{F}_{12} = \vec{F}_{21} = \vec{F}$$

Then, magnitude of magnetic force between two long and uniform cross-sectional area of parallel wires is

$$\boxed{F = \frac{\mu_0 I_1 I_2 l}{2\pi d}}$$

Or,
Force per unit length of parallel conductor in magnitude is

$$\boxed{\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}}$$

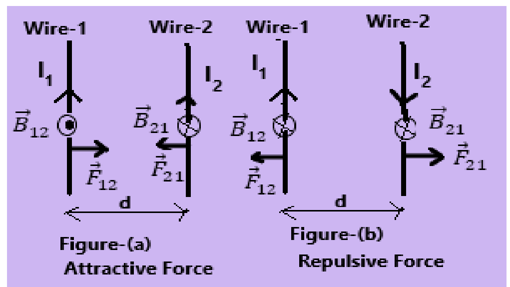


Figure-11: Magnetic Force between two wires

Nature of the Force

Currents in the Same Direction

- The conductors **attract** each other.

Currents in Opposite Directions

- The conductors **repel** each other.

Memory Tip

Same Direction → Attraction
Opposite Direction → Repulsion

Solved Example-1

A long solenoid has turns per metre = 1000, current = 2 A. Find the magnetic field inside the solenoid.

Solution

Using

$$B = \mu_0 n I$$

Or

$$B = 4\pi \times 10^{-7} \times 1000 \times 2 = 8\pi \times 10^{-4}\,T = 2.51 \times 10^{-3} T$$

Answer:

$\boxed{2.51 \times 10^{-3} T}$

Solved Example-2

Two long parallel conductors carry currents of 5 A and 8 A in the same direction. They are separated by 10 cm.

Find the force per metre between them.

Solution

Using

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Let $I_1 = 5\ A$, and $I_2 = 8A$. Distance d = 10 cm = 0.1 m

Therefore,

$$\frac{F}{l} = \frac{2 \times 10^{-7} \times 5 \times 8}{0.1} = 8 \times 10^{-5}\ N/m$$

Since the currents are in the same direction, the force is **attractive**.

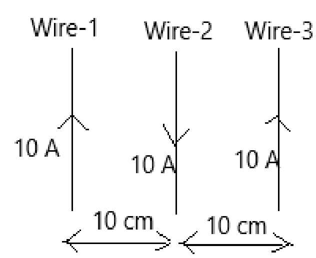
Answer:

$\boxed{8 \times 10^{-5} N/m}$

Solved Example-3

Three long straight wires are placed vertically in the horizontal plane. They carry the current 10 A, 10 A and 10 A respectively in the direction as shown. They are separated by 10 cm. Find the net force on wire-2 of 10 cm length.

Solution:



Using the expression of force between two long parallel wires as

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

Given, $I_1 = I_2 = I_3 = 10\ A$,d = 10 cm, l =10 cm of wire-2

Therefore,

Force on wire-2 due to wire-1

$$F_{21} = 2 \times 10^{-7} \left(\frac{I_2 I_1 l}{d_{21}}\right)$$

Or

$$F_{21} = 2 \times 10^{-7} \times \left(\frac{10 \times 10 \times 10}{10}\right) = 2 \times 10^{-5}\ N$$

(Towards wire-3)

Force on wire-2 due to wire-3

$$F_{23} = 2 \times 10^{-7} \left(\frac{I_2 I_3 l}{d_{23}}\right)$$

Or

$$F_{23} = 2 \times 10^{-7} \times \left(\frac{10 \times 10 \times 10}{10}\right) = 2 \times 10^{-5} N$$

(Towards wire-1)
Since both are of equal magnitude and in opposite direction, then
Net force

$$F_2 = F_{21} - F_{23} = 0$$

Answer:0 N

Exam Tips

- Ampere's Circuital Law is applicable mainly to highly symmetrical current distributions.
- Inside a long solenoid, the magnetic field is **uniform**.
- Outside an ideal solenoid or toroid, the magnetic field is nearly zero.
- Remember the attraction–repulsion rule for parallel conductors.

Common Mistakes

□ Using the toroid formula for a solenoid.
□ Forgetting that n=N/L in the solenoid formula.
□ Using the outer radius instead of the mean radius for a toroid.
□ Forgetting to convert centimetres into metres.

JEE Insight (Beyond NCERT)

1. Field Inside a Toroid

The magnetic field is **not exactly uniform** because
$B \propto 1/r$.
Thus, the field is slightly stronger near the inner radius than near the outer radius. This subtle point is frequently tested in **JEE Advanced**.

2. Choosing Between Biot–Savart Law and Ampere's Law

- **Biot–Savart Law** is general and can be applied to any current distribution, though calculations may be lengthy.
- **Ampere's Circuital Law** is preferred when there is sufficient symmetry (straight wire, solenoid, toroid), making derivations much simpler.

Part – 6: Moving Coil Galvanometer, Ammeter & Voltmeter

Learning Objectives

After studying this part, students will be able to:

- Explain the construction and working of a Moving Coil Galvanometer.
- Derive the expression for the deflecting torque.
- Define current sensitivity and voltage sensitivity.
- Convert a galvanometer into an ammeter.
- Convert a galvanometer into a voltmeter.
- Solve Board, NEET, and JEE numerical problems.

4.35 Moving Coil Galvanometer (MCG)

A **Moving Coil Galvanometer** is a sensitive instrument used to detect and measure **small electric currents**.
It works on the principle that:
A current-carrying coil placed in a uniform magnetic field experiences a torque.

4.36 Construction

A Moving Coil Galvanometer consists of:

- A light rectangular coil of N turns.
- Fine insulated copper wire wound on an aluminium frame.
- A strong horseshoe permanent magnet.
- Soft iron cylindrical core to produce a **strong radial magnetic field**.
- Two phosphor-bronze spiral springs.
- A pointer attached to the coil moving over a calibrated scale.

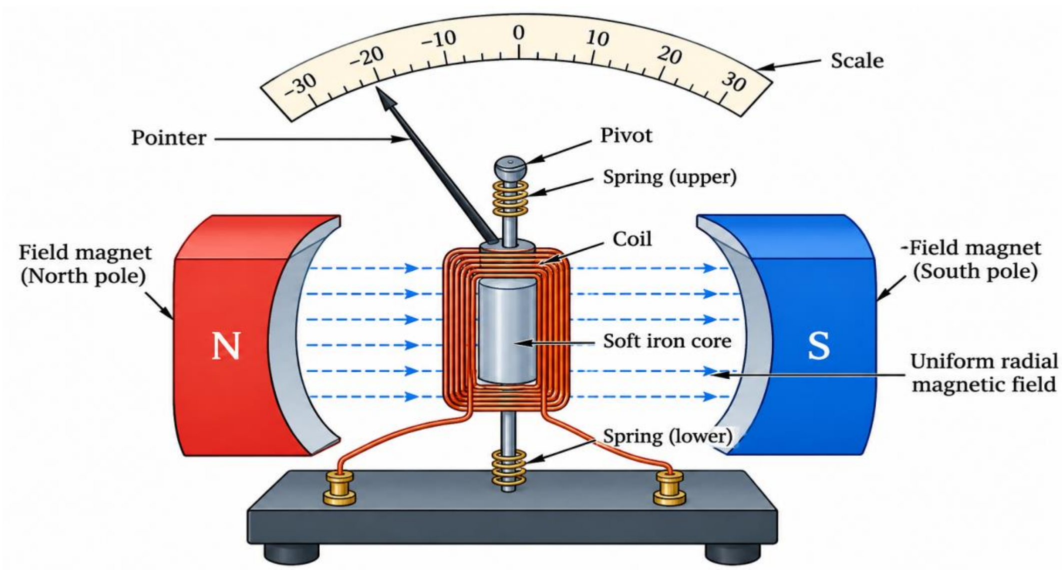


Figure 12 — Moving Coil Galvanometer

Functions of Different Parts

Part	Function
Permanent Magnet	Produces a strong magnetic field
Soft Iron Core	The soft iron core helps produce a strong radial magnetic field.
Coil	Carries current and experiences torque
Springs	Provide restoring torque and current path
Pointer	Indicates deflection
Aluminium Frame	Provides electromagnetic damping

4.37 Principle of Working

When current flows through the coil, each side experiences a magnetic force.
These forces form a couple, producing a **deflecting torque**.
The restoring springs oppose this torque.
At equilibrium,

$$\tau_d = \tau$$

where

- τ_d = deflecting torque
- τ = restoring torque

4.38 Deflecting Torque

The magnetic torque acting on the coil is

$$\boxed{\tau_d = NIAB \sin\theta}$$

Since the magnetic field is radial, sin θ = 1 for all positions of the coil. Plane of the coil is parallel to the magnetic field.
Therefore,

$$\boxed{\tau_d = NIAB}$$

Where

- N = number of turns
- B = magnetic field
- I = current
- A = area of the coil

4.39 Restoring Torque

The restoring torque due to the springs is

$$\boxed{\tau_r = C\varphi}$$

Where

- C = torsional constant of the spring
- φ = angular deflection

4.40 Equation of the Galvanometer

At equilibrium,

$$NIAB = C\varphi$$

Therefore,

$$I = \frac{C\varphi}{NAB}$$

Hence,

$$I \propto \varphi$$

Conclusion: The deflection is directly proportional to the current.

4.41 Current Sensitivity

Current sensitivity is the deflection produced per unit current.

$$\boxed{I_S = \frac{NAB}{C}}$$

To increase current sensitivity:

- Increase the number of turns (N).
- Increase the area of the coil (A).
- Increase the magnetic field (B).
- Decrease the torsional constant (C).

4.42 Voltage Sensitivity

Voltage sensitivity is the deflection produced per unit applied voltage.
Since V=IG, where G is the galvanometer resistance,

$$\boxed{S_V = \frac{\varphi}{V} = \frac{NBA}{CG}}$$

4.43 Conversion of a Galvanometer into an Ammeter

An ammeter measures **large currents** and should have **very low resistance**.
This is achieved by connecting a **low resistance (shunt)** in parallel with the galvanometer.
Let

- G = galvanometer resistance
- Ig = full-scale deflection current
- I = maximum current to be measured
- S = shunt resistance

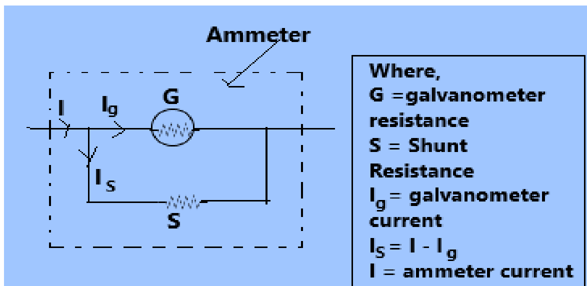


Figure-13: Conversion of a galvanometer into an ammeter

Then,

$$\boxed{S = \frac{GI_g}{I - I_g}}$$

Or,

$$SI = GI_g + SI_g \Rightarrow I = I_g + \frac{GI_g}{S}$$

Or

$$\boxed{I = I_g + \frac{GI_g}{S}}$$

Ammeter Resistance (R_A)

$$\boxed{R_A = \frac{GS}{G+S} = \frac{1}{\frac{1}{S}+\frac{1}{G}}}$$

This shows that

- The higher the shunt resistance, the higher will be ammeter resistance and vise-versa.
- Lower value of ammeter will have higher shunt resistance.

For example,
100 mA ammeter will have lower shunt resistance than 5 A ammeter.

Characteristics of an Ammeter

- Connected in **series**.
- Very low internal resistance.
- Should not significantly change the circuit current.

4.44 Conversion of a Galvanometer into a Voltmeter

A voltmeter measures **potential difference** and should have **very high resistance**.
A **high resistance (multiplier)** is connected in series with the galvanometer.
Let

- R = series resistance
- V = maximum voltage to be measured

Then,

$$\boxed{R = \frac{V}{I_g} - G}$$

Or

$$\boxed{V = (G+R)I_g}$$

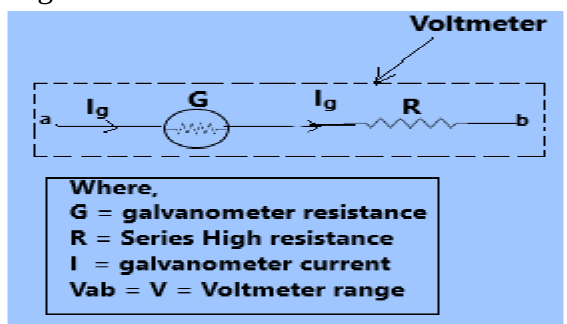


Figure-14: Conversion of a galvanometer into a voltmeter

This shows that

- For higher value of voltmeter range, the higher will be the series resistance required.

Characteristics of a Voltmeter

- Connected in **parallel** across the component.
- Very high internal resistance.
- Draws negligible current from the circuit.

4.45 Comparison: Galvanometer, Ammeter & Voltmeter

Instrument	Measures	Connection	Internal Resistance
Galvanometer	Small current	Series	Moderate
Ammeter	Current	Series	Very low
Voltmeter	Potential difference	Parallel	Very high

4.46 Damping in a Galvanometer

The aluminium frame moves in the magnetic field and induced currents (eddy currents) are produced.
These currents oppose the motion, causing the pointer to settle quickly without oscillations.
This is called **electromagnetic damping**.

Solved Example 1

A galvanometer has:

- Resistance G=100 Ω
- Full-scale current Ig=2 mA

It is to be converted into an ammeter of range 5 A.
Find the required shunt resistance.

Solution

Using

$$S = \frac{GI_g}{I - I_g}$$

Therefore,

$$S = \frac{100 \times 2 \times 10^{-3}}{5 - 0.002} = \frac{0.2}{4.998} = 0.040\ \Omega$$

Answer:

$\boxed{0.040\ \Omega}$

Solved Example 2

A galvanometer has:

- G=100 Ω
- Ig=1 mA

It is to be converted into a voltmeter of range 10 V. Calculate series resistance.

Solution

Using

$$R = \frac{V}{I_g} - G$$

Therefore,

$$R = \frac{10}{10^{-3}} - 100 = 9900 = 9.9\ k\Omega$$

Answer: $\boxed{9.9k\Omega}$

Exam Tips

- A galvanometer **detects** small currents but cannot measure large currents directly.
- An ammeter is always connected **in series**.
- A voltmeter is always connected **in parallel**.
- Remember: **Low resistance → Ammeter**, **High resistance → Voltmeter**.

Common Mistakes

 Connecting an ammeter in parallel.
 Connecting a voltmeter in series.
 Confusing shunt resistance with series (multiplier) resistance.
 Forgetting to convert mA into A before calculations.

JEE Insight (Beyond NCERT)

1. Why is a Radial Magnetic Field Used?

A radial magnetic field ensures that the plane of the coil is always parallel to the magnetic field lines. Therefore, the angle between the **normal to the coil** and the magnetic field remains 90^0,, making the torque:

$$\tau = NBIA$$

independent of the coil's position. This gives a **linear scale**, an important conceptual point for JEE.

2. Galvanometer Sensitivity

- Increasing the number of turns or coil area increases sensitivity.
- Increasing the spring constant decreases sensitivity.
- Increasing the galvanometer resistance decreases **voltage sensitivity**, though it does not directly affect **current sensitivity**.

Advanced Concepts, NCERT Exemplar & JEE Insights

1. Concept Map of the Chapter

```
Moving Charges
        ↓
Magnetic Force
        ├──────────→ Circular Motion
        ├──────────→ Helical Motion
        ↓
Current Carrying Conductor
        ↓
Torque on Current Loop
        ↓
Magnetic Dipole Moment
        ↓
Biot-Savart Law
        ↓
Ampere's Circuital Law
        ↓
Straight Wire
Circular Loop
Solenoid
Toroid
        ↓
Moving Coil Galvanometer
```

2. Frequently Confused Concepts

(a) Electric Field vs Magnetic Field

Electric Field	Magnetic Field
Produced by charges	Magnetic fields are produced by electric currents/moving charges and, more generally, by time-varying electric fields.
Acts on stationary and moving charges	Acts only on moving charges
Unit: $N\,C^{-1}$	Unit: Tesla (T)
Can do work	Cannot do work on a free charged particle

(b) Electric Force vs Magnetic Force

Electric Force	Magnetic Force
F = qE	F = qvB sin θ
Changes speed	A magnetic field alone does no work on a charged particle; therefore, it does not change the particle's speed or kinetic energy, although it can change the direction of its velocity.
Can increase kinetic energy	Does not change kinetic energy

(c) Coulomb's Law vs Biot–Savart Law

Coulomb's Law	Biot–Savart Law
Electric field	Magnetic field
Static charges	Moving charges
Inverse-square dependence	Inverse-square for current element

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3. High-Yield JEE Concepts

Concept 1

If $\vec{v} \parallel \vec{B}$ then F = 0
The particle moves in a straight line.

Concept 2

If $\vec{v} \perp \vec{B}$, the particle moves in a circle.

Concept 3

If velocity has both parallel and perpendicular components,
the particle follows a **helical path**.

Concept 4

Magnetic field never changes

- speed,
- kinetic energy,
- momentum magnitude.

It changes only the **direction of momentum**.

Concept 5

The magnetic force is always perpendicular to velocity.
Hence,
$\vec{F} \cdot \vec{v} = 0$
Therefore,
Power delivered by a magnetic field = 0.
This is one of the most frequently asked conceptual questions in **NEET** and **JEE Main**.

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4. NCERT Exemplar Concepts

Q1: Why does a magnetic field not do work on a charged particle?
Answer:
Because the magnetic force is always perpendicular to the particle's displacement.

Q2: Can the magnetic force change the momentum of a charged particle?
Answer:
Yes.
The **direction** of momentum changes, but its **magnitude** remains constant.

Q3: Why is the path circular?
Answer:
The magnetic force acts as the centripetal force.

Q4: Why is the magnetic field inside a solenoid almost uniform?
Answer:
Because the magnetic field lines inside a long solenoid are parallel and closely spaced.

Q5: Why is the magnetic field outside an ideal solenoid negligible?
Answer:
The fields due to adjacent turns largely cancel outside the solenoid.

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5. Advanced JEE Concepts

Radius Comparison

$r=mv/qB$
Hence, $r \propto mv$
A heavier or faster particle travels in a larger circular path.

Time Period

$T=2\pi m/qB$
Notice that T does **not** depend on velocity.
This is a favourite conceptual question.

Frequency

$f=qB/2\pi m$,
Increasing the magnetic field increases the frequency of revolution.

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6. NEET Important Areas

- Radius
- Time period
- Frequency
- Helical motion
- Biot–Savart law

- Straight conductor
- Circular loop
- Solenoid
- Galvanometer

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7. JEE Advanced Focus

- Vector form of Lorentz force
- Cross product applications
- Superposition of magnetic fields
- Circular and helical motion derivations
- Ampere's law applications
- Magnetic field due to finite conductors
- Combined electric and magnetic fields (velocity selector)

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8. Common Mistakes

- Mixing **Right-Hand Thumb Rule** with **Fleming's Left-Hand Rule**.
- Forgetting the angle in $F = qvB \sin \theta$
- Using total velocity instead of the perpendicular component in helical motion.
- Confusing the formulas for a **solenoid** and a **toroid**.
- Connecting an ammeter in parallel or a voltmeter in series.

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9. Memory Tricks

Magnetic Force

"QVB" Rule

Charge × Velocity × Magnetic Field

Same Direction Currents

Same → Attract

Opposite Direction Currents

Opposite → Repel

Fleming's Left-Hand Rule

F – B – I

- **Thumb** → Force
- **Forefinger** → Magnetic Field
- **Middle Finger** → Current

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1. Numerical Problem-Solving Strategy

Type–1: Magnetic Force

Identify:

- Charge (q)
- Velocity (v)
- Magnetic field (B)

- Angle (θ)

Use

$F = qvB \sin\theta$

Remember

- $0^0 \rightarrow F = 0$
- $90^0 \rightarrow F = qvB\sin 90^0 \rightarrow F = qvB$

Type–2: Circular Motion

Given

- Mass
- Charge
- Velocity
- Magnetic field

Required

Radius

Use

$r = mv/(qB)$

Type–3 : Time Period

Use

$T = 2\pi m/(qB)$

Important

Time period is **independent of velocity**.

Type–4 : Straight Conductor

$B = \mu_0 I/(2\pi r)$

Always convert **cm** → **m** before substitution.

Type–5 : Circular Loop

$B = \mu_0 NI/(2R)$

Check carefully:

- Number of turns (N)
- Radius (R)

Type–6 : Solenoid

$B = \mu_0 nI$ where, $n = N/L$

Type–7: Galvanometer

Ammeter

$$GI_g = S(I - I_g) = SI_S$$

Voltmeter

$$R = \frac{V}{I_g} + G$$

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2. Most Common Mistakes

Mistake–1

Using $\sin\theta$ instead of $\cos\theta$ while resolving velocity into parallel and perpendicular components.

Mistake–2

Not converting
$cm \rightarrow m$

Mistake–3

Confusing μ_0 with ε_0

Mistake–4

Using $B = \mu_0 NI / (2R)$ for a straight conductor.

Mistake–5

Connecting

- Ammeter in parallel
- Voltmeter in series

Mistake–6

Forgetting the number of turns NN in a circular coil.

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3. Concept-Based MCQ Traps

Trap 1

Does a magnetic field do work?
□ **Answer:** No.

Trap 2

Can a magnetic field change speed?
□ **Answer:** No.

Trap 3

Can a magnetic field change direction?
□ **Answer:** Yes.

Trap 4

When is magnetic force maximum?
□ **Answer:** When
$\theta = 90°$

Trap 5

When is magnetic force zero?
□ **Answer:** When
$\theta = 0°$ or $180°$