PART–1: AC Fundamentals, AC Voltage and Current

7.1 Introduction

In many electrical systems, the direction and magnitude of current change periodically with time. Such current is called **alternating current (AC)**.

In contrast, direct current (DC) flows in one direction with constant polarity under ideal steady conditions.

Alternating current is especially important because AC voltage can be conveniently transformed to different voltage levels using transformers, making it suitable for electrical power transmission and distribution.

7.2 Periodic Current

A current is called **periodic** if its value repeats itself after a fixed interval of time.

The time after which the current repeats its complete pattern is called the **time period**, T.

$$\boxed{f = \frac{1}{T}}$$

Where

- f = frequency
- T = time period

SI Unit of Frequency

Hz

One hertz means one complete cycle per second.

7.3 Angular Frequency

The angular frequency of an AC quantity is related to its frequency by

$$\boxed{\omega = 2\pi f}$$

Therefore,

$$\boxed{\omega = \frac{2\pi}{T}}$$

Where

- ω = angular frequency
- f = frequency
- T = time period

Important NEET Point

Frequency and angular frequency are **not the same physical quantity**.

$$[\omega] = \text{rad s}^{-1}$$

While

$$[f] = \text{s}^{-1} = \text{Hz}$$

7.4 Sinusoidal Alternating Current

A sinusoidal alternating current can be represented mathematically as

$$\boxed{i = I_m \sin(\omega t + \varphi)}$$

Where

- i = instantaneous current
- Im = peak value of current
- ω = angular frequency
- t = time
- ϕ = initial phase

For zero initial phase,

$$\boxed{i = I_m \sin \omega t}$$

7.5 Instantaneous Value

The value of AC at a particular instant of time is called its **instantaneous value**.
For current:

$$i = I_m \sin \omega t$$

For voltage:

$$v = V_m \sin \omega t$$

Where Im and Vm are their respective peak values.

7.6 Peak Value

The maximum magnitude attained by an alternating current or voltage is called its **peak value** or **maximum value**.
For current:

$$\boxed{i_{max} = I_m}$$

For voltage:

$$\boxed{v_{max} = V_m}$$

7.7 Complete Cycle of AC

During one complete cycle, sinusoidal AC passes through:

- Positive maximum
- Zero
- Negative maximum
- Zero
- Positive maximum again

The complete cycle corresponds to an angular displacement of

$$\boxed{2\pi}$$

radians.

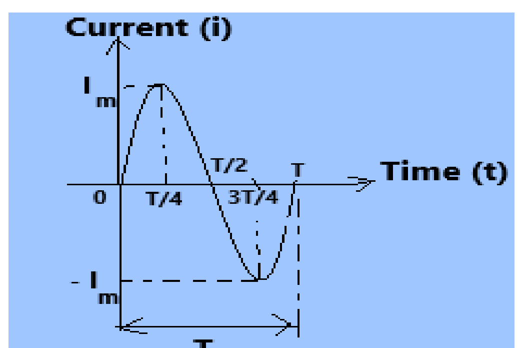


Figure 7.1: Sinusoidal AC current Wave

7.8 Phase

The quantity

$$\omega t \pm \varphi$$

is called the **phase** of the sinusoidal quantity.
For

$$i = I_m \sin(\omega t + \varphi)$$

the phase is

$$\boxed{\omega t + \varphi}$$

and ϕ is called the **initial phase** or **phase constant**.

7.9 Phase Difference

Suppose two alternating quantities are

$$v = V_m \sin(\omega t \pm \theta_1)$$

and

$$i = I_m \sin(\omega t \pm \theta_2)$$

Then their phase difference is

$$\boxed{\varphi = (\omega t \pm \theta_1) - (\omega t \pm \theta_2)}$$

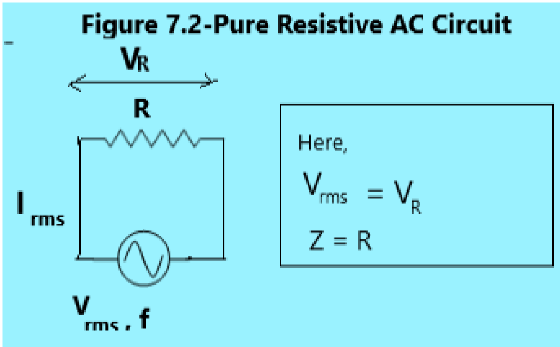
That is,

$$\boxed{Phase\ difference = Voltage\ Phase - Current\ Phase}$$

If ϕ = 0, they are **in phase**.
If ϕ = π /2, they are said to differ in phase by 90^0.

7.10 AC Voltage Applied to a Resistor

Consider a pure resistance R connected to an AC voltage.



Let

$$v = V_m \sin \omega t$$

By Ohm's law,

$$i = \frac{v}{R}$$

Therefore,

$$i = \frac{V_m}{R} \sin \omega t$$

Since

$$I_m = \frac{V_m}{R}$$

We obtain

$$\boxed{i = I_m \sin \omega t}$$

Thus, for a pure resistor:
Voltage and current are in phase

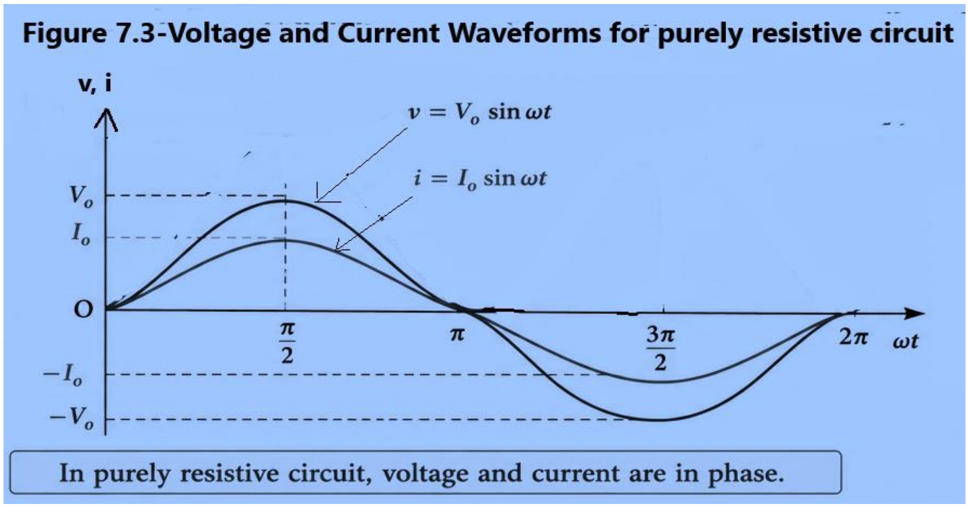
Important Result

For a pure resistance,

$$\boxed{\varphi = 0}$$

And therefore the power factor is unity:

$$\boxed{\cos\varphi = 1}$$



7.11 Peak Current in a Pure Resistor

From

$$V_m = I_m R$$

We get

$$\boxed{I_m = \frac{V_m}{R}}$$

This is the peak-current form of Ohm's law for a pure resistance.

7.12 RMS Value of AC

The **root mean square (RMS) value** of an alternating current is the value of steady DC current that would produce the same average heating effect in a resistor over a complete cycle.

For a sinusoidal current,

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

Similarly, for voltage,

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

7.13 Peak and RMS Values

Therefore,

$$\boxed{I_m = \sqrt{2} I_{rms}}$$

And

$$\boxed{V_m = \sqrt{2} V_{rms}}$$

NEET Shortcut

Whenever a sinusoidal AC value is given as RMS value:

$$\boxed{Peak\ Value = \sqrt{2} \times RMS\ value}$$

Whenever peak value is given:

$$\boxed{RMS\ value = \frac{Peak\ value}{\sqrt{2}}}$$

7.14 Average Value of AC

For a complete cycle of a sinusoidal current,

$$\boxed{I_{avg} = 0}$$

because the positive and negative halves cancel each other.
However, the average value over a **positive half-cycle** is not zero.
For

$$i = I_m \sin \omega t$$

the average value over the positive half-cycle is

$$\boxed{I_{avg} = \frac{2I_m}{\pi}}$$

or approximately

$$\boxed{I_{avg} \approx 0.637 I_m}$$

7.15 Average Voltage

Similarly, for a sinusoidal voltage, the average value over the positive half-cycle is

$$\boxed{V_{avg} = \frac{2V_m}{\pi}}$$

Therefore,

$$\boxed{V_{avg} \approx 0.637 V_m}$$

7.16 Important Comparison

Quantity	Complete Cycle	Positive Half-Cycle
Average current	0	$2I_m/\pi$
Average voltage	0	$2V_m/\pi$
RMS current	$I_m/\sqrt{2}$	Not normally used in this form
RMS voltage	$V_m/\sqrt{2}$	Not normally used in this form

Form factor:
It is defined as the ratio of rms current/voltage and the average current/voltage.

$$\boxed{Form\ factor = \frac{I_{rms}}{I_{avg}} = 1.11}$$

Crest factor
It is the ratio of maximum value of current/voltage and maximum value of current/voltage.

$$\boxed{Crest\ Factor = \frac{Peak\ Value}{RMS\ Value} = 1.414}$$

7.17 Power in a Pure Resistor
For an AC current through a resistance,

$$p = i^2R$$

The average power over a complete cycle is

$$\boxed{P_{avg} = {I_{rms}}^2R = \frac{1}{2}{I_m}^2R}$$

Using

$$V_{rms} = I_{rms}R$$

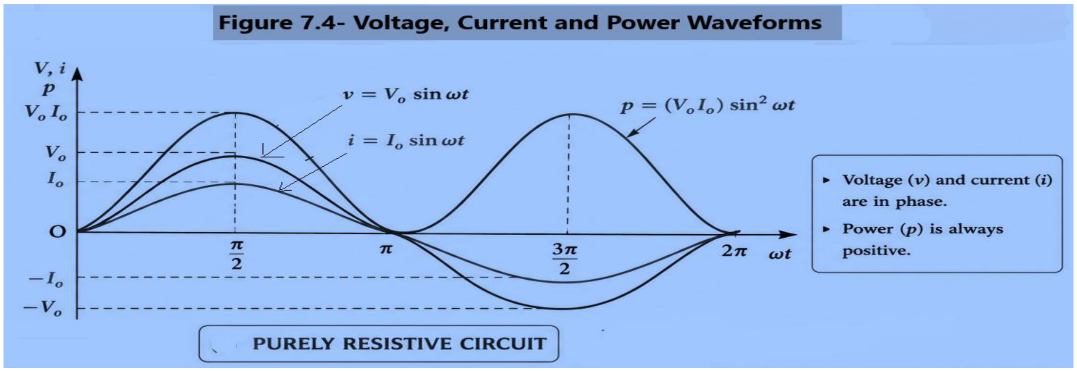
We can also write

$$\boxed{P_{avg} = V_{rms}I_{rms}}$$

And

$$\boxed{P_{avg} = \frac{{V_{rms}}^2}{R}}$$

(We can take $V_0 = V_m$ and $I_0 = I_m$)



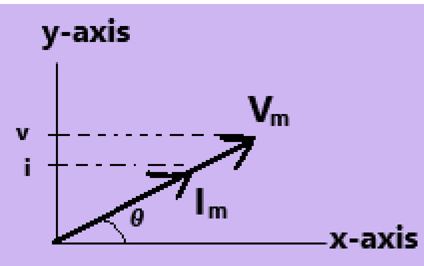


Figure 7.5-Phasor Diagram in Purely Resistive Circuit

Board Examination Point

For a pure resistor:

$$\boxed{P_{avg} = V_{rms} I_{rms}}$$

Because voltage and current are in phase.

Solved Numerical 7.1

An AC voltage has an RMS value of 220V. Find its peak value.

Solution

$$V_m = \sqrt{2} \times V_{rms} = 1.414 \times 220 \approx 311\ V$$

Solved Numerical 7.2

A sinusoidal current has a peak value of 10A. Find its RMS value.

Solution

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} \approx 7.07\ A$$

Solved Numerical 7.3

A 100Ω resistor is connected to an AC supply of 200V RMS. Find the average power consumed.

Solution

$$P_{avg} = \frac{{V_{rms}}^2}{R} = \frac{200 \times 200}{100} = 400\ W$$

NEET / JEE Concept Check

Question-1

An AC current has peak value Im. What is its RMS value?

Answer:

$$\boxed{\frac{I_m}{\sqrt{2}}}$$

Question-2
What is the average value of a sinusoidal AC over a complete cycle?
Answer:

$$\boxed{0}$$

Question-3
For a pure resistor, what is the phase difference between current and voltage?
Answer:
0

Derivation-1: Derive the relation between RMS value of current and Maximum current
Answer:
Since, root mean square current is the square root of mean of square of instantaneous current.
Let instantaneous current of a sinusoidal wave is

$$i = I_m \sin \omega t ----(1)$$

The rms current is

$$I_{rms} = \sqrt{average\ of\ i^2} ----(2)$$

Squaring both sides of equation (2), we get

$$I_{rms}{}^2 = mean\ value\ of\ i^2 --- (3)$$

RMS current is taken over one complete ac cycle of sinusoidal wave.
Therefore,

$$I_{rms}{}^2 = \frac{\int_0^T i^2 dt}{\int_0^T dt} = \frac{\int_0^{\frac{2\pi}{\omega}} I_m{}^2 sin^2\omega t\ dt}{\int_0^{\frac{2\pi}{\omega}} dt} = \left(\frac{I_m{}^2}{\frac{2\pi}{\omega}}\right)\left[\int_0^{\frac{2\pi}{\omega}} \frac{(1 - cos2\omega t)}{2} dt\right]$$

Since,

$$T = \frac{2\pi}{\omega}$$

And,

$$\cos 2\omega t = 1 - 2sin^2\omega t$$

Therefore,

$$I_{rms}{}^2 = \left(\frac{I_m{}^2}{2 \times \frac{2\pi}{\omega}}\right)\left(\int_0^{\frac{2\pi}{\omega}} dt - \int_0^{\frac{2\pi}{\omega}} cos2\omega t\ dt\right) = \left(\frac{I_m{}^2}{2 \times \frac{2\pi}{\omega}}\right)\left(\frac{2\pi}{\omega} - \frac{sin4\pi}{2\omega}\right)$$

Thus, we get

$$I_{rms}{}^2 = \left(\frac{I_m{}^2}{2 \times \frac{2\pi}{\omega}}\right)\left(\frac{2\pi}{\omega}\right) = \frac{I_m{}^2}{2} ----(4)$$

Taking square root of equation (4) both sides, we have

$$I_{rms} = \frac{I_m}{\sqrt{2}} ----(5)$$

Since,

$$\int \cos 2\omega t\ dt = \frac{\sin 2\omega t}{2\omega}\ over\ 0\ to\ \frac{2\pi}{\omega} and \sin 4\pi = 0$$

Hence,

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m}$$

Similarly, it can be derived for rms value of voltage as

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707 V_m}$$

Derivation -2: Derive the relation between average value of current and maximum current over half cycle of AC sinusoidal wave

Answer:

Using the formula for average value of a physical quantity say, A with respect to time, is

$$A_{avg} = \frac{\int_{t_1}^{t_2} A dt}{\int_{t_1}^{t_2} dt}$$

Since, average value of current and voltage over one complete cycle is zero. So, average current over half cycle is

$$I_{avg} = \frac{\int_0^{\frac{T}{2}} i dt}{\int_0^{\frac{T}{2}} dt} ----(1)$$

Let instantaneous current is

$$i = I_m \sin \omega t ----(2)$$

Putting the value of i in equation (1), we have

$$I_{avg} = \frac{\int_0^{\frac{T}{2}} I_m \sin \omega t \, dt}{\int_0^{\frac{T}{2}} dt} = \left(\frac{I_m}{\omega}\right) \frac{(-cos\ \omega t)_0^{\frac{\pi}{\omega}}}{(t)_0^{\frac{\pi}{\omega}}} = \left(\frac{I_m}{\omega}\right)\left(\frac{-\cos \pi + \cos 0^0}{\frac{\pi}{\omega}}\right) = \frac{2I_m}{\pi}$$

Since

$$\cos \pi = -1 \ and \ \cos 0^0 = 1$$

Therefore,

$$\boxed{I_{avg} = \frac{2I_m}{\pi} = 0.637 I_m}$$

Similarly,

Average value of voltage over half cycle is

$$\boxed{V_{avg} = \frac{2V_m}{\pi} = 0.637 V_m}$$

Derivation-3: Derive the average power expression in Purely Resistive ac circuit

Answer:

Since, instantaneous power of ac wave is

$$p = vi --- (1)$$

Let

$$v = V_m \sin \omega t$$

And

$$i = I_m \sin \omega t$$

Substituting the instantaneous value of voltage and current in equation (1), we get Instantaneous power, p is

$$p = V_m I_m sin^2 \omega t = V_m I_m \left(\frac{1 - \cos 2\omega t}{2}\right) - - - (2)$$

Now, average value of power will be as

$$P_{avg} = \frac{\int_0^T p\, dt}{\int_0^T dt} = V_m I_m \frac{\int_0^{\frac{2\pi}{\omega}} \left(\frac{1 - \cos 2\omega t}{2}\right) dt}{\int_0^{\frac{2\pi}{\omega}} dt} = \left(\frac{V_m I_m}{2}\right) \left(\frac{\left(t - \frac{sin 2\omega t}{2\omega}\right)_0^{\frac{2\pi}{\omega}}}{(t)_0^{\frac{2\pi}{\omega}}} \right) = \frac{V_m I_m}{2}$$

Since, $sin 2\omega t$ over one complete cycle is zero.

Therefore

$$\boxed{P_{avg} = \frac{V_m I_m}{2} = \frac{1}{2}\left(\frac{{I_m}^2}{R}\right) = V_{rms} I_{rms}}$$

PART–2: Phasor, AC through a Pure Inductor and Inductive Reactance

7.18 Representation of AC Quantities by Phasor

A sinusoidally varying voltage or current can be represented conveniently by a rotating vector called a **phasor**.

In this chapter, phasor magnitudes are taken as peak values unless stated otherwise. RMS phasors may also be used by convention.

A phasor is a vector whose:

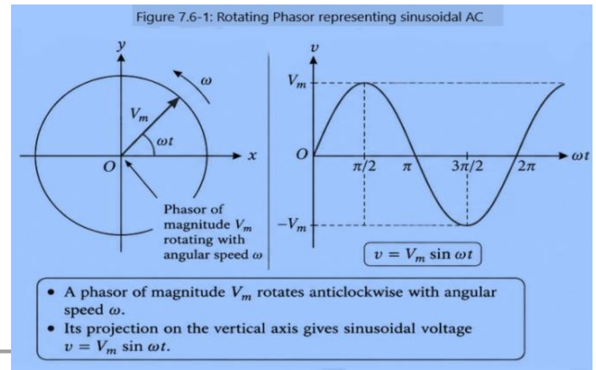
- angular speed represents the angular frequency,
- angular position represents its phase.

For example, consider

$$v = V_m \sin \omega t$$

The rotating vector associated with this voltage has magnitude Vm and rotates with angular velocity ω.

The projection of the rotating vector on a suitable axis gives the instantaneous value of the voltage.



7.19 Phase Relationship between Voltage and Current

The phase relationship between voltage and current is extremely important in AC circuits.

Depending on the circuit element:

Pure resistor

Voltage and current are **in phase**.

$$\varphi = 0^0$$

Pure inductor

Current **lags voltage by 90⁰**.

$$\varphi = \frac{\pi}{2}$$

Pure capacitor

Current **leads voltage by 90⁰**

$$\varphi = -\frac{\pi}{2}$$

The sign/direction of the phase relation must be interpreted carefully according to the chosen convention.

Note-Leading/Lagging AC circuit:

Let the phase angle be defined by

$$v = V_m \sin \omega t$$

And

$$i = I_m \sin(\omega t \pm \varphi)$$

Then ϕ > 0 means current lags voltage, while ϕ < 0 means current leads voltage.

Nature of AC circuit:

- If current leads the voltage by 90^0, the circuit is Purely Capacitive..
- If current lags the voltage by 90^0, the circuit is Purely Inductive.

- If current and voltage are in same phase, the circuit is Purely Resistive.

7.20 AC Voltage Applied to a Pure Inductor

Consider an ideal inductor of inductance L connected to an AC voltage source.

Derivation-4: Show that current lags the voltage by $\frac{\pi}{2}$

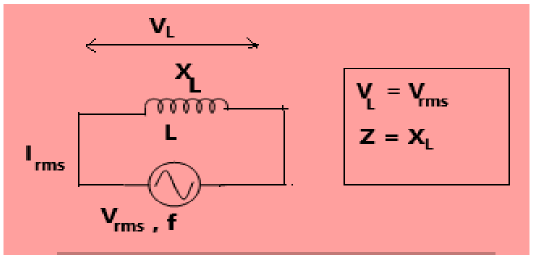


Figure 7.6: Purely Inductive circuit

Let the applied voltage be

$$v = V_m \sin \omega t$$

For an ideal inductor,

$$v = L\frac{di}{dt}$$

Or

$$di = \frac{1}{L} v dt \;---\; (1)$$

Integrating equation (1),

$$i = \int \frac{1}{L} v dt$$

Or

$$i = \frac{1}{L}\int V_m \sin \omega t \, dt = -\frac{V_m}{\omega L}\cos \omega t$$

Define the peak current as

$$I_m = \frac{V_m}{X_L} = \frac{V_m}{\omega L}$$

Hence,

$$i = -I_m \cos \omega t = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

This proves that current is behind the voltage by 90^0 or $\frac{\pi}{2}$.

Important Result

For a pure inductor:

Current lags voltage by 90^0.

Therefore,

$$\varphi = +90^0$$

Where, the current is taken as lagging behind the voltage.

7.21 Inductive Reactance

From

$$I_m = \frac{V_m}{\omega L}$$

We can write

$$V_m = I_m \omega L$$

Comparing this with Ohm's-law form,

$$V_m = I_m X_L$$

we obtain

$$X_L = \omega L$$

This quantity is called **inductive reactance**.

We can also write

$$X_L = 2\pi f L$$

7.22 SI Unit of Inductive Reactance

Inductive reactance has the same unit as resistance: Ω
Because

$$X_L = \frac{V_m}{I_m}$$

Therefore,
$[X_L] = \Omega$

7.23 Dependence of Inductive Reactance

From

$$X_L = 2\pi f L$$

We can see:

$$X_L \propto f$$

And

$$X_L \propto L$$

Therefore:

- increasing frequency increases inductive reactance;
- increasing inductance increases inductive reactance.

NEET Shortcut

If frequency becomes **double**:

$$X_L{}' = 2X_L$$

If inductance becomes **double**:

$$X_L{}' = 2X_L$$

If both frequency and inductance become double:

$$X_L{}' = 4X_L$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

7.24 Phase Difference in a Pure Inductive Circuit

For an ideal inductor:

$$v = V_m \sin \omega t$$

while

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Hence the current lags behind the voltage by 90^0.
Now,
Phase difference
$\varphi = voltage\ phase - current\ phase$
Therefore,

$$\varphi = \omega t - \left(\omega t - \frac{\pi}{2}\right)$$

Or

$$\varphi = \frac{\pi}{2}$$

radians

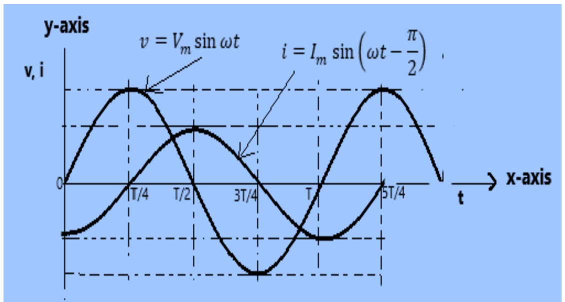


Figure 7.7: Voltage and Current waveforms in Purely inductive circuit

And, power factor = $\cos\frac{\pi}{2} = 0$

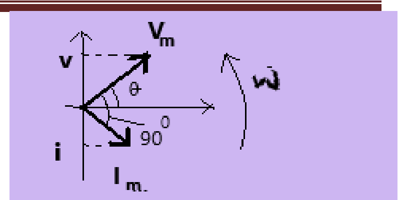


Figure 7.8: Phasor diagram of Pure L ac circuit

7.25 Phasor Diagram for a Pure Inductor

In a pure inductive circuit:

- voltage phasor leads current phasor by 90^0;
- equivalently, current phasor lags voltage phasor by 90^0.

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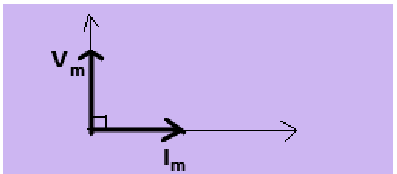
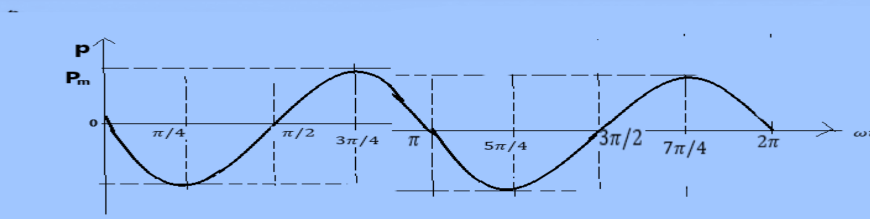


Figure 7.8-1Phasor of Pure L AC circuit

(Use either Figure 7.8 or Figure 7.8-1)

7.26 Instantaneous Power in a Pure Inductor

Figure 7.9: Instantaneous power variation in pure inductor



$P_m = \frac{V_m I_m}{2}$ is the maximum (or peak) value of instantaneous power.

Instantaneous power is

$$p - vi$$

For the pure inductor:

$$v = V_m \sin\omega t$$

And

$$i = I_m \sin(\omega t - 90^0) = -I_m \cos\omega t$$

Since
Therefore,

$$p = -V_m I_m \sin\omega t \cos\omega t = -\frac{V_m I_m}{2}\sin 2\omega t$$

Therefore, instantaneous power is

$$\boxed{p = -\frac{V_m I_m}{2}\sin 2\omega t}$$

Since,

$$\sin 2\omega t = 2\sin\omega t\cos\omega t$$

The power alternates between positive and negative values.

7.27 Average Power in a Pure Inductor

Over a complete cycle, the positive and negative contributions cancel.

Therefore,

$$\boxed{P_{avg} = 0}$$

Important Concept

An ideal inductor **does not consume average power** from the AC source.
A practical inductor has some resistance and therefore consumes some real power.
It temporarily stores energy in its magnetic field and then returns that energy to the circuit.

7.28 Energy Stored in an Inductor

The energy stored in an inductor carrying current i is

$$\boxed{U = \frac{1}{2} L i^2}$$

The maximum energy is therefore

$$\boxed{U = \frac{1}{2} L {I_m}^2}$$

When, $\omega t = 0^0 \Rightarrow \cos 0^0 = 1 \Rightarrow i = I_m$
This is an important conceptual connection between electromagnetic induction and alternating current.

7.29 Why Does an Inductor Oppose AC?

When current through an inductor changes, the magnetic flux linked with the coil changes.
According to Faraday's law, an induced emf is produced.
According to Lenz's law, this induced emf opposes the change in current.
Thus, an inductor offers **reactance** to alternating current.
This opposition is called **inductive reactance**.

$$\boxed{X_L = \omega L}$$

7.30 Effect of Frequency on a Pure Inductor

Since

$$X_L = \omega L = 2\pi f L$$

if f increases, X_L increases.
Therefore, a pure inductor offers:

- **small opposition to low-frequency AC**
- **large opposition to high-frequency AC**

NEET Concept

An ideal inductor behaves approximately like a short circuit for **DC** in the **steady state** because

$$f = 0$$

Therefore,

$$X_L = 0$$

For AC, however,

$$X_L = 2\pi f L$$

and hence the opposition depends on frequency.

7.31 Important Comparison: Resistor vs Inductor

Property	Pure Resistor	Pure Inductor
Opposition	Resistance R	Reactance X_L
Formula	R	$X_L=\omega L$
Phase difference	0^0	90^0
Current-voltage relation	In phase	Current lags
Average power	Non-zero	Zero for ideal inductor
Energy storage	No	Magnetic field
Frequency dependence	Ideal R independent of f	$X_L \propto f$

7.32 Solved Numerical 7.4

An inductor of inductance 0.20H is connected to a 50Hz AC supply. Find its inductive reactance.

Solution

Given:

$$X_L = 0.20\ H, f = 50\ Hz$$

We use:

$$X_L = 2\pi f L$$

Therefore,

$$X_L = 2 \times 3.14 \times 50 \times 0.2 = 62.8\ \Omega$$

Hence,

$X_L \approx 62.8\ \Omega$

7.33 Solved Numerical 7.5

An AC voltage of RMS value 100V and frequency 50Hz is applied to a pure inductor of 0.5H. Find the RMS current.

Solution

Inductive reactance:

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.5 = 50\ \pi\ \Omega$$

For RMS quantities:

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{100}{50\pi} = \frac{2}{\pi}\ A$$

Hence,

Answer: $I_{rms} = \frac{2}{\pi} \approx 0.637\ A$

7.34 Solved Numerical 7.6 — NEET Level

An ideal inductor has inductive reactance 100 Ω at a frequency of 50 Hz. Find its inductance.

We know:

$$X_L = 2\pi f L$$

Or

$$L = \frac{X_L}{2\pi f} --- (1)$$

Substituting the values, therefore,

$$L = \frac{100}{2\pi \times 50} = \frac{1}{\pi} \approx 0.318\ H$$

Hence,
L≈ 0.318 H

NEET/JEE Concept Points

1. If frequency increases, what happens to X_L?

$$X_L = 2\pi f L$$

Therefore, **X_L increases**.

---

2. In a pure inductor, which leads?

Voltage leads current by 90^0.
Equivalently:
Current lags voltage by 90^0.

---

3. Average power consumed by an ideal inductor?

$$P_{avg} = 0$$

---

4. Does an ideal inductor dissipate energy?

No. It **stores and returns energy**.

---

5. What is the opposition offered by an ideal inductor to DC in steady state?

For DC: `f=0`,
Hence:

$$X_L = 0$$

PART–3: AC Voltage Applied to a Pure Capacitor

7.35 AC Voltage Applied to a Pure Capacitor

Derivation-5:

Show that current leads the voltage by 90^0 in an ideal capacitor.

Consider an ideal capacitor of capacitance C connected to an AC voltage source.
Let the applied voltage be

$$v = V_m \sin \omega t ----(1)$$

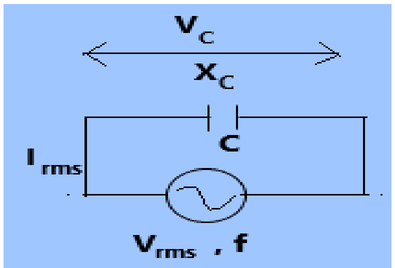


Figure 7.10: Purely Capacitive Circuit

For a capacitor,

$$q = CV ----(2)$$

Differentiating both sides of equation (2) with respect to time, we get

$$\frac{dq}{dt} = C\frac{dV}{dt} --- (3)$$

Since, current is rate of flow of charge,
Hence,

$$i = \frac{dq}{dt}$$

Therefore,

$$i = C\frac{dV}{dt} --- (4)$$

Putting the value of v from equation (1) in equation (4),
Hence,

$$i = C\frac{d(V_m \sin \omega t)}{dt}$$

Since C and Vm are constants,

$$i = CV_m\omega \cos \omega t = \frac{V_m}{\frac{1}{\omega C}} \cos \omega t$$

Since,
$X_C = \frac{1}{\omega C}$ and $\cos \omega t = \sin(90^0 + \omega t) = \sin(\omega t + 90^0)$ and $I_m = \frac{V_m}{X_C}$
Therefore, we obtain

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) --- (5)$$

This proves that current leads the voltage by 90^0 or $\pi/2$.

7.36 Important Result: Phase Relation

For a pure capacitor,

$$i = I_m \sin(\omega t + 90^0)$$

While

$$v = V_m \sin \omega t$$

Therefore, **Current leads voltage by 90^0.** Or **Voltage lags current by 90^0.**
Hence,

$$\varphi = -90^0$$

This is one of the most important facts for NEET/JEE questions on AC.

The instantaneous voltage and current equations are as:
$v = V_m \sin \omega t$
$i = I_m \sin(\omega t + \pi/2) = I_m \cos \omega t$
When voltage is zero, current is maximum and
when current is zero, voltage is maximum.
A table is shown below how to plot this corresponding graph.

ωt	t	$i = I_m \cos \omega t$	$v = V_m \sin \omega t$
0	0	I_m	0
$\pi/2$	T/4	0	V_m
π	T/2	$-I_m$	0
$3\pi/2$	3T/4	0	$-V_m$
2π	T	I_m	0

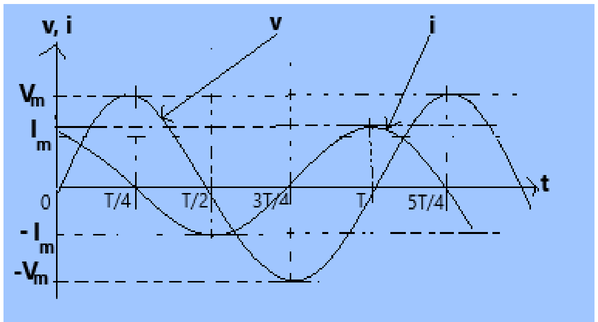


Figure 7.11-Voltage and Current waveforms in pure capacitor

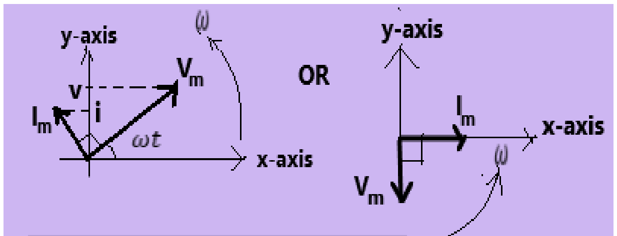


Figure 7.12: Phasor diagram of purely Inductive Circuit

7.37 Capacitive Reactance

From

$$I_m = V_m \omega C$$

We get

$$V_m = \frac{I_m}{\omega C}$$

Comparing with the AC form

$$V_m = I_m X_C$$

We obtain

$$X_C = \frac{1}{\omega C}$$

7.38 Meaning of Capacitive Reactance

Definition:

The opposition offered by a capacitor to alternating current is called **capacitive reactance**.
It is represented by X_C.

$$X_C = \frac{1}{2\pi f C}$$

Its SI unit is: Ω

7.39 Dependence of Capacitive Reactance

From

$$X_C = \frac{1}{2\pi f C}$$

We obtain:

$$X_C \propto \frac{1}{f}$$

And

$$X_C \propto \frac{1}{C}$$

Therefore:

- increasing frequency **decreases** capacitive reactance;
- increasing capacitance **decreases** capacitive reactance.

NEET Shortcut

For capacitor:

$$X_C \propto \frac{1}{C}$$

For inductor:

$$X_L \propto L$$

So remember:
Inductor → frequency increases → opposition increases
Capacitor → frequency increases → opposition decreases

7.40 RMS Current Through a Pure Capacitor

For RMS quantities:

$$I_{rms} = \frac{V_{rms}}{X_C}$$

Using

$$X_C = \frac{1}{\omega C}$$

We get

$$I_{rms} = V_{rms}\omega C$$

Therefore:

$$\boxed{I_{rms} = \omega C V_{rms}}$$

7.41 Effect of Frequency

Since

$$X_C = \frac{1}{2\pi f C}$$

If frequency increases, capacitive reactance decreases.
Therefore a capacitor offers:

- relatively large opposition to low-frequency AC;
- relatively small opposition to high-frequency AC.

7.42 Capacitor in a DC Circuit

For DC:

$$f = 0$$

Therefore the limiting behaviour of capacitive reactance is:

$$X_C \to \infty$$

Thus, after the capacitor becomes fully charged in a steady DC circuit, it behaves as an **open circuit**.

Important

For an ideal capacitor:

DC → blocked in steady state

AC → can pass, with reactance depending on frequency

7.43 Capacitor as a Frequency-Dependent Element

Since

$$X_C = \frac{1}{2\pi f C}$$

A capacitor allows higher-frequency variations more readily than lower-frequency variations. This property is important in:

- filtering circuits,
- coupling circuits,
- signal processing,
- electronic circuits.

7.44 Instantaneous Power in a Pure Capacitor

Instantaneous power is

$$p = vi$$

For the capacitor:

$$v = V_m \sin \omega t$$

and

$$i = I_m \cos \omega t$$

Therefore,

$$p = \frac{V_m I_m}{2} \sin 2\omega t$$

Thus, instantaneous power continuously changes between positive and negative values.

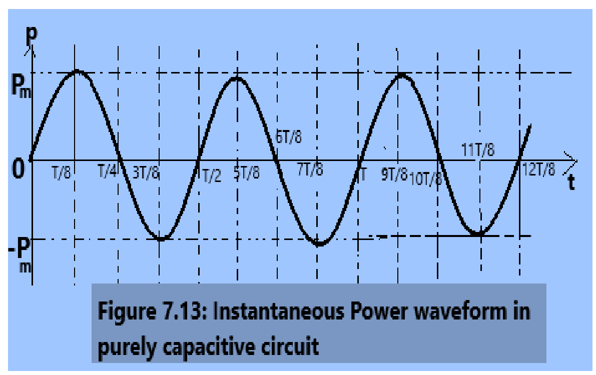


Figure 7.13: Instantaneous Power waveform in purely capacitive circuit

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7.45 Average Power in a Pure Capacitor

Over a complete cycle, the positive and negative contributions cancel.

Therefore:

$$\boxed{P_{avg} = 0}$$

Thus, an ideal capacitor consumes **zero average power** from an AC source.

7.46 Energy Stored in a Capacitor

The energy stored in a capacitor charged to potential difference V is

$$U = \frac{1}{2} C v^2$$

For maximum voltage V_m:

$$U = \frac{1}{2} C {V_m}^2$$

The capacitor alternately stores energy in its electric field and returns it to the circuit.

7.47 Pure Capacitor: Complete Summary

Quantity	Pure Capacitor
Voltage	$v = V_m \sin \omega t$
Current	$i = I_m \sin(\omega t + 90^0)$
Phase relation	Current leads voltage by 90^0
Reactance	$X_C = 1/\omega C$
Frequency dependence	$X_C \propto 1/f$
RMS current	$I_{rms} = V_{rms}/X_C$
Average power	0
Energy storage	Electric field
DC steady state	Open circuit

7.48 Inductor vs Capacitor — Very Important

Feature	Pure Inductor	Pure Capacitor
Reactance	$X_L = \omega L$	$X_L = 1/\omega C$
Frequency effect	Increases with f	Decreases with f
Current relation	Current lags voltage	Current leads voltage
Phase difference	90^0	90^0
Average power	0	0
Energy stored	Magnetic field	Electric field
Steady DC	Short-circuit behaviour	Open-circuit behaviour

7.49 Solved Numerical 7.7

A capacitor of capacitance 10μF is connected to an AC source of frequency 50Hz. Calculate its capacitive reactance.

Given

$C = 10\mu F = 1 \times 10^{-5} F, f = 50\ Hz$

Using

$$X_C = \frac{1}{2\pi f C}$$

Therefore,

$$X_C = \frac{1}{2\mu \times 50 \times 10^{-5}} = \frac{2 \times 500}{\pi} = 0.637 \times 500 = 318.3\ \Omega$$

Hence,

$X_C \approx 318.3\ \Omega$

7.50 Solved Numerical 7.8

An AC voltage of 100V RMS is applied to a capacitor whose capacitive reactance is 200Ω. Find the RMS current.

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{100}{200} = 0.5A$$

Therefore,
Hence,
Irms = 0.5A

7.51 Solved Numerical 7.9 — NEET Level

The frequency of an AC source is doubled while the capacitance remains unchanged. What happens to capacitive reactance?
Given,

$$f' = 2f$$

We know:

$$X_C = \frac{1}{2\pi f C} - - - - (1)$$

Hence,

$$X_C{}' = \frac{1}{2\pi f' C} = \frac{1}{4\pi f C} - - - (2)$$

Dividing equation (2) by (1), we get

$$X_C{}' = \frac{X_C}{2}$$

Answer
So, capacitive reactance becomes **half**.

7.52 Solved Numerical 7.10 — NEET/JEE Level

The capacitance of a capacitor is doubled while the frequency of the AC source remains unchanged. What happens to the RMS current?
We know:

$$I_{rms} = \frac{V_{rms}}{X_C} = 2\pi f C V_{rms}$$

For constant ω and Vrms, and $C' = 2C$

$$I_{rms} \propto C$$

then

$$I_{rms}{}' = I_{rms} \times \frac{C'}{C} = 2I_{rms}$$

Answer: Double

7.53 Conceptual Question — Board/NEET

Why does capacitive reactance decrease with increasing frequency?
Because

$$X_C = \frac{1}{2\pi f C}$$

Therefore, X_C is inversely proportional to frequency.

As frequency increases, the capacitor charges and discharges more rapidly, so its opposition to the changing current becomes smaller.

7.54 Conceptual Question

Why does a capacitor block steady DC but allow AC?

For steady DC, after charging is complete, the voltage across the capacitor becomes constant and the current becomes zero.
For AC, the voltage continuously changes with time, so the capacitor continuously charges and discharges, allowing alternating current to flow.

7.55 Important Board/JEE Point

Do not say simply:
"A capacitor has infinite resistance."
The more accurate statement is:
An ideal capacitor has infinite capacitive reactance in the zero-frequency/steady-DC limit and therefore behaves as an open circuit in steady state.
For AC:

$$X_C = \frac{1}{2\pi f C}$$

and it is finite for f > 0.

PART–4: Series LCR Circuit, Impedance, Phase Angle & Resonance

7.56 Series LCR Circuit

A **series LCR circuit** consists of:

- Resistance R
- Inductance L
- Capacitance C

connected in series with an AC source.
The same current flows through all three components.

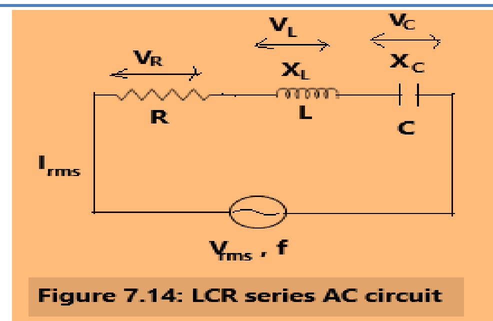


Figure 7.14: LCR series AC circuit

7.57 Reactance in a Series LCR Circuit

The three circuit elements offer different types of opposition.

Resistance, R

Inductive reactance, X_L

Capacitive reactance, X_C

The net reactance is

$$X = X_L - X_C$$

Therefore,

$$\boxed{X = X_L - X_C}$$

Or

$$\boxed{X = \omega L - \frac{1}{\omega C}}$$

7.58 Phase Relationship in Series LCR Circuit

The voltage across the resistor is in phase with current.
The voltage across the inductor leads the current by 90∘.
The voltage across the capacitor lags the current by 90∘.
Therefore, the net reactive voltage is determined by the difference: $V_L - V_C$
The resultant applied voltage is obtained by phasor addition.

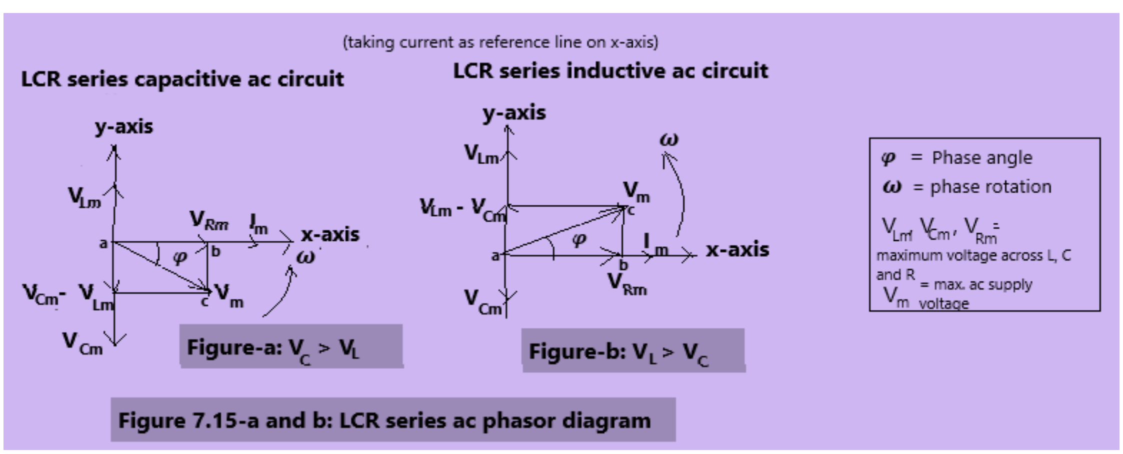


Figure 7.15-a and b: LCR series ac phasor diagram

7.59 Impedance of Series LCR Circuit

Definition:
The total opposition offered by a series LCR circuit to AC is called its **impedance**.
It is represented by Z.
Hence,

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

SI Unit: Ω

7.60 RMS Current in Series LCR Circuit

Using the AC equivalent of Ohm's law,

$$I_{rms} = \frac{V_{rms}}{Z}$$

Similarly, for peak values:

$$I_m = \frac{V_m}{Z}$$

Therefore,

$$\boxed{I_{rms} = \frac{V_{rms}}{Z}}$$

7.61 Phase Angle

From the phasor triangle,

$$\boxed{\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}}$$

7.62 Power Factor of Series LCR Circuit

The power factor is

$$\boxed{\cos\varphi = \frac{V_R}{V_{rms}} = \frac{R}{Z}}$$

Derivation-6

Derive resultant voltage, Impedance, Phase angle and Power factor from the Phasor diagram

Since,

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{V_m}{I_m}$$

(As $V_{rms} = \frac{V_m}{\sqrt{2}}$ and $I_{rms} = \frac{I_m}{\sqrt{2}}$)
Or

$$V_{rms} = I_{rms}Z - - - (1)$$

From Phasor diagram in Figure 7.15-a or b, In right angle triangle abc,

(i) Resultant or effective ac voltage

By Pythagoras Theorem,

$$ac^2 = ab^2 + bc^2$$

Or,

$$v_m{}^2 = V_{Rm}{}^2 + (V_{Cm} - V_{Lm})^2 - - - (2)$$

Since,

$$V_m = \sqrt{2}V_{rms}, V_{Rm} = \sqrt{2}V_R, V_{Cm} = \sqrt{2}V_C, V_{Lm} = \sqrt{2}V_L$$

Substituting these values in equation (2), we obtain

$$V_{rms}{}^2 = V_R{}^2 + (V_C - V_L)^2 - - - (3)$$

Or,

$$V_{rms}{}^2 = V_R{}^2 + (V_L - V_C)^2 - - - (4)$$

Or,

$$V_{rms} = \sqrt{V_R{}^2 + (V_L - V_C)^2} - - - (5)$$

Hence, effective or rms source voltage is

$$\boxed{V_{rms} = \sqrt{V_R{}^2 + (V_L - V_C)^2}}$$

(ii) Impedance of LCR series ac circuit

From equation (1), and (5),we have

$$I_{rms}Z = I_{rms}\sqrt{R^2 + (X_L - X_C)^2}$$

Or

$$Z = \sqrt{R^2 + (X_L - X_C)^2} - - - (6)$$

Hence,

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

(iii)Phase difference or Phase angle between voltage and current in LCR series ac circuit

From the phasor diagram, in triangle abc,

$$\tan\varphi = \frac{bc}{ab} = \frac{V_{Lm} - V_{Cm}}{V_{Rm}} = \frac{V_L - V_C}{V_R}$$

Therefore

$$\tan\varphi = \frac{V_L - V_C}{V_R} - - - (7)$$

Since,

$$V_R = I_{rms}R;\ V_L = I_{rms}X_L;\ V_C = I_{rms}X_C$$

So,

$$\tan\varphi = \frac{X_L - X_C}{R} - - - - (8)$$

Since,

$$X_L = \omega L;\ X_C = \frac{1}{\omega C}$$

Hence,

$$\boxed{\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}}$$

(iv) Power factor of the LCR series ac circuit

From the phasor diagram, in right angle triangle abc,

$$\cos\varphi = \frac{ab}{ac} = \frac{V_{Rm}}{V_m} = \frac{\sqrt{2}V_R}{\sqrt{2}V_{rms}} = \frac{V_R}{V_{rms}} = \frac{I_{rms}R}{I_{rms}Z} = \frac{R}{Z} = Powerfactor$$

Hence,

$$\boxed{Power\ Factor = \cos\varphi = \frac{V_R}{V_{rms}} = \frac{R}{Z}}$$

7.63 Nature of the Circuit

The nature of a series LCR circuit depends on the relative values of XL and XC.

Case 1: $X_L > X_C$

Then:

$$X_L - X_C > 0$$

The circuit is **inductive**.

Current **lags** voltage.

Case 2: $X_C > X_L$

Then:

$$X_C - X_L > 0$$

The circuit is **capacitive**.

Current **leads** voltage.

Case 3: $X_L = X_C$

Then:

$$X_L - X_C = 0$$

The circuit is at **resonance**.

Current and voltage are in phase.

7.64 Resonance in Series LCR Circuit

Resonance occurs when the inductive reactance becomes equal to the capacitive reactance.

Therefore,

$$X_L = X_C$$

Using:

$$X_L = \omega L$$

And

$$X_C = \frac{1}{\omega C}$$

We obtain:

$$\omega L = \frac{1}{\omega C}$$

This angular frequency $\omega = \omega_0$
Therefore,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Hence,

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

Where ω_0 is the **angular resonant frequency**.

7.65 Resonant Frequency

Since

$$\omega_0 = 2\pi f_0$$

We obtain:

$$2\pi f_0 = \frac{1}{\sqrt{LC}}$$

Therefore,

MOST IMPORTANT FORMULA

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

This is the **resonant frequency of a series LCR circuit**.

7.66 Impedance at Resonance

At resonance,

$$X_L = X_C$$

Therefore,

$$X_L - X_C = 0$$

Hence,

$$Z = \sqrt{R^2}$$

Therefore,

$$\boxed{Z = R}$$

Thus the impedance of a series LCR circuit is **minimum at resonance**.

7.67 Current at Resonance

Since

$$I_{rms} = \frac{V_{rms}}{Z}$$

and at resonance

$$Z = R$$

We get:

$$\boxed{I_{rms} = \frac{V_{rms}}{R}}$$

Therefore, current is **maximum at resonance**.
Similarly,

$$\boxed{I_m = \frac{V_m}{R}}$$

7.68 Phase Angle at Resonance

At resonance,

$$X_L = X_C$$

Therefore,

$$\tan\varphi = 0$$

Hence,

$$\varphi = 0$$

Therefore:

Voltage and current are in phase.

The power factor is:

$$\boxed{\cos\varphi = 1}$$

7.69 Important Characteristics at Resonance

At resonance:

$$X_L = X_C$$

$$Z = R = \text{Minimum}$$

$$Power\ factor = 1$$
$$Current\ is\ Maximum$$

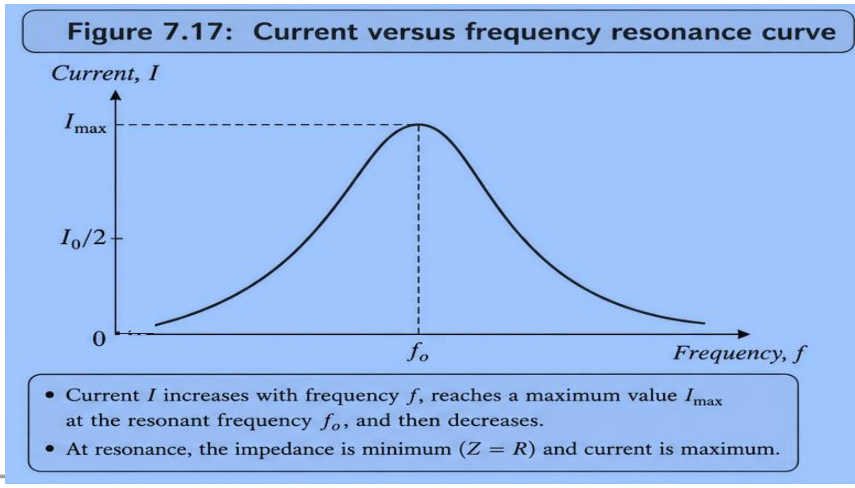
Thus, a series LCR circuit behaves like a **pure resistance** at resonance.

7.70 Resonance Curve

The current in a series LCR circuit is

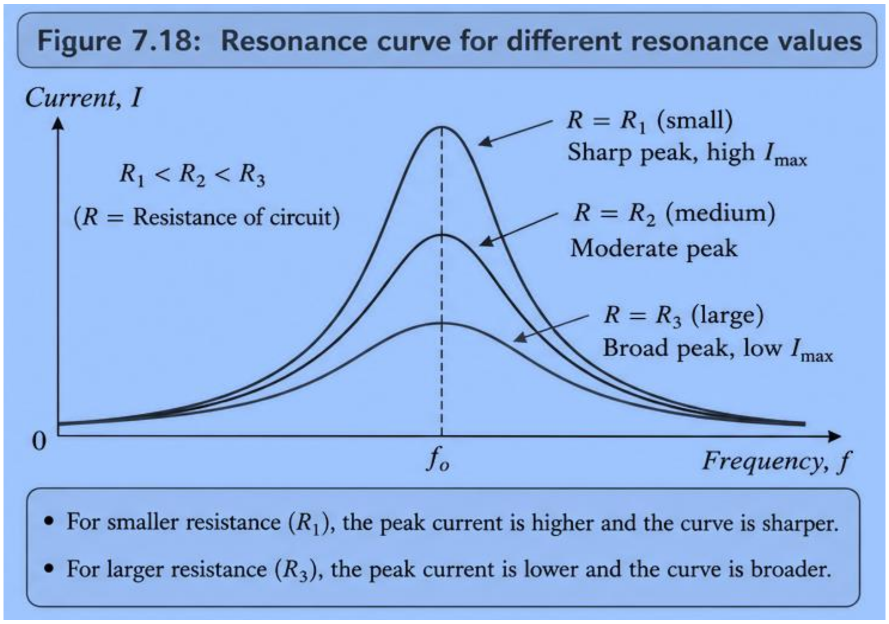
$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

At the resonant frequency f_0, the impedance is minimum and current is maximum.
As frequency moves away from f_0, current decreases.



7.71 Effect of Resistance on Resonance Curve

For a series LCR circuit, increasing resistance increases the minimum impedance and therefore decreases the maximum current.
The resonance curve becomes broader and less sharp.
For smaller resistance, the resonance peak is sharper.



7.72 Selectivity (NEET/JEE Competitive Enrichment)

The ability of a resonant circuit to strongly respond to a particular frequency while suppressing frequencies away from it is called **selectivity**.
A sharper resonance curve indicates better selectivity.
Therefore:
Lower resistance → sharper resonance → better selectivity
This principle is important in tuning circuits.

7.73 Quality Factor (NEET/JEE Competitive Enrichment)

The quality factor Q describes the sharpness of resonance.
For a series LCR circuit,

$$Q = \frac{\omega_0 L}{R}$$

Using the resonance condition,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

We obtain:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Thus,

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

A larger Q means a **sharper resonance**.

7.74 Bandwidth (NEET/JEE Competitive Enrichment)

The bandwidth of a resonant circuit is related to the difference between the upper and lower half-power frequencies.
If these frequencies are f_2 and f_1, then:

$$\Delta f = f_2 - f_1$$

For a series LCR circuit,

$$Q = \frac{f_0}{\Delta f}$$

Therefore:

$$\Delta f = \frac{f_0}{Q} = \frac{f_0}{\omega_0 L/R} = \frac{f_0 R}{2\pi f_0 L} = \frac{R}{2\pi L}$$

Hence,

$$\boxed{\Delta f = \frac{R}{2\pi L}}$$

NEET/JEE Insight
High Q → narrow bandwidth → sharp resonance → good selectivity.

7.75 Half-Power Frequencies (NEET/JEE Competitive Enrichment)

At the half-power frequencies, the current is:

$$I = \frac{I_m}{\sqrt{2}}$$

Since power is proportional to I^2,

$$P = \frac{P_m}{2}$$

These two frequencies define the bandwidth.

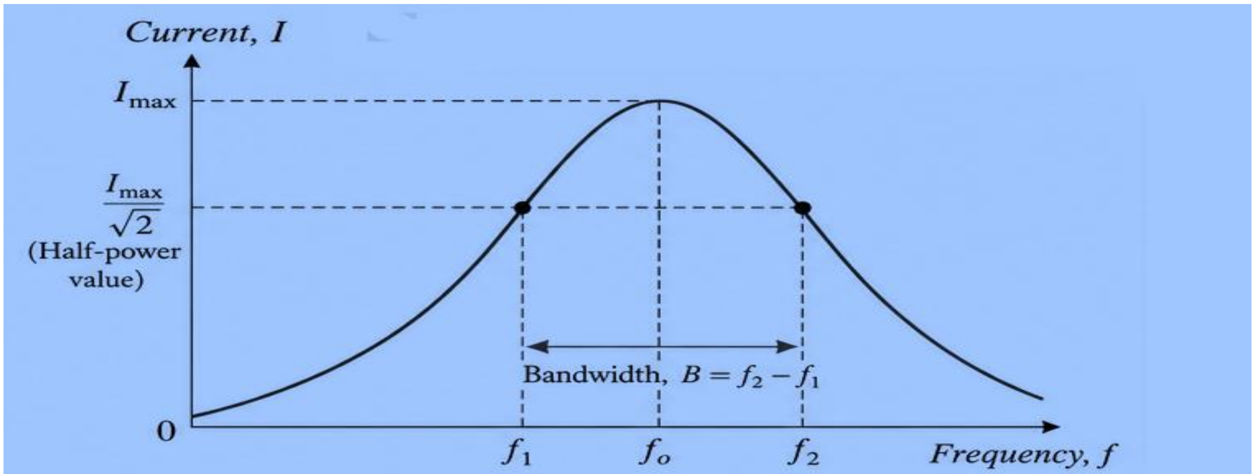


Figure 7.19: Current versus frequency half power points and bandwidth

7.76 Series LCR Circuit — Important Relationships

Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Current

$$I_{rms} = \frac{V_{rms}}{Z}$$

Phase angle

$$\tan\varphi = \frac{X_L - X_C}{R}$$

Power factor

$$\cos\varphi = \frac{R}{Z}$$

Resonance

$$X_L = X_C$$

Resonant angular frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

7.77 Solved Numerical — Impedance

A series LCR circuit has: R = 10 Ω, X_L = 30 Ω, X_C = 20 Ω. Find its impedance.

Solution

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Therefore,

$$Z = \sqrt{(10)^2 + (30 - 20)^2} = 10\sqrt{2}\,\Omega$$

Hence,
$Z = 10\sqrt{2}\,\Omega$
or approximately
Z≈14.14 Ω
Since $X_L > X_C$, the circuit is **inductive**.

7.78 Solved Numerical — Resonant Frequency

A series LCR circuit has: L= 0.1 H, and C=10 μF, Find the resonant frequency.
Using:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Or

$$f_0 = \frac{1}{2\pi\sqrt{0.1 \times 10 \times 10^6}} = \frac{1000}{2\pi} = 0.636 \times 250 = 159.2\ Hz$$

Therefore,

$$f_0 = 159.2\ Hz$$

7.79 Solved Numerical — Resonance

A series LCR circuit is at resonance at 500Hz. What happens to the phase angle?
At resonance:

$$X_L = X_C$$

Therefore,

$$\tan\varphi = \frac{X_L - X_C}{R} = 0$$

Hence,

$$\varphi = 0^0$$

Therefore voltage and current are in phase.

7.80 Solved Numerical — Current at Resonance

An AC source of RMS voltage 100 V is connected to a series LCR circuit having resistance 20 Ω. At resonance, calculate the RMS current.
At resonance:

$$Z = R$$

Therefore,

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R}$$

Or

$$I_{rms} = \frac{100}{20} = 5\ A$$

Hence,
Irms = 5A

7.81 NEET/JEE High-Yield Concepts

Concept 1
At resonance, which quantity is minimum?
Impedance
Zmin = R

Concept 2
At resonance, which quantity is maximum?
Current

$$I_{rms} = \frac{V_{rms}}{R}$$

Concept 3
At resonance, what is the power factor?
1

Concept 4
At resonance, what is the phase difference?
0

Concept 5
If L is increased while C remains constant: then what about resonant frequency?

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Therefore f_0 **decreases**.

Concept 6
If C is increased while L remains constant, then
F_0 decreases

Concept 7
If R is increased:

- maximum current decreases;
- resonance becomes less sharp;
- bandwidth increases;
- quality factor decreases.

7.82 Fast Revision Table

Quantity	Series LCR Circuit
Inductive reactance	$X_L = \omega L$
Capacitive reactance	$X_C = 1/\omega C$
Impedance	$Z = \sqrt{R^2 + (X_L - X_C)^2}$
Current	$I_{rms} = V_{rms}/Z$
Phase angle	$\tan\varphi = (X_L - X_C)/R$
Power factor	$\cos \quad = R/Z$
Resonance condition	$X_L = X_C$
Resonant angular frequency	$\omega_0 = 1/\sqrt{LC}$
Resonant frequency	$f_0 = 1/2\pi\sqrt{LC}$
Impedance at resonance	Z = R
Current at resonance	Maximum
Power factor at resonance	1

NEET/JEE Memory Box

Series LCR:

$\mathbf{X_L > X_C}$ → Inductive → Current **lags**
$\mathbf{X_C > X_L}$ → Capacitive → Current **leads**
$\mathbf{X_L = X_C}$ → Resonance → Current **maximum** → Z **minimum** → Power factor **1**
Higher Q → Sharper resonance → Smaller bandwidth → Better selectivity.
$\omega > \omega_0$ LCR circuit is Inductive.
$\omega < \omega_0$ → LCR circuit is capacitive.

Graphs in LCR AC Series Circuit

(a) Between resistance R and frequency (f): R is independent of frequency
(b) Between inductive reactance (X_L) and frequency (f): $X_L \propto f$
(c) Between capacitive reactance (X_C) and frequency (f): $X_C \propto 1/f$
(d) Between impedance (Z) and frequency (f): $Z = \sqrt{R^2 + (X_L - X_C)^2}$

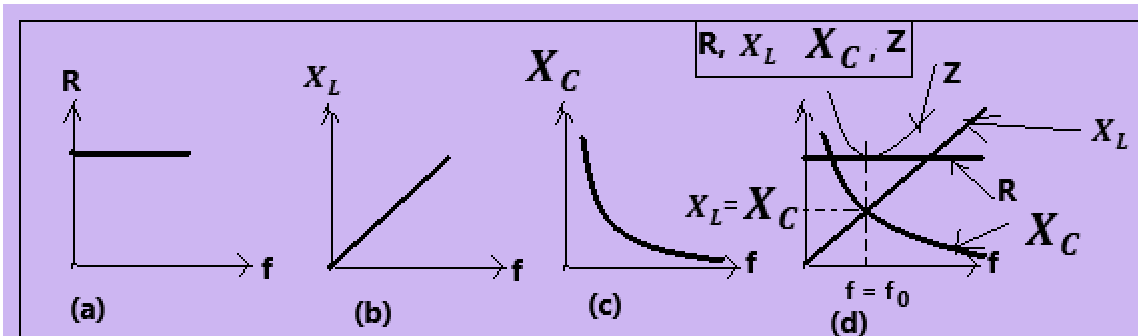


Figure-7.19-1: Graphs between circuit elements and frequency

PART–5: POWER IN AC CIRCUIT, POWER FACTOR & TRANSFORMER

7.56 Power in an AC Circuit

Derivation:

Consider an AC circuit in which the instantaneous voltage and current are

$$v = V_m \sin \omega t$$

And

$$i = I_m \sin(\omega t + \varphi)$$

Where, ϕ is the phase difference between current and voltage.
The instantaneous power supplied to the circuit is

$$p = vi$$

Therefore,

$$p = V_m I_m \sin \omega t \sin(\omega t + \varphi)$$

Using the trigonometric identity

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

We obtain

$$p = \frac{V_m I_m}{2} [\cos \varphi - \cos(2\omega t + \varphi)]$$

Over one complete cycle, the average value of the second term is zero.
Hence,

$$P_{avg} = \frac{V_m I_m}{2} \cos \varphi$$

Since

$$V_{rms} = V_m/\sqrt{2}$$

And

$$I_{rms} = I_m/\sqrt{2}$$

We get:

$$\boxed{P_{avg} = V_{rms} I_{rms} \cos \varphi}$$

This is the **average power consumed by an AC circuit**.

7.57 Power Factor

It is the cosine of phase difference between voltage and current.
The quantity

$$\cos \varphi$$

is called the **power factor** of the AC circuit.
Therefore,

$$\boxed{Power\ Factor = \cos \varphi}$$

It has **no unit and no dimension**.

Physical Meaning

Power factor tells us how effectively the electrical power supplied to an AC circuit is converted into useful average power.

A high power factor means that a larger fraction of the apparent power contributes to useful average power.

7.58 Power Factor and Average Power of Different Circuits

Pure Resistor

For a pure resistor:

$$\varphi = 0^0$$

Therefore,

$$\cos\varphi = 1$$

Hence,

$$P_{avg} = V_{rms}I_{rms}$$

Pure Inductor

For a pure inductor:

$$\varphi = \frac{\pi}{2}$$

Therefore,

$$\cos\varphi = 0$$

Hence,

$$P_{avg} = 0$$

Pure Capacitor

For a pure capacitor:

$$\varphi = -\frac{\pi}{2}$$

Therefore,

$$\cos\varphi = 0$$

and

$$P_{avg} = 0$$

Thus both an ideal inductor and an ideal capacitor consume **zero average power**.

7.59 Power Factor of a Series LCR Circuit

For a series LCR circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

and

$$\cos\varphi = \frac{R}{Z}$$

Therefore,

$$\boxed{Power\ Factor = \frac{R}{Z}}$$

Hence the average power is

$$\boxed{P_{avg} = V_{rms}I_{rms}\cos\varphi}$$

Therefore:

VERY IMPORTANT

$$\boxed{P_{avg} = \frac{{I_m}^2 R}{2}}$$

Only the resistance consumes average electrical power in an ideal series LCR circuit.

7.60 Wattless Current

In a purely inductive or purely capacitive AC circuit:

$$\cos\varphi = 0$$

Therefore,

$$P_{avg} = 0$$

The current is then called **wattless current**, because it produces zero average power consumption.

Important

The current is not necessarily zero.
Rather,
Average power = 0, because voltage and current are 90∘ out of phase.

7.61 Apparent Power

The product

$$V_{rms} I_{rms}$$

is called **apparent power**.
It is represented by S.

FORMULA

$$S = V_{rms} I_{rms}$$

Its unit is **volt-ampere (VA)**.
Average/real power/actual power:

$$P = V_{rms} I_{rms} \cos\varphi$$

Thus,

$$P = S\cos\varphi$$

7.62 Power Triangle

For a series AC circuit:

$$P = V_{rms} I_{rms} \cos\varphi$$

and reactive power is associated with the quadrature component.
The power triangle is represented by:

$$S^2 = P^2 + Q^2$$

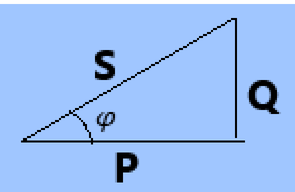


Figure 7.20: Power Triangle

Where:

- P = active/real power
- Q = reactive power
- S = apparent power

Thus,

$$\boxed{\cos\varphi = \frac{Actual\ Power}{Apparent\ Power}}$$

7.63 Why High Power Factor is Desirable

For a given real power:

$$P = V_{rms} I_{rms} \cos\varphi$$

Therefore,

$$I_{rms} = \frac{P}{V_{rms}\cos\varphi}$$

If the power factor is low, the current required for the same useful power becomes larger.
Larger current causes greater transmission losses because:

$$P = \frac{{I_m}^2 R}{2}$$

Hence, electrical power systems aim to maintain a **high power factor**.

7.64 Solved Numerical — Power Factor

An AC circuit has $V_{rms} = 200\ V$, $I_{rms} = 5\ A$, and $\cos\varphi = 0.8$. Find the average power.

Solution

$$P_{avg} = V_{rms} I_{rms} \cos\varphi$$

Or

$$P_{avg} = 200 \times 5 \times 0.8 = 800\ W$$

7.65 Solved Numerical — Pure Inductor

An ideal inductor is connected to an AC source. What is its average power consumption?
For a pure inductor:

$$\cos\varphi = 0$$

Therefore:

$$P_{avg} = V_{rms} I_{rms} \cos\varphi = 0$$

7.66 TRANSFORMER

A transformer is an electrical device used to **increase or decrease AC voltage**.
It works on the principle of:

Mutual electromagnetic induction

A transformer cannot operate with steady DC in the normal transformer sense because a steady current does not produce continuously changing magnetic flux.

7.67 Basic Construction of a Transformer

An ideal transformer consists mainly of:

1. Primary coil
2. Secondary coil
3. Laminated soft-iron core

The primary coil is connected to the AC input.
The secondary coil provides the output voltage.

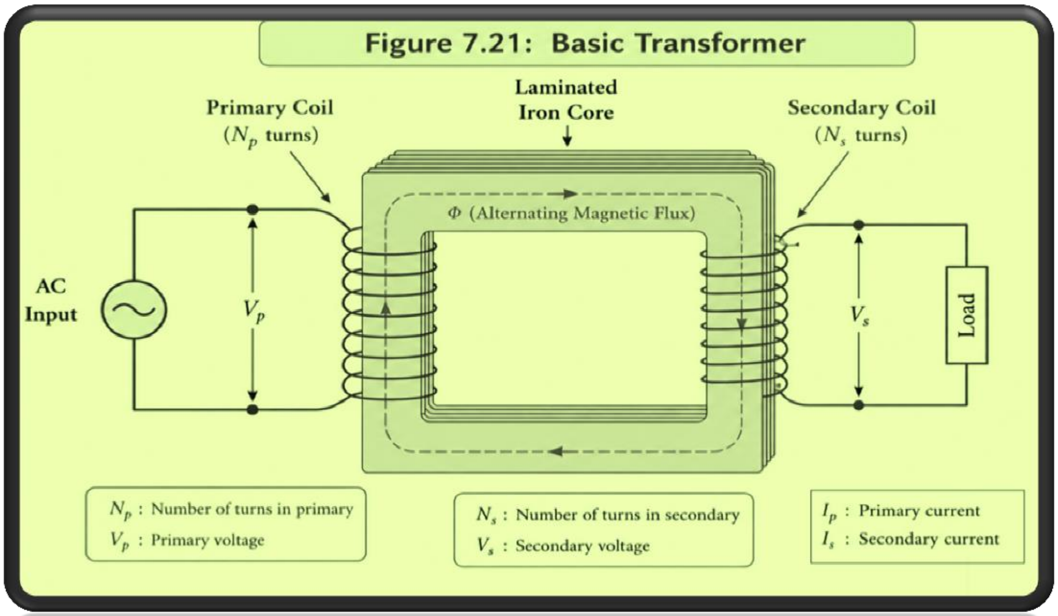


Figure 7.21: Basic Transformer

7.68 Working Principle

When an alternating voltage is applied to the primary coil, an alternating current flows through it.

This produces a changing magnetic flux in the iron core.

The changing flux links the secondary coil.

According to Faraday's law, an emf is induced in the secondary coil.

Thus electrical energy is transferred from the primary circuit to the secondary circuit through electromagnetic induction.

7.69 Transformer Equation

For an ideal transformer,

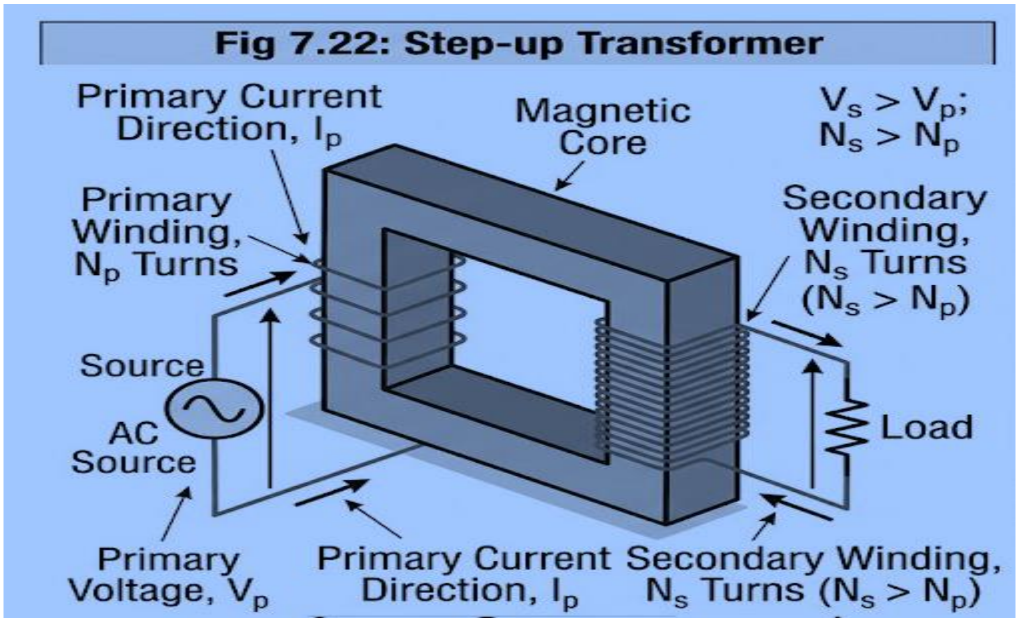
$$\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S}$$

Therefore:

$$\boxed{\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S}}$$

This is one of the most important transformer relations for NEET/JEE.

7.70 Step-Up Transformer



If

$$N_S > N_P$$

then

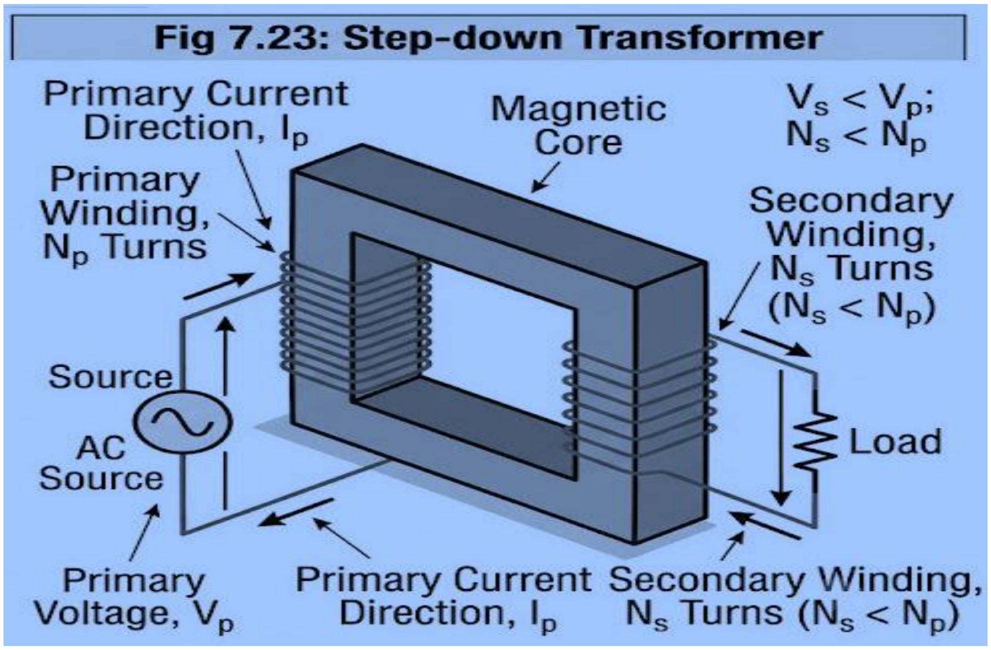
$$V_S > V_P$$

Such a transformer is called a **step-up transformer**.

Uses

- Increasing voltage for transmission of electrical power.
- High-voltage applications.

7.71 Step-Down Transformer



If

$$N_S < N_P$$

Then

$$V_S < V_P$$

Such a transformer is called a **step-down transformer**.

Uses

- Reducing voltage for suitable electrical/electronic devices.
- Power distribution.

7.72 Current Relation in an Ideal Transformer

For an ideal transformer, input power equals output power:

$$V_P I_P = V_S I_S$$

Therefore,

$$\frac{V_S}{V_P} = \frac{I_P}{I_S}$$

Using the voltage-turn relation:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

We obtain:

$$\boxed{\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S}}$$

Thus:

Voltage ratio is directly proportional to turns ratio.
Current ratio is inversely proportional to turns ratio.

7.73 Important Transformer Relations

For an ideal transformer:

$$\frac{V_S}{V_P} = \frac{I_P}{I_S}$$

Therefore,

$$\boxed{V_S I_S = V_P I_P}$$

7.74 Transformer Efficiency

In a practical transformer, some energy is lost.
Efficiency is:

$$\eta = \frac{Output\ Power}{Input\ Power} \times 100$$

Therefore,

$$\eta = \frac{V_S I_S}{V_P I_P} \times 100 - - - (1)$$

For an ideal transformer:

$$\eta = 100\%$$

A practical transformer has:

$$\eta < 100\%$$

If transformer is not ideal and let it is having efficiency η

In this case, from the equation (1) above for transformer efficiency,

$$V_S I_S = \eta V_P I_P$$

Or,

$$\boxed{\frac{V_S}{V_P} = \eta \frac{I_P}{I_S}}$$

7.75 Major Energy Losses in a Practical Transformer

1. Copper Loss

Resistance of the coils causes heating.

$$P = {I_{rms}}^2 R$$

Reduction: Use a thick copper wire.

2. Eddy Current Loss

Changing magnetic flux induces unwanted currents in the iron core.
Reduction: Use a laminated core.

3. Hysteresis Loss

Repeated magnetisation and demagnetisation of the core causes energy loss.
Reduction: Use suitable soft magnetic material.

4. Flux Leakage

Not all magnetic flux produced by the primary links the secondary.
Reduction: Proper core design and close magnetic coupling.

7.76 Why Transformer Core is Laminated

The changing magnetic flux induces eddy currents in the conducting core.
Laminating the core increases the electrical resistance of the possible eddy-current paths and therefore reduces eddy-current losses.

□ Board Question

Why are transformer cores laminated?
Answer:
To reduce energy loss due to eddy currents.

7.77 Why Transformer Does Not Work on DC

A transformer requires continuously changing magnetic flux.
With steady DC after the switching transient:

$$\frac{d\varphi}{dt} = 0$$

Therefore, no continuously induced emf is produced in the secondary.
Hence:
A transformer is designed for AC, not steady DC.

7.78 Solved Numerical — Transformer

A transformer has 1000 turns in its primary coil and 200 turns in its secondary coil. If the primary voltage is 220V, calculate the secondary voltage.

Given

$N_S = 200,\ \ N_P = 1000,\ \ V_P = 220\ V$

Using:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Therefore,

$$V_S = \frac{N_S V_P}{N_P} = \frac{200 \times 220}{1000} = 44\ V$$

Hence,

Vs = 44V

It is a **step-down transformer**.

7.79 Solved Numerical — Current

An transformer of efficiency 90% changes 220V into 1100V. If the secondary current is 2A, find the primary current.

For transformer:

$$\frac{V_S}{V_P} = \frac{\eta \times I_P}{I_S}$$

Therefore,

$$I_P = \frac{V_S I_S}{\eta V_P} = \frac{1100 \times 2}{0.9 \times 220} = \frac{100}{9} = 11.11\ A$$

Hence,

Ip = 11.11A

For an ideal transformer, we obtain

$I_P = 10\ A$

Notice: In the case of an ideal transformer,

Voltage increased by factor 5, current decreased by factor 5.

7.80 NEET/JEE Concept Check

Question

A transformer has: $N_S = 5N_P$

Then: $V_S = 5V_P$

For an ideal transformer:

$$\frac{I_P}{I_S} = \frac{V_S}{V_P} = 5$$

Therefore:

$$I_P = 5I_S$$

7.81 Very Important Comparison

Feature	Step-Up	Step-Down
Turns	Ns > Np	Ns < Np
Voltage	Vs > Vp	Vs < Vp
Current	Is < Ip	Is > Ip
Ideal power	Same input/output	Same input/output

MASTER COMPARISON TABLE

Property	Resistor	Inductor	Capacitor
Reactance	—	$X_L = \omega L$	$X_C = 1/\omega C$
Phase	0^0	Current lags 90^0	Current leads 90^0
Power factor	1	0	0
Average power	Maximum	0	0
Energy	Heat	Magnetic field	Electric field
DC steady state	Conducts	Short-circuit behaviour	Open-circuit behaviour

MASTER RESONANCE TABLE

Quantity	At Resonance
X_L	X_C
Net reactance	0
Impedance	R
Current	Maximum
Phase angle	0
Power factor	1
Average power	Maximum
Voltage-current phase	In phase

NEET/JEE QUICK TRAPS

Trap 1

Increasing frequency in an inductor

$X_L \uparrow$

Trap 2

Increasing frequency in a capacitor

$X_C \downarrow$

Trap 3

At resonance, **current is maximum**, not minimum.

Trap 4

At resonance, **impedance is minimum**.

Trap 5

At resonance:

Z = R

not Z=0.

Trap 6
A pure L or pure C circuit has:
Pavg= 0
but its current is **not necessarily zero**.

Trap 7
For an ideal transformer:
Pin = Pout
but voltage and current individually need not remain the same.

Numerical For Practice

1. An LCR series AC circuit has resistance 10 Ω. The supply voltage is 200 V and frequency is 50 Hz. The circuit is in resonance. The inductance is $10/\pi$ mH.
   Determine
   (a) Current in the circuit
   (b) Potential difference across the capacitance
   (c) Power factor of the circuit
   (d) Potential difference across the resistance
2. In an R-C series AC circuit, the voltages across R and C are given 100 V each.
   Calculate
   (a) Power factor of the circuit
   (b) Resultant effective voltage
3. A transformer has turn ratio 100: 1 (secondary to primary). The efficiency of the transformer is 80%. If the primary current is 50 A,
   Calculate
   (a) Secondary current
   (b) Input power, if primary voltage is 200 V
   (c) Is the transformer Step up or Step down?
4. In an LCR series AC circuit, if capacitance is removed, current lags the voltage by 30^{0}. If inductance is removed, the current leads the voltage by the same phase angle. If the resistance of the circuit is 20 Ω and applied AC voltage is 200 V,
   Determine
   (a) The power factor of the circuit
   (b) Current in the circuit
   (c) Average power
5. In the circuit shown below:
6. The source voltage is 220 V. It draws maximum current 11 A.
   The values of inductance and capacitance are 200 mH and 2 μF respective
   Calculate
   (a) Value of resistance
   (b) Resonant frequency
   (c) Potential difference across L and C.

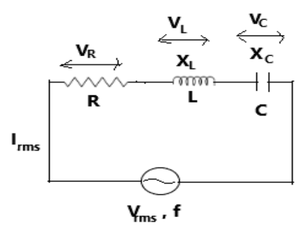


Figure 7.14: LCR series AC circuit

7. From the phasor diagram of LCR series AC circuit, $V_m = 100\sqrt{2}V$, $V_{Cm} = 200\ V$, $V_{Lm} = 100V$, Power factor = $1/\sqrt{2}$

   Find:

   (a) rms voltage across R

   (b) Phase difference

   (c) Impedance of the circuit

   (d) Nature of the circuit