Part – 1: Chapter-6: Electromagnetic Induction
Introduction, Magnetic Flux & Faraday's Experiments

Learning Objectives

After studying this part, students will be able to:

- Understand the phenomenon of electromagnetic induction.
- Explain Faraday's experiments.
- Define induced emf and induced current.
- Understand magnetic flux and its physical significance.
- Apply the concept of magnetic flux in numerical problems.

6.1 Introduction

Electromagnetic Induction (EMI) is the phenomenon of **production of an electromotive force (emf)** in a conductor whenever the **magnetic flux linked with it changes**.

This phenomenon was discovered by **Michael Faraday (1831)** and independently by **Joseph Henry**.

Electromagnetic induction is the fundamental principle behind:

- Electric Generators
- Transformers
- Induction Motors
- Bicycle Dynamos
- Wireless Charging
- Induction Cooktops

6.2 Historical Background

In 1820, **Ørsted** discovered that an electric current produces a magnetic field.

Faraday then asked an important question:

"Can a changing magnetic field produce electricity?"

After many experiments, he proved that the answer is **Yes**, leading to the discovery of **Electromagnetic Induction**.

6.3 Faraday's First Experiment

Experimental Setup

A coil is connected to a **sensitive galvanometer**.

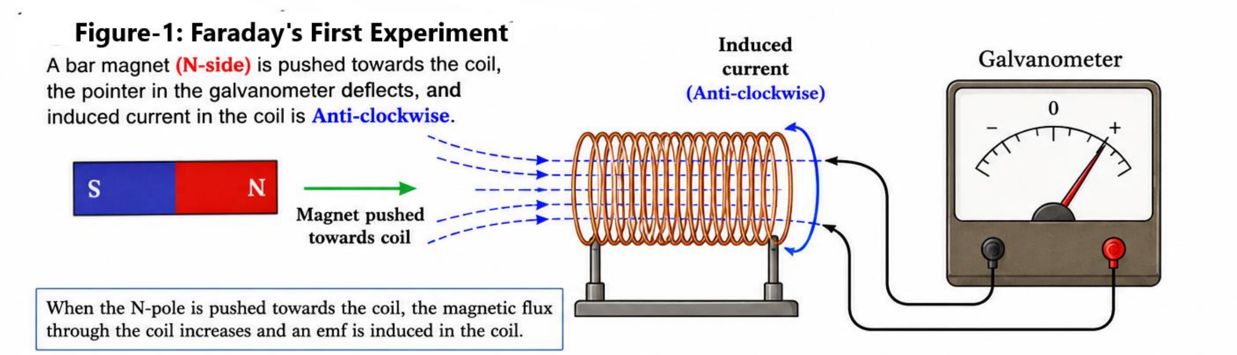
A bar magnet is moved **towards** and **away from** the coil.

Observations

- Magnet moving towards the coil → galvanometer shows deflection.
- Magnet moving away from the coil → galvanometer deflects in the opposite direction.
- Magnet kept stationary → no deflection.

Conclusion

An emf is induced **only when the magnetic flux linked with the coil changes**.



6.4 Faraday's Second Experiment

Experimental Setup

Two coils are wound on a common soft iron core.

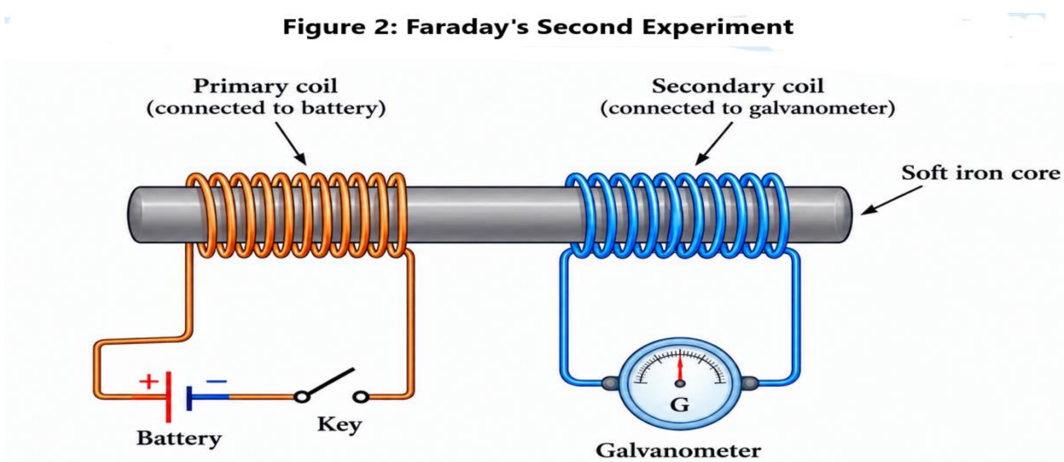
- Primary coil → connected to a battery through a key.
- Secondary coil → connected to a galvanometer.

Observations

- On closing the key, the galvanometer shows a momentary deflection.
- On opening the key, the galvanometer deflects in the opposite direction.
- When the current becomes steady, there is no deflection.

Conclusion

A changing current in the primary coil changes the magnetic field, inducing an emf in the secondary coil.
This is the principle of **mutual induction**.



6.5 Electromagnetic Induction

Definition

The phenomenon of inducing an emf in a conductor due to a change in magnetic flux linked with it is called **electromagnetic induction**.

6.6 Induced emf

The emf produced due to electromagnetic induction is called the **induced emf**.

SI Unit

Volt (V)

6.7 Induced Current

If the induced circuit is closed, the induced emf drives an electric current called the **induced current**.

6.8 Magnetic Flux

Definition

Magnetic flux through a surface is a measure of the magnetic field passing through that surface. It tells us how much magnetic field is linked with the surface.

$$\varphi_B = \int \vec{B} \cdot d\vec{A}$$

For Uniform magnetic field,

$$\varphi = BA\cos\theta$$

Formula

For a uniform magnetic field,

$$\varphi = BA\cos\theta$$

Where:

- Φ = Magnetic Flux
- B = Magnetic Field
- A = Area
- θ = Angle between the magnetic field and the **normal** to the surface

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6.9 SI Unit of Magnetic Flux

SI Unit: Weber (Wb)

Definition

One weber is the magnetic flux that produces an emf of **1 volt** when it changes uniformly to zero in **1 second**.

Or,

As per NIST definition,

One weber is the magnetic flux which, when reduced uniformly to zero through a one-turn circuit in 1 s, induces an emf of 1 V.

$$1Wb = 1Vs$$

6.10 Special Cases of Magnetic Flux

Case 1

Magnetic field **perpendicular** to the surface

$$\theta = 0^0$$

Therefore,

$$\varphi = BA$$

Maximum flux

Case 2
Magnetic field **parallel** to the surface

$$\theta = 90^0$$

Therefore,

$$\varphi = 0$$

No magnetic flux passes through the surface.

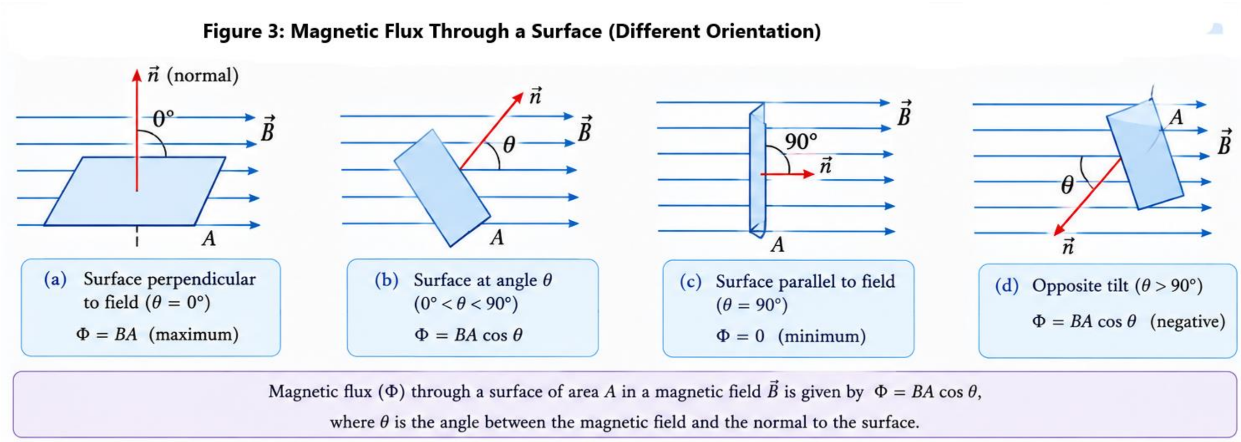


Figure 3: Magnetic Flux Through a Surface (Different Orientation)

6.11 Factors Affecting Magnetic Flux
Magnetic flux changes if:
- Magnetic field strength changes.
- Area of the loop changes.
- Orientation of the loop changes.
- The loop moves into or out of the magnetic field.

These are the four common ways to produce electromagnetic induction.

Solved Example 1
A square coil of side **20 cm** is placed in a uniform magnetic field of **0.5 T**. The magnetic field is perpendicular to the plane of the coil.
Find the magnetic flux.

Solution
Area, A = (0.2)2 = 0.04 m^2
Using

$$\Phi = BA = 0.5 \times 0.04 = 0.02 \text{ Wb}$$

Answer
0.02 Wb

Solved Example 2
A magnetic field is parallel to the surface of a circular loop. Find the magnetic flux.

Solution
Since the angle between the magnetic field and the **normal** is 90^0,

$$\Phi = BA\cos 90^0 = 0$$

Answer: 0 Wb

Exam Tips

✔ Remember that the angle in $\Phi = BA\cos\theta$ is measured with the **normal to the surface**, **not** with the surface itself.
✔ Electromagnetic induction occurs **only when magnetic flux changes**.
✔ A stationary magnet near a stationary coil produces **no induced emf**.

Common Mistakes

□ Taking the angle with the plane instead of the normal.
□ Confusing magnetic field (Tesla) with magnetic flux (Weber).
□ Assuming induced current exists without a closed circuit.
□ Thinking that a constant magnetic field always induces emf.

Quick Revision Box

Magnetic Flux

$$\Phi = BA\cos\theta$$

SI Unit = Weber (Wb)

Electromagnetic Induction

Change in magnetic flux → Induced emf

JEE Insight

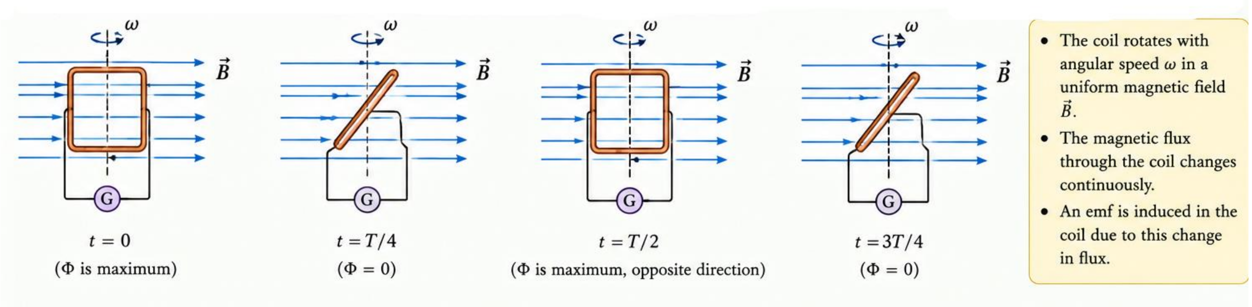
Why is a Changing Magnetic Flux Necessary?

A magnetic field alone does **not** produce an induced emf. The **flux linked with the circuit must change**. This change may occur due to:

- Motion of the conductor,
- Change in magnetic field strength,
- Rotation of the coil,
- Change in the area enclosed by the loop.

This unified idea is the foundation of **Faraday's Law**.

Figure 4: Coil rotating in uniform magnetic field (Conceptual Illustration)



Part – 2: Faraday's Laws of Electromagnetic Induction & Lenz's Law

Learning Objectives

After studying this part, students will be able to:

- State Faraday's First and Second Laws.
- Derive the mathematical expression for induced emf.
- Explain Lenz's Law.
- Understand the negative sign in Faraday's law.
- Apply the laws to conceptual and numerical problems.

6.12 Faraday's First Law of Electromagnetic Induction

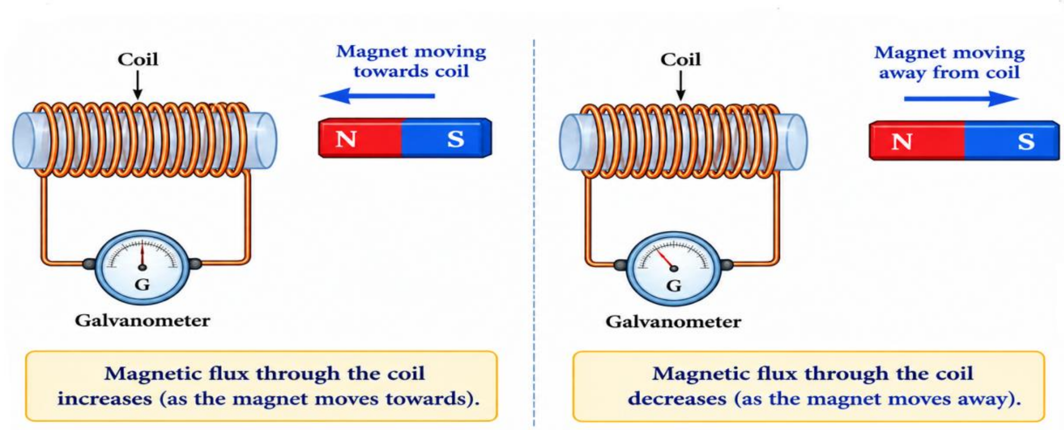
Statement

Whenever the magnetic flux linked with a circuit changes, an induced emf is produced in the circuit. If the circuit is closed, an induced current also flows.

Important Point

- Constant magnetic flux → No induced emf.
- Changing magnetic flux → Induced emf.

Figure 5: Change in Magnetic Flux Through a Coil



6.13 Faraday's Second Law

Statement

The magnitude of the induced emf is directly proportional to the **rate of change of magnetic flux** linked with the circuit.

For a single-turn coil,

$$e \propto \frac{d\varphi}{dt}$$

Therefore,

$$e = \frac{d\varphi}{dt}$$

Ignoring direction, this gives the magnitude of the induced emf.

Actually,

$$\boxed{e = -\frac{d\varphi}{dt}}$$

Here, negative sign shows that induced emf opposes the changing flux.

6.14 Induced emf for N Turns

If a coil has **N turns**, each turn experiences the same change in magnetic flux.
Hence,

$$\boxed{e = N\frac{d\varphi}{dt}}$$

The direction is obtained from **Lenz's Law**.

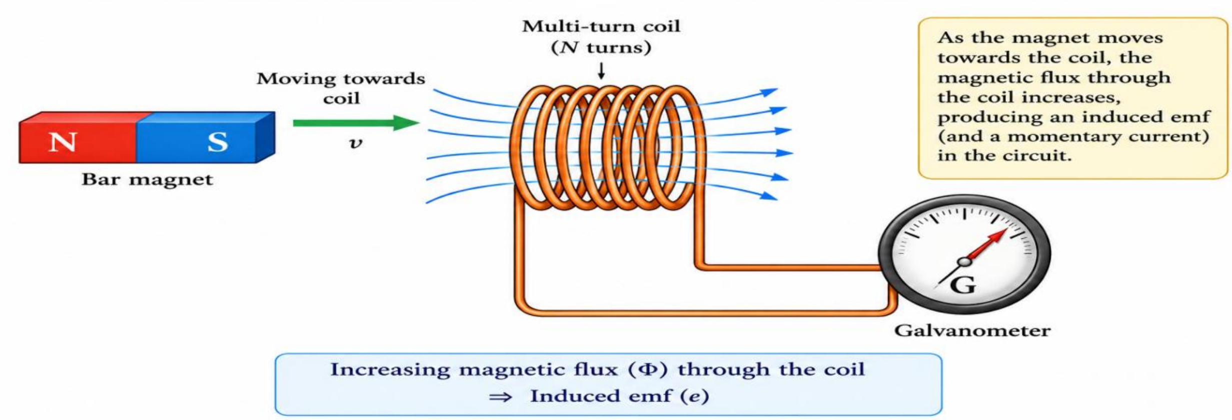
6.15 Faraday-Lenz Law

Combining Faraday's law with Lenz's law,

$$\boxed{e = -N\frac{d\varphi}{dt}}$$

The **negative sign** indicates the direction of the induced emf.

Figure 6: Coil With Changing Magnetic Flux producing induced emf



6.16 Physical Meaning of the Negative Sign

The negative sign does **not** indicate a negative voltage.
It signifies that:
The induced current always opposes the change in magnetic flux that produces it.
This is the mathematical expression of **Lenz's Law**.

6.17 Lenz's Law

Statement

The direction of the induced current is always such that the magnetic field produced by it **opposes the change in magnetic flux** responsible for its production.
Important: The induced magnetic field opposes the change in magnetic flux, not necessarily the original magnetic field itself.

6.18 Why Does Nature Oppose the Change?

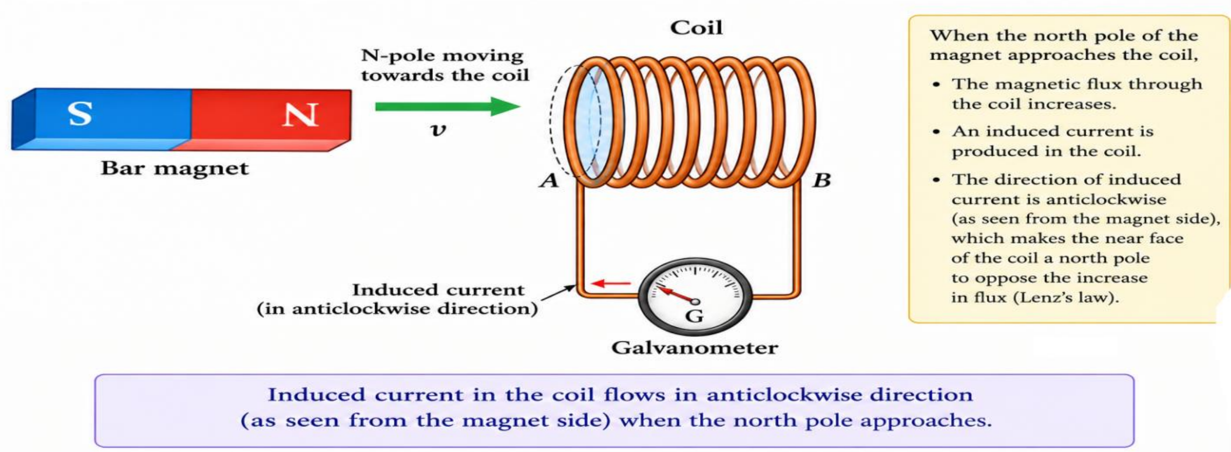
Lenz's Law is a direct consequence of the **Law of Conservation of Energy**.
If the induced current supported the change instead of opposing it, energy would be produced without any external work, which is impossible.
Thus, the induced current always opposes the change causing it.

6.19 Cases Based on Lenz's Law

Case 1: Magnet Approaching the Coil

- Magnetic flux through the coil increases.
- The induced current produces a magnetic field opposing the increase.
- The face of the coil near the approaching pole behaves like the **same pole**, causing repulsion.

Figure 7: North Pole Approaching a coil



Case 2: Magnet Moving Away from the Coil

- Magnetic flux decreases.
- The induced current produces a magnetic field to oppose the decrease.
- The face of the coil becomes the **opposite pole**, causing attraction.

6.20 Methods of Changing Magnetic Flux

An induced emf can be produced by:

1. Moving a magnet towards or away from a coil.
2. Moving the coil in a magnetic field.
3. Rotating the coil in a magnetic field.
4. Changing the strength of the magnetic field.
5. Changing the area enclosed by the coil.

Solved Example 1

A coil of **200 turns** experiences a change in magnetic flux from **0.50 Wb** to **0.10 Wb** in **0.20 s**. Find the magnitude of the induced emf.

Solution

Given:

- N=200
- $\Delta\Phi = 0.50 - 0.10 = 0.40$ Wb
- $\Delta t = 0.20$ s

Using

$$e = N\frac{\Delta\varphi}{\Delta t} = \frac{200 \times 0.40}{0.20} = 400\ V$$

Answer: 400 V

Solved Example 2

Will an induced emf be produced if:

- A magnet is held stationary near a coil?

Answer

No.

Since the magnetic flux linked with the coil does not change,

$$\frac{d\varphi}{dt} = 0$$

Therefore,

$$e = 0$$

Exam Tips

✔ Learn both Faraday's Laws exactly as stated.

✔ Always explain the **negative sign** using Lenz's Law.

✔ In derivations, write the magnitude first and then introduce the negative sign using Lenz's Law.

✔ Mention that Lenz's Law is based on the **Law of Conservation of Energy**.

Common Mistakes

 Ignoring the number of turns N in the formula
 Forgetting the negative sign in Faraday-Lenz law
 Writing that induced current supports the change in magnetic flux
 Confusing magnetic flux with magnetic field

Quick Revision Box

Faraday's Law (Magnitude)

$$e = N(d\Phi/dt)$$

Faraday-Lenz Law

$$e = -N(d\Phi/dt)$$

Lenz's Law

Induced current always opposes the change in magnetic flux.

JEE Insight

A changing magnetic flux may result from:

- Variation in magnetic field (B)
- Variation in area (A)
- Rotation of the loop (change in θ)
- Combination of all three

Since

$$\Phi = BA \cos \theta$$

a change in **any** of these quantities can induce an emf. This concept is frequently tested in **JEE Advanced**.

Part – 3: Motional EMF, Fleming's Right-Hand Rule & Derivation of e = Blv

Learning Objectives

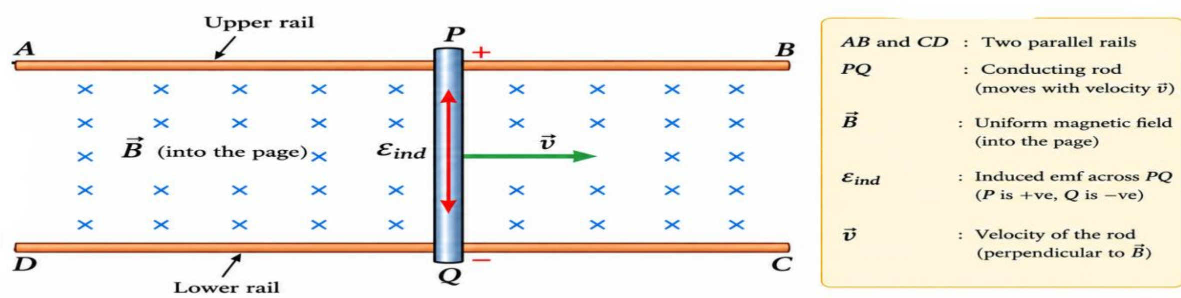
After studying this part, students will be able to:

- Understand the concept of motional emf.
- Explain the origin of induced emf in a moving conductor.
- Apply Fleming's Right-Hand Rule.
- Derive the expression for motional emf.
- Solve Board, NEET and JEE numerical problems.

6.21 Motional Electromotive Force (Motional EMF)

When a conductor moves through a magnetic field, the free electrons inside the conductor experience a magnetic force.
This causes a separation of charges, producing a potential difference across the ends of the conductor.
This induced potential difference is called **Motional EMF**.

Figure 8: A conducting rod moving perpendicular to a uniform magnetic field



When the conducting rod PQ moves perpendicular to a uniform magnetic field $\vec{B}$, an emf is induced across the rod (P is positive, Q is negative).

6.22 Origin of Motional EMF

Consider a conductor of length **l** moving with velocity **v** perpendicular to a uniform magnetic field **B**.
Each charge **q** experiences the magnetic (Lorentz) force

$$F = qvB$$

The electrons accumulate at one end until the electric force balances the magnetic force.
At equilibrium,

$$qE = qvB$$

or

$$E = v\,B$$

Since

$$V = El$$

Therefore,

$$e = Blv$$

Derivation of Motional EMF

Using

$$V=El$$

and

$$E=vB$$

Hence,

$$e=Blv$$

where

- B = Magnetic field (Tesla)
- l = Length of conductor (m)
- v = Velocity (m/s)

Derivation-1:

We need to obtain the induced emf in the moving rod PQ.
As shown in the Figure 6.10, the velocity vector is perpendicular to the rod.
Since the rod is moving with velocity vector

$$\vec{v} = v\hat{\imath}$$

The magnetic field is uniform and

$$\vec{B} = B(-\hat{k})$$

The rod direction and magnitude is

$$\vec{l} = l\hat{\jmath}$$

Using Lorentz force,

$$\vec{F} = \vec{F}_e + \vec{F}_B = q\vec{E} + q(\vec{v} \times \vec{B}) - - - (1)$$

At the equilibrium, $\vec{F} = 0$ and electrons in the rod moves downward. Replacing q by –e, we get

$$\vec{E} = -\vec{v} \times \vec{B} - - - -(2)$$

Using,

$$dV = -\vec{E} \cdot d\vec{l} - - - -(3)$$

We get

$$dV = (\vec{v} \times \vec{B}) \cdot d\vec{l} - - - -(4)$$

Integrating equation (4) both sides, we have

$$\int_0^e dV = \int_0^l (\vec{v} \times \vec{B}) \cdot d\vec{l} - - - -(5)$$

Or

$$e = \int_0^l [v(i) \times B(-\hat{k})] \cdot dl(\hat{\jmath}) \Rightarrow e = vB\int_0^l dl$$

Or

$$\boxed{e = vBl}$$

6.23 General Form

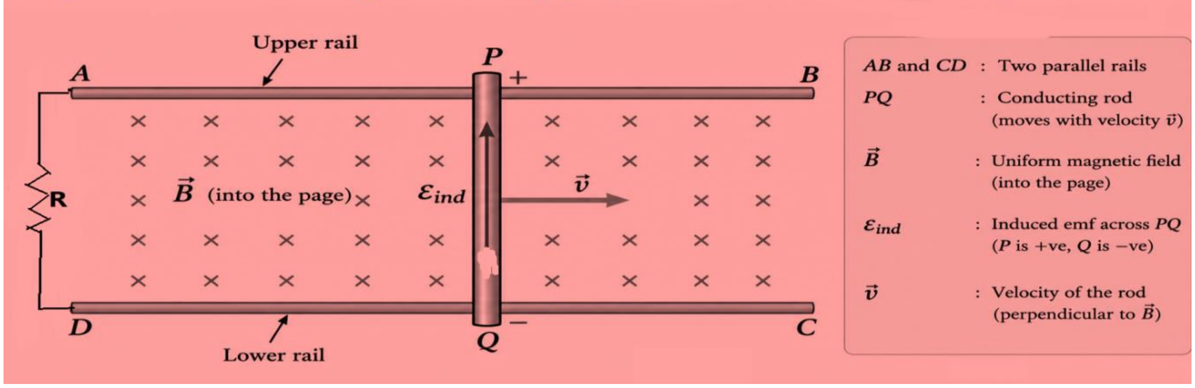
If the conductor moves at an angle θ to the magnetic field,

$$\boxed{e = Bvl\sin\theta}$$

Special Cases

When $\theta = 90^0$, e = Blv, Maximum emf.
When $\theta = 0^0$, e = 0, No induced emf.

Figure 8-1: A conducting rod moving perpendicular to a uniform magnetic field

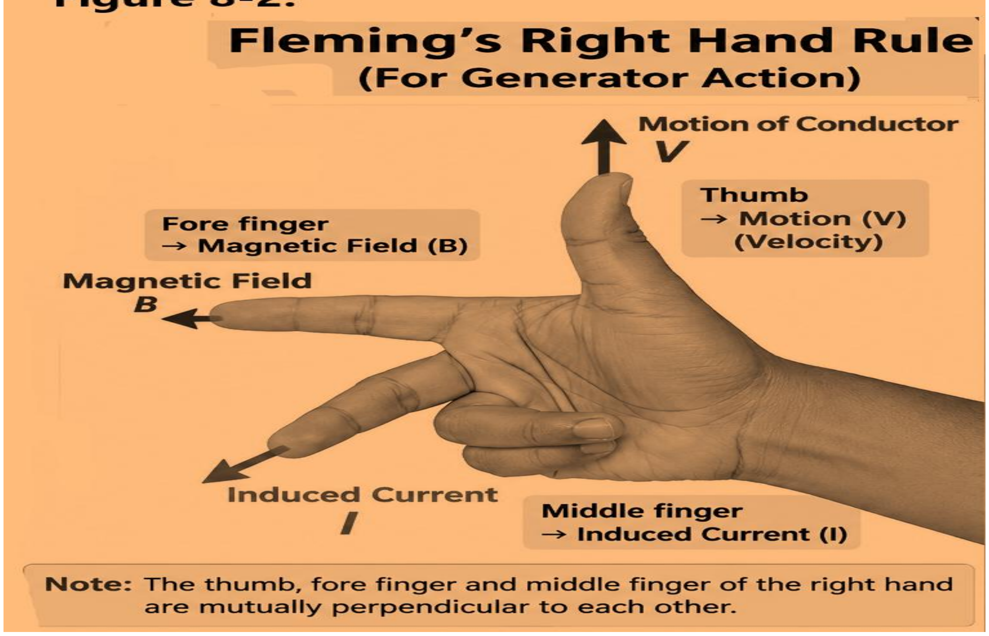


6.24 Fleming's Right-Hand Rule

This rule is used to determine the direction of the induced current and polarity of induced emf. Stretch the thumb, forefinger and middle finger of the **right hand** so that they are mutually perpendicular.

- **Forefinger** → Direction of magnetic field (**B**)
- **Thumb** → Direction of motion (**v**)
- **Middle finger** → Direction of induced current (**I**)

Figure 8-2:



6.25 Conditions for Motional EMF

Induced emf is produced only if:

- The conductor moves relative to the magnetic field.
- Motional emf arises when the conductor has a velocity component perpendicular to the magnetic field, so that magnetic force acts on its charges.

$$\boxed{e = BvL\sin\theta}$$

Note:

$$e = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

This is scalar triple product.
No Motional EMF if

$$\boxed{\vec{v} \| \vec{B} \text{ or } \vec{B} \| \vec{l} \text{ or } \vec{v} \| \vec{l}}$$

6.26 Applications of Motional EMF

- Electric generators
- Bicycle dynamo
- Railway signalling systems
- Electromagnetic flow meters
- Linear induction devices

Induced Current and its direction:

The induced current in the closed circuit of resistance R is

$$i = \frac{e}{R} - - - (1)$$

Since, induced emf in the rod is

$$e = vBL - - - (2)$$

Where L = length of rod PQ
Therefore,

$$i = \frac{vBL}{R} - - - (3)$$

Its direction is anticlockwise, using Fleming Right hand rule.

Electric Power and Force in the Magnetic Circuit

Since power dissipated in the resistor is

$$P = i^2R = \left(\frac{v^2B^2L^2}{R^2}\right)R = \frac{v^2B^2L^2}{R}$$

Hence,

$$\boxed{P = \frac{v^2B^2L^2}{R}}$$

Force exerted on the rod

Since

$$P = Fv$$

Or,

$$F = \frac{P}{v} = \frac{vB^2L^2}{R}$$

Graph

Using Faraday's law of electromagnetic induction,

$$e = -\frac{d\varphi}{dt}$$

Since, for a closed circuit,

$$e = iR$$

Therefore,

$$i = -\frac{1}{R}\frac{d\varphi}{dt} - - - - (1)$$

A graph is plotted between induced current and $d\varphi/dt$. Its slope = -1/R

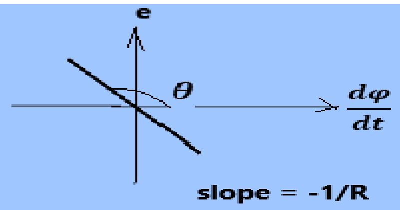


Figure 8-2: induced emf vs.slope

Derivation-2:

To derive motional emf

$$e = \frac{1}{2}B\omega L^2$$

When a rod is moving in circular motion.

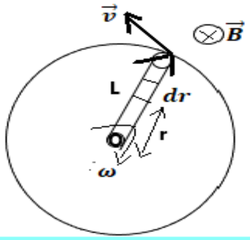


Figure 8.3: Angular rotation of a rod

Suppose, a thin rod of length L is moving in the uniform (cross) Magnetic field.

Let r be the distance from the centre O up to dr. (a small portion of length).

By using

$$dV = -Edr$$

We get

$$\int_0^e dV = \int_0^L Edr - - - (1)$$

Since,

$$E = v \times B - - - - (2)$$

And,

$$v = r\omega - - - - (3)$$

Using equations (1), (2) and (3),we have

$$\int_0^e dV = \int_0^L (r\omega)B\, dr$$

Or,

$$e = B\omega\left(\frac{r^2}{2}\right)_0^L = \frac{1}{2}B\omega L^2 - - - - (4)$$

Hence

$$\boxed{e = \frac{1}{2} B\omega L^2}$$

From the above equation (4), $\omega = 2\pi f$
Therefore,

$$\boxed{e = \pi f B L^2}$$

Solved Example 1

A conductor of length **0.50 m** moves with a speed of **8 m/s** perpendicular to a magnetic field of **0.40 T**.
Find the induced emf.

Solution

Given:

- B = 0.40 T
- L = 0.50 m
- v = 8 m/s

Using

$$e = Blv$$

Or

$$e = 0.40 \times 0.50 \times 8$$

Or

$$e = 1.6\ V$$

Answer
1.6 V

Solved Example 2

A conductor moves **parallel** to a magnetic field. Find the induced emf.

Solution

Since $\theta = 0^0$,

$$E = Blv \sin 0^0$$

Or

$$e = 0$$

Answer
No emf is induced.

Concept Check

Can a stationary conductor produce motional emf?

Answer:
No.
Relative motion between the conductor and the magnetic field is necessary.

Can a moving conductor fail to produce emf?
Answer:
Yes.
If it moves parallel to the magnetic field lines, it does not cut magnetic flux, so no emf is induced.

Exam Tips

✔ Always use **Fleming's Right-Hand Rule** for generators (induced current).
✔ Do **not** confuse it with **Fleming's Left-Hand Rule**, which is used for motors.
✔ In numerical problems, check the angle between the velocity and the magnetic field before applying the formula.

Common Mistakes

□ Using Left-Hand Rule instead of Right-Hand Rule.
□ Forgetting the sinθ factor in the general expression.
□ Confusing magnetic force with electric force.
□ Writing e=Bv and forgetting the conductor length.

Quick Revision Box

Motional EMF

$$e = vbl$$

General Formula

$$\boxed{e = vBl\sin\theta}$$

Fleming's Right-Hand Rule

- Thumb → Motion
- Forefinger → Magnetic Field
- Middle Finger → Current

JEE Insight

The expression e=Blv is a special case of **Faraday's Law**.
As the conductor moves, the area enclosed by the circuit changes, causing a change in magnetic flux.
For a conductor moving with velocity v,

$$\frac{dA}{dt} = lv$$

Using Faraday's Law,

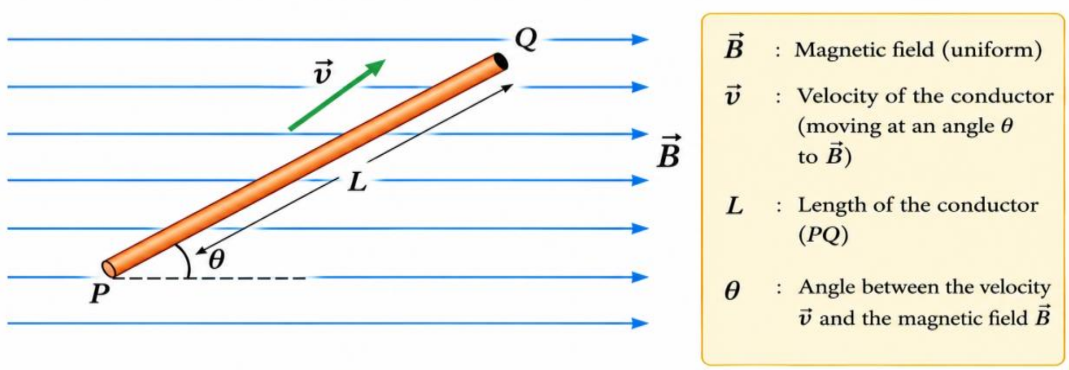
$$e = B\frac{dA}{dt}$$

Or

$$e = Blv$$

This derivation is frequently asked in **JEE Main** and **JEE Advanced**.

Figure 9: Conductor moving at an angle θ to the uniform magnetic field



A conductor of length L moves with velocity $\vec{v}$ at an angle θ to a uniform magnetic field $\vec{B}$.

Part – 4: Self-Induction, Mutual Induction & Inductance

Learning Objectives

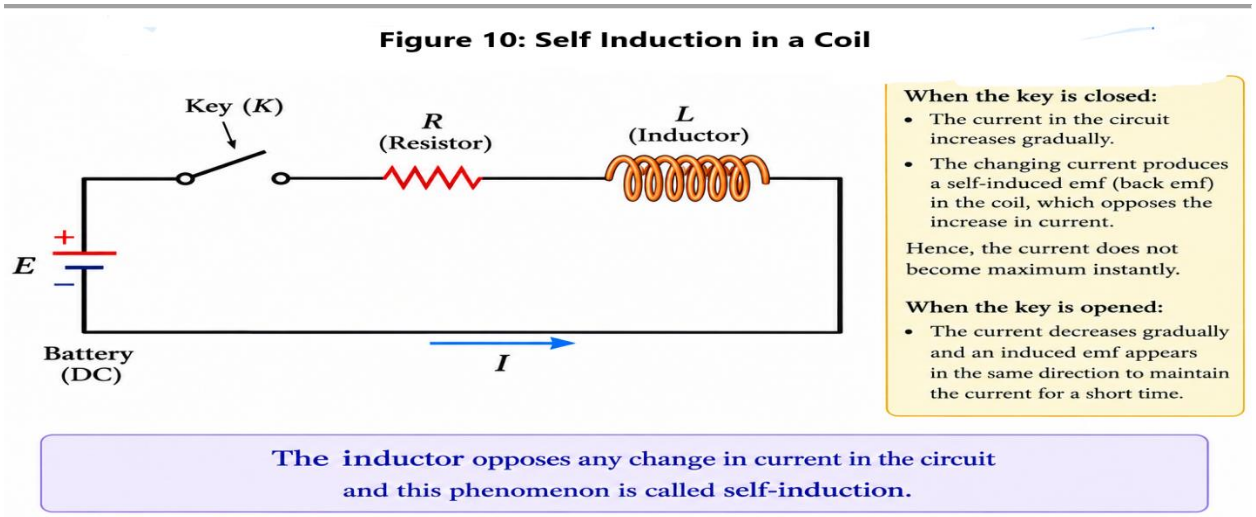
After studying this part, students will be able to:

- Explain self-induction and mutual induction.
- Define self-inductance and mutual inductance.
- Derive the expression for self-induced emf.
- Understand the SI unit of inductance.
- Explain energy stored in an inductor.
- Solve Board, NEET and JEE numerical problems.

6.27 Self-Induction

Definition

Self-induction is the phenomenon in which a changing current in a coil induces an emf **in the same coil** due to the change in its own magnetic flux.
The induced emf always opposes the change in current (Lenz's Law).



6.28 Self-Induced EMF

When the current through a coil changes,

- the magnetic field changes,
- the magnetic flux linked with the same coil changes,
- an emf is induced in that coil.

This emf is called **self-induced emf**.

Mathematical Expression

From Faraday's law,

$$e = -N\frac{d\varphi}{dt}$$

For a linear magnetic medium, the magnetic flux linked with the coil is proportional to the current. $\varphi \propto I$
Therefore

$$\varphi = LI$$

Where, L is the **self-inductance**.
For N turns,

$$\boxed{L = \frac{N\varphi}{I}}$$

Or

$$N\varphi = LI$$

Differentiating,

$$\boxed{e = -L\frac{dI}{dt}}$$

Note:
Magnetic flux per turn is φ.
Flux linkage is $N\varphi$,
Therefore,

$$L = \frac{N\varphi}{I}$$

6.29 Self-Inductance (L)

Definition

Self-inductance is the property of a coil by virtue of which it opposes any change in the current flowing through it.
It is also called the **coefficient of self-induction**.

Expression

$$L = \frac{N\varphi}{I}$$

6.30 SI Unit of Self-Inductance

SI Unit: Henry (H)

Definition of One Henry

A coil has an inductance of **1 henry** if a current changing at the rate of **1 A s⁻¹** induces an emf of **1 V**.

$$\boxed{1H = 1Wb/A}$$

6.31 Factors Affecting Self-Inductance

The inductance of a coil depends upon:

- Number of turns (N)
- Area of cross-section (A)
- Length of the coil (l)
- Nature of the core material (μ)

For a long solenoid,

$$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

Derivation-3:

To derive the self-inductance of a solenoid

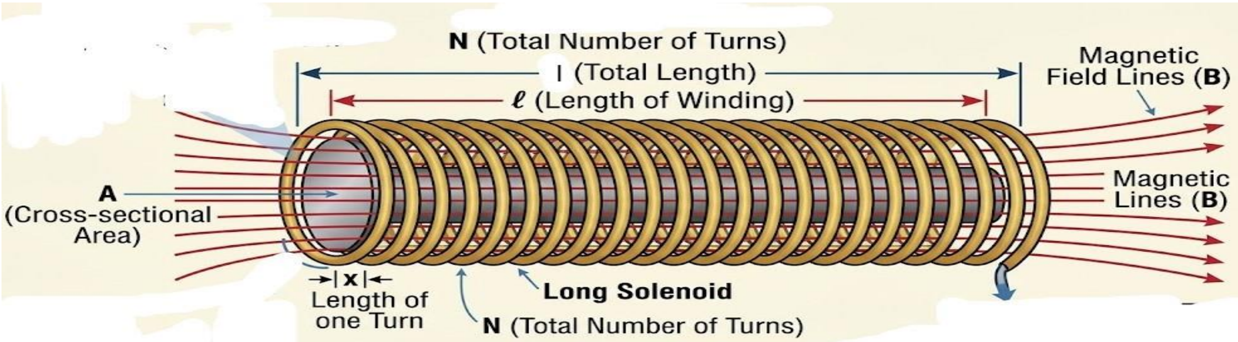


Figure 11: Long Solenoid

Assuming a long solenoid has length l, number of turns N and area A.
Let the current I flows in the solenoid.
The magnetic field at the centre of solenoid on the axis is

$$B = \mu_0 nI = \mu_0 \frac{N}{l} I - - - (1)$$

The magnetic flux link the coil is

$$\varphi = BA - - - - - (2)$$

(Here angle $\theta = 0^0$ as magnetic field direction is same as area vector.)
Substituting B from equation (1) in equation (2), we have

$$\varphi = \frac{\mu_0 NIA}{l} - - - (3)$$

Now, the self- inductance is

$$L = \frac{N\varphi}{I} - - - (4)$$

Substituting φ from equation (3) in equation (4), we get

$$L = \frac{\mu_0 N^2 IA}{(l)I} = \frac{\mu_0 N^2 A}{l} - - - (5)$$

Therefore, self inductance of a solenoid is

$$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

6.32 Mutual Induction

Definition

Mutual induction is the phenomenon in which a changing current in one coil induces an emf in a nearby coil.

The two coils are called:

- Primary coil
- Secondary coil

6.33 Mutual Induced EMF

The induced emf in the secondary coil is

$$e_2 = -M\frac{dI_1}{dt}$$

Or

$$e_1 = -M\frac{dI_2}{dt}$$

Where

- M = Mutual inductance

6.34 Mutual Inductance

Definition

Mutual inductance is the property by virtue of which a change in current in one coil induces an emf in another coil.

Expression

$$M = \frac{N_2\varphi_2}{I_1}$$

Or

$$M = \frac{N_1\varphi_1}{I_2}$$

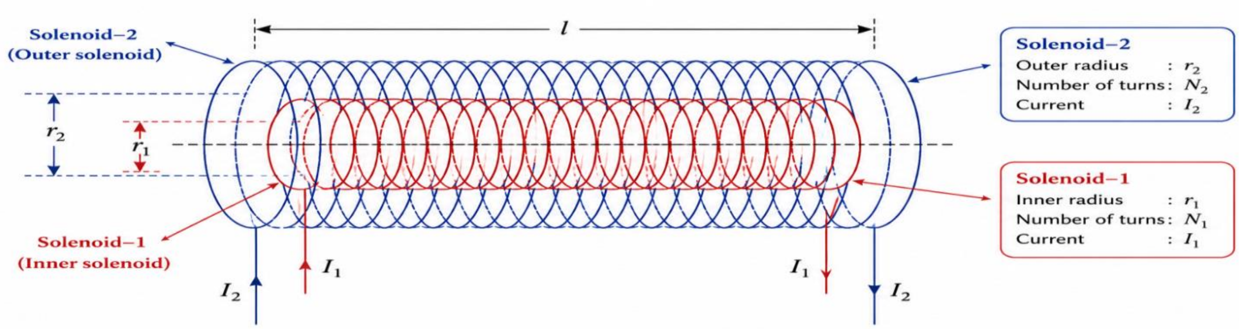
Where:

- N_2 = Number of turns in the secondary coil
- Φ_{21} = Magnetic flux linked with the secondary due to current in the primary
- I_1 = Current in the primary

Derivation-4:

Derive the mutual inductance of two long solenoids co-axially placed.

Figure 12: Two Long Solenoids Co-axially Placed



The current induced in coil-1 will link the flux in coil-2.
As shown in the figure, let the current I_1 flows in the inner coil of radius r_1, having number of turns N_1 and length l.
The magnetic field at the centre of inner solenoid-1 is

$$B_1 = \mu_0 n_i I_1 = \mu_0 \frac{N_1}{l} I_1 - - - (1)$$

The magnetic flus link with solenoid-2 is

$$\varphi_2 = B_1 A_2 - - - (2)$$

The mutual inductance between two coils is

$$M = \frac{N_2 \varphi_2}{I_1} - - - - (3)$$

Using equation (2) and (3), we get

$$M = \frac{\mu_0 N_1 N_2 A_2}{l} - - - - - - (4)$$

Since, solenoid has its cross-section area, $A_2 = \pi {r_2}^2$,
Hence, we obtain

$$\boxed{M = \frac{\mu_0 N_1 N_2 \pi {r_2}^2}{l}}$$

6.35 Energy Stored in an Inductor

When current increases in an inductor, work has to be done against the self-induced emf.
This work is stored as magnetic energy.
The energy stored is

$$U = \frac{1}{2} L I^2$$

Derivation-5:

To derive energy stored in the inductor coil in the uniform magnetic field

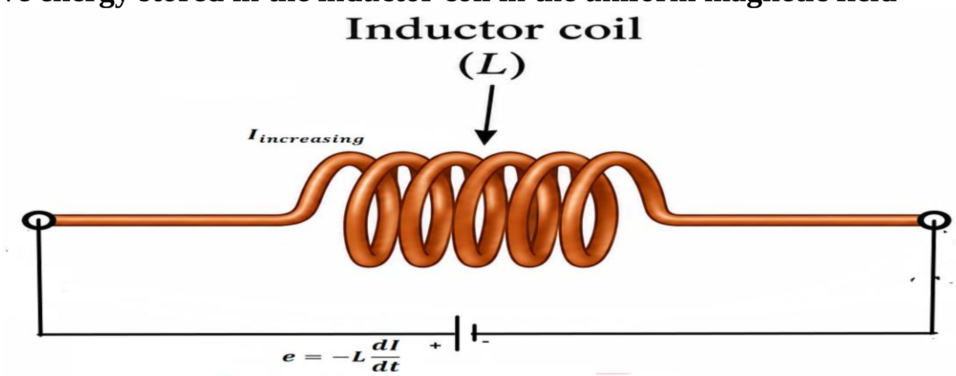


Figure 13: Inductor Coil

Let a coil has self-inductance L and current increases from 0 to I in time dt.
Therefore, induced emf in the coil is

$$e = -L \frac{dI}{dt} - - - (1)$$

Since, power delivered by induced emf is

$$P = eI - - - - (2)$$

From equation (1) and (2), we have

$$P = -LI\frac{dI}{dt} - - - (3)$$

Now, work done to increase current from 0 to I is

$$dW = Pdt = -LIdI - - - - (4)$$

The external source does work against the self-induced emf, and this energy is stored in the magnetic field of the inductor.

$$dW = -dU$$

Therefore,

$$dU = LIdI - - - (5)$$

Integrating equation (5) both sides, we get

$$U = \int dU = \int_0^I LIdI = \frac{1}{2}LI^2 - - - - (6)$$

Hence, energy stored (energy stored as potential energy U) in the coil is

$$\boxed{U = \frac{1}{2}LI^2}$$

6.36 Energy Density (Advanced Concept)

For a magnetic field,

$$\boxed{u_B = \frac{B^2}{2\mu_2}}$$

Where

- u = Energy density
- B = Magnetic field
- μ = Permeability

Derivation-6:

To obtain the expression of energy density in the magnetic field

Since, energy density is the energy sored per unit volume.
Therefore, energy density u is

$$u = \frac{U}{Volume} - - - (1)$$

Let a solenoid has length l and area of cross-section A. Therefore, volume of solenoid is

$$Volume = Area \times Length = Al - - - - (2)$$

The energy stored in the solenoid is

$$U = \frac{1}{2}LI^2 - - - (3)$$

From equation (1) ,(2) and (3), we have

$$u = \frac{\frac{1}{2}LI^2}{Al} - - - (4)$$

The seld inductance in free space of a solenoid is

$$L = \frac{\mu_0 N^2 A}{l} - - - - (5)$$

Using equation (4) and (5), we get

$$u = \frac{\mu_0 N^2 A I^2}{2Al^2} = \frac{\mu_0 N^2 I^2}{2l^2} = \left(\frac{1}{2}\right)\left(\frac{\mu_0{}^2}{\mu_0}\right)\left(\frac{N^2 I^2}{l^2}\right) - - - (6)$$

We know that, magnetic field at the centre of a solenoid is

$$B = \mu_0 n I = \frac{\mu_0 N I}{l} - - - (7)$$

From equation (6) and (7), we obtain energy density is

$$\boxed{u = \frac{B}{2\mu_0}}$$

This is similar to energy density in electric field in the capacitor as

$$\boxed{u = \frac{\epsilon_0 E^2}{2}}$$

JEE Advanced Note: This formula is beyond the core CBSE syllabus but is useful for competitive examinations.

Solved Example 1

A coil has an inductance of **2 H**. The current through it changes from **1 A** to **5 A** in **0.2 s**. Find the induced emf.

Solution

Using

$$e = -L\frac{dI}{dt} = -\frac{2 \times (5-1)}{0.2} = -40\ V$$

Magnitude of induced emf

40 V

Solved Example 2

An inductor has L = 4 H, Current through it is I = 3 A , Find the energy stored.

Solution

Using

$$U = \frac{LI^2}{2} = \frac{4 \times (3)^2}{2} = 18\ J$$

Answer: 18 J

Exam Tips

✔ Remember the distinction:

- **Self-induction** → Same coil.
- **Mutual induction** → Two different coils.

✔ The **negative sign** in both induction formulas is due to **Lenz's Law**.

✔ Energy stored in an inductor is always **positive**.

Common Mistakes

- Confusing self-inductance L with mutual inductance M.
- Forgetting the factor 21 in the energy formula.
- Omitting the negative sign in the induced emf expression.
- Confusing inductance (henry) with resistance (ohm).

Quick Revision Box

Self-Induced EMF

$$e = -L\,(dI/dt)$$

Mutual-Induced EMF

$$e = -M\,(dI/dt)$$

Self-Inductance

$$L = N\Phi/I$$

Inductance of a Long Solenoid

$$L = (\mu N^2 A)/l$$

Energy Stored

$$U = (1/2)\,LI^2$$

JEE Insight

The expression

$$U = \frac{1}{2}LI^2$$

is completely analogous to the electrostatic energy stored in a capacitor,

$$U = \frac{1}{2}CV^2$$

This comparison is frequently used in **JEE Main** and **JEE Advanced** to relate electric and magnetic energy storage.

Part – 6: Applications of Electromagnetic Induction, AC Generator & its Principle & Chapter Summary

Learning Objectives

After studying this part, students will be able to:

- Understand important applications of electromagnetic induction.
- AC generator.
- Relate EMI to everyday devices.
- Revise all key concepts of the chapter.
- Prepare for Board, NEET and JEE examinations.

6.37 Practical Applications of Electromagnetic Induction

Electromagnetic induction is widely used in modern electrical and electronic systems.
Major applications include:

- Electric generators
- Transformers
- Induction motors
- Bicycle dynamos
- Induction cooktops
- Wireless charging systems
- Metal detectors
- Electromagnetic brakes
- Induction furnaces
- Electric guitar pickups
- RFID systems

6.38 Electric Generator (Basic Principle)

An electric generator converts **mechanical energy into electrical energy**.

Principle

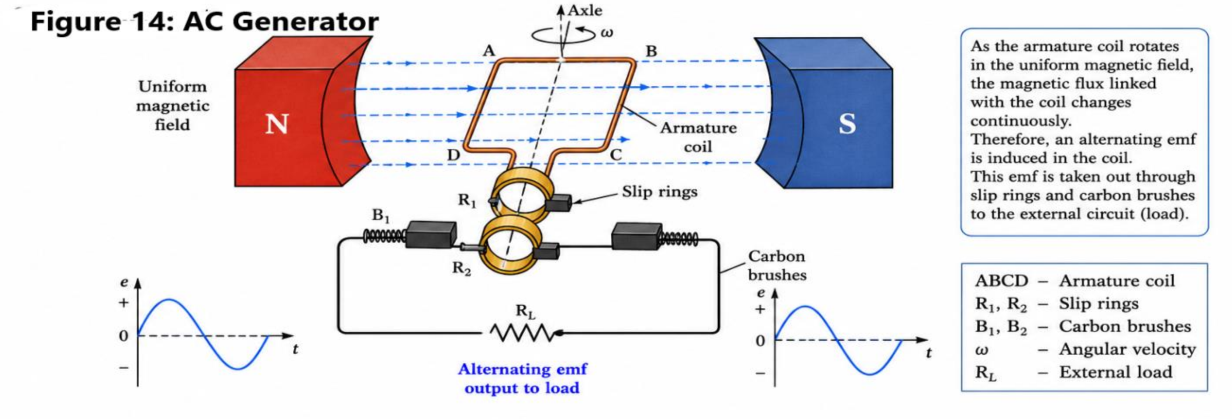
When a coil rotates in a magnetic field, the magnetic flux linked with the coil changes continuously.
According to Faraday's Law, an emf is induced.
The detailed construction and working of AC and DC generators will be studied in later chapters.
It works on the principle of electromagnetic induction.

6.39 Why Electromagnetic Induction is Important

Electromagnetic induction has revolutionised power generation and transmission.
Without EMI:

- Power stations would not generate electricity.
- Transformers would not work.
- Long-distance power transmission would be impractical.
- Many modern electrical devices would not function.

Construction Diagram:



Derivation

To obtain the output voltage in AC generator and to prove that

$$E_m = NBA\omega$$

Let an armature coil has N number of turns, area of its cross-section A and moving in the uniform magnetic field $\vec{B}$. It makes an angle θ between magnetic field $\vec{B}$ and area vector which is normal to the plane of the coil.

The coil moves with angular speed ω and hence,

$$\theta = \omega t$$

According to Faraday's law of electromagnetic induction, Emf induced in the coil is

$$e = -N\frac{d\varphi}{dt} - - - (1)$$

Since, magnetic flux is

$$\varphi = BA\cos\theta = BA\cos\omega t - - - (2)$$

Therefore, equation (1) becomes as

$$e = -N\frac{d(BA\cos\omega t)}{dt} = -N(-BA\sin\omega t)(\omega)$$

Or,

$$e = NBA\omega\sin\omega t - - - -(3)$$

Or,

$$\boxed{e = E_m\sin\omega t - - - (4)}$$

Equation (4) is the instantaneous induced emf or output voltage from AC generator.

Where,

$$\boxed{E_m = NBA\omega}$$

Also, $\omega = 2\pi f$, therefore

$$\boxed{E_m = 2\pi f NBA}$$

6.40 Summary of Key Concepts

Throughout this chapter, we studied:

- Magnetic Flux
- Faraday's Laws
- Lenz's Law
- Motional EMF
- Fleming's Right-Hand Rule
- Eddy Currents **(Competitive / Application Enrichment)**
- Self-Induction
- Mutual Induction
- Inductance
- Energy Stored in an Inductor
- Practical Applications

6.41 Comparison Table

Electromagnetic Induction	Electrostatics
Produced by changing magnetic flux	Produced by electric charges
Unit of Flux: Weber	Unit of Charge: Coulomb
Faraday's Law	Coulomb's Law
Lenz's Law decides direction	Electric field is produced by electric charges; its direction at a point depends on the net electric field due to the source charges.

6.44 NEET Focus Points

✔ Definition of magnetic flux
✔ SI unit of Weber
✔ Faraday's Laws
✔ Lenz's Law
✔ Motional EMF
✔ Fleming's Right-Hand Rule
✔ Eddy currents
✔ Self-inductance
✔ Mutual induction
✔ Energy stored in an inductor

6.45 JEE Main & Advanced Focus

Students should be able to:

- Apply Faraday's Law in non-standard situations.
- Analyse changing area, changing magnetic field and changing orientation together.
- Solve multi-step numerical problems involving motional EMF.
- Apply energy conservation using Lenz's Law.
- Solve problems involving self and mutual inductance.

Common Mistakes

 Using Fleming's Left-Hand Rule instead of Right-Hand Rule for generators
 Forgetting the negative sign in Faraday-Lenz law
 Confusing magnetic flux (Weber) with magnetic field (Tesla)
 Ignoring the angle between the magnetic field and the **normal** to the surface while calculating magnetic flux

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JEE Insight

Almost every problem in Electromagnetic Induction can be approached by asking:

1. **Is the magnetic flux changing?**
2. **Why is it changing?** (change in B, area or orientation)
3. **Which law applies?** (Faraday's Law + Lenz's Law)

Developing this approach helps students solve unfamiliar problems efficiently.

Teacher's Note

Concept First, Formula Next: Students should first understand *why* an emf is induced (change in magnetic flux) before memorising equations. Once the concept is clear, derivations, numerical and MCQs become much easier.
