# Part –1: Introduction, System of Particles, Position Vector and Centre of Mass (Basics)

## Learning Objectives

After studying this part, students should be able to:

- Understand the concept of a **system of particles**.
- Distinguish between a **particle** and a **rigid body**.
- Define the **position vector**.
- Explain the physical meaning of the **Centre of Mass (COM)**.
- Calculate the COM for simple systems.
- Apply COM concepts to elementary NEET/JEE problems.

## 1.1 Introduction

In previous chapters, we studied the motion of a **single particle**. However, many real-life objects—such as a car, a cricket bat, the Earth–Moon system, or the human body—consist of **many particles**.

To analyse such objects, we study them as a **system of particles**.

## 1.2. What Is a System of Particles?

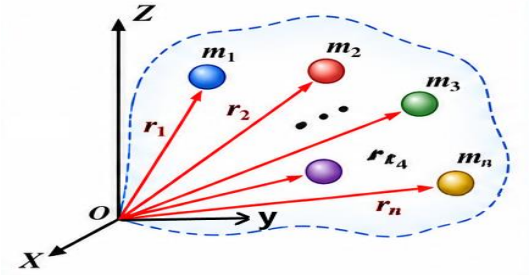
### Definition

A **System of Particles** is a collection of two or more particles considered together for the study of their motion.

Examples:

- Solar system
- Molecules
- Cricket bat
- Bicycle
- Human body
- Earth–Moon system

**Figure 6.1: System of Particles**




A system of particles consists of a large number of particles ($m_1, m_2, m_3, \ldots, m_n$) having different masses and positions.

## 1.3 Particle (Point Mass)

A **particle** is an idealized body whose dimensions are negligible compared with the distance involved in the motion.

Examples:

- Earth revolving around the Sun (approximation)
- A bullet in flight
- A football moving across a field (approximation)

## 1.4 Rigid Body

### Definition

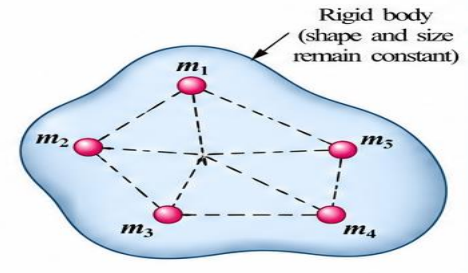
A **Rigid Body** is an ideal body in which the distance between any two particles remains constant even when external forces act.

In reality, perfectly rigid bodies do not exist, but many solids behave approximately as rigid bodies.

Examples:

- Metal rod
- Flywheel
- Disc
- Pulley

**Figure 6.2: Rigid Body**

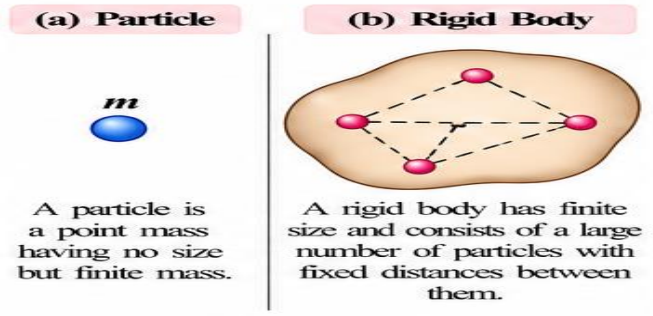



A rigid body is a system of a large number of particles in which the distances between any two particles remain constant.

## 1.5 Difference Between a Particle and a Rigid Body

| Particle | Rigid Body |
|---|---|
| Size is neglected | Size cannot be neglected |
| Only translational motion is considered | Translational and rotational motions are possible |
| Shape is ignored | Shape is important |

**Figure 6.3: Difference Between a Particle and a Rigid Body**




## 1.6 Position Vector

### Definition

The **position vector** of a particle is the vector drawn from the origin of the coordinate system to the position of the particle. The position vector is denoted by $\vec{r}$.

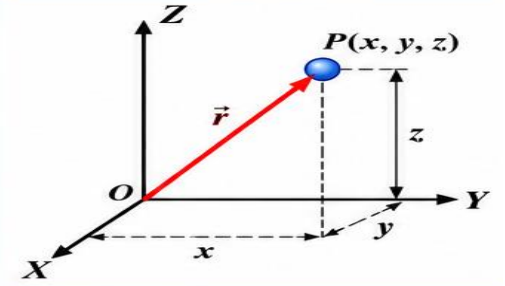
## 1.7 Mathematical Representation

For a particle at coordinates (x, y, z), the position vector is

$$\boxed{\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}}$$

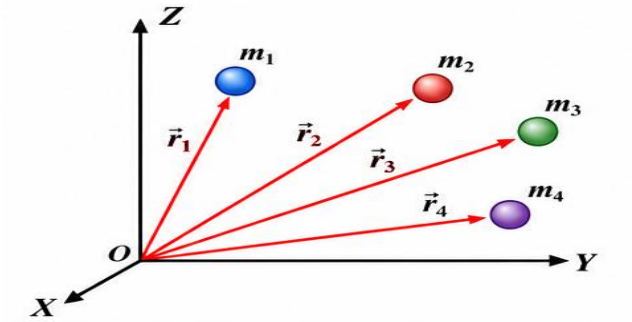
Where, $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are unit vectors along $x, y$ and $z$ directions respectively.

**Figure 6.4: Position Vector of a Particle**




The position vector of a particle at point $P(x, y, z)$ with respect to origin $O$ is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

**Figure 6.5: Position Vectors of Particles in a System**




Position vectors of different particles ($m_1$, $m_2$, $m_3$, $m_4$) in a system with respect to a common origin $O$.

## 1.8 Importance of Position Vector

The position vector helps us determine:

- Displacement
- Velocity
- Acceleration
- Centre of Mass
- Angular Momentum
- Torque

## 1.9 Centre of Mass (COM)

### Definition

The **Centre of Mass (COM)** of a system is the point at which the entire mass of the system may be assumed to be concentrated for the purpose of describing its translational motion.
It is the **mass-weighted average position** of all the particles in the system.

## 1.10 Physical Significance

If the line of action of a force passes through the centre of mass, that force produces zero torque about the centre of mass. Therefore, it does not produce rotational acceleration about the centre of mass itself.
For many problems, the motion of the whole system can be studied by analysing the motion of its COM.

## 1.11 Everyday Examples of Centre of Mass

- A cricket bat balances at its centre of mass.
- A hammer's COM lies closer to the heavier head.
- Tight-rope walkers use a long pole to lower the effective centre of mass and improve stability.
- High jumpers change body posture so that their centre of mass can pass below the bar while the body passes over it.

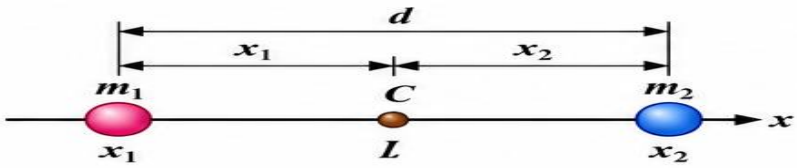
## 1.12 Centre of Mass of Two Particles

Consider two particles of masses $m_1$ and $m_2$ placed on the x-axis.

The position of the centre of mass is

$$\boxed{x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}}$$

**Figure 6.6: Centre of Mass of Two Particles**




$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Centre of mass of two particles of masses $m_1$ and $m_2$ located at $x_1$ and $x_2$ on a straight line. Here $d = x_1 + x_2$ is the distance between the particles.

### Vector Form

$$\boxed{\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}}$$

This formula is valid in one, two and three dimensions.

## 1.13 Special Cases

### Equal Masses

If $m_1=m_2$, then

$$\boxed{x_{com} = \frac{x_1 + x_2}{2}}$$

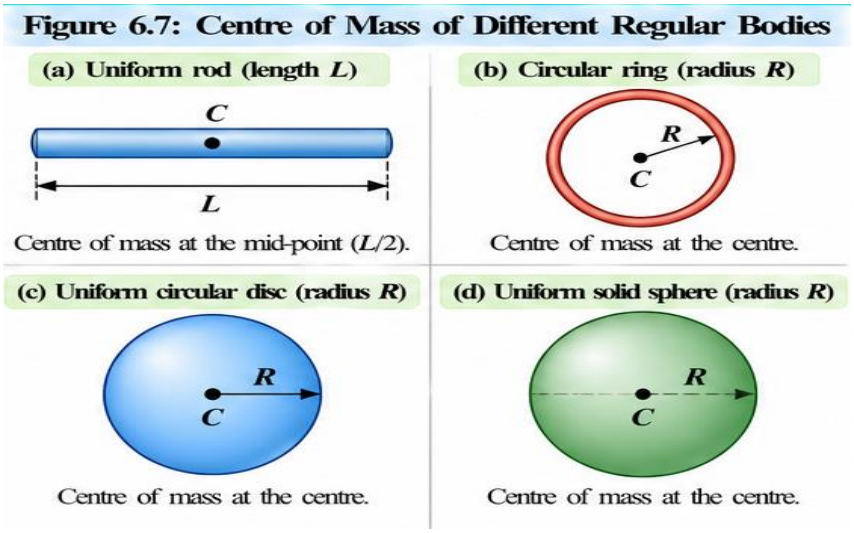
The COM lies exactly at the midpoint.

### Unequal Masses

The COM always lies **closer to the heavier mass**.
This is an important conceptual question in NEET and JEE.

## 1.14 Properties of the Centre of Mass

1. It may lie **inside** the body.
2. It may lie **outside** the body (e.g., a ring).
3. It depends on the distribution of mass.
4. It is unique for a given mass distribution.
5. For a uniform and symmetric body, the COM coincides with the geometric centre.




## Solved Numerical 1

Two particles of masses **2 kg** and **3 kg** are placed at x = 0 m and x = 5 m respectively. Find the position of the centre of mass.

### Solution

$$x_{cm} = \frac{2 \times 0 + 3 \times 5}{2 + 3} = 3\ m$$

**Answer:** $\boxed{x_{cm} = 3m}$

## Solved Numerical 2

Two equal masses are placed at x = 2 m and x = 8 m. Find the centre of mass.

### Solution

Since the masses are equal,

Using

$$x_{com} = \frac{x_1 + x_2}{2}$$

Or

$$x_{com} = \frac{2 + 8}{2} = 5\ m$$

**Answer:** = 5 m

## Solved Numerical 3 (NEET Level)

Two particles of masses **4 kg** and **1 kg** are placed at x = 0 m and x = 10 m respectively. Find the position of the centre of mass.

### Solution

Since,

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Or

$$x_{cm} = \frac{4 \times 0 + 1 \times 10}{4 + 1} = 2\ m$$

The COM lies much closer to the heavier mass.

**Answer:** $x_{cm}$ = 2 m

### NEET/JEE Concept Box

### Can the Centre of Mass Lie Outside the Body?

**Yes.**

Examples:

- Circular ring
- Hollow sphere
- Boomerang (depending on shape)

This is a frequently tested conceptual point.

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## Chapter Summary (Part–1)

- A **system of particles** consists of two or more particles studied together.
- A **particle** is an idealized point mass, whereas a **rigid body** has finite dimensions and can rotate.
- The **position vector** specifies the location of a particle relative to the origin.
- The **centre of mass** is the mass-weighted average position of a system.
- For two particles, the COM lies closer to the heavier particle and is given by the weighted-average formula.

# Part – 2: Centre of Mass (Advanced Concepts, Continuous Bodies & Applications)

## 2.1 Centre of Mass of Many Particles

Consider a system of **n particles** having masses $m_1$, $m_2$, $m_3$,…, $m_n$ located at position vectors

$$\vec{r}_1, \vec{r}_2, \vec{r}_3, - - - - - -$$

The position vector of the centre of mass is

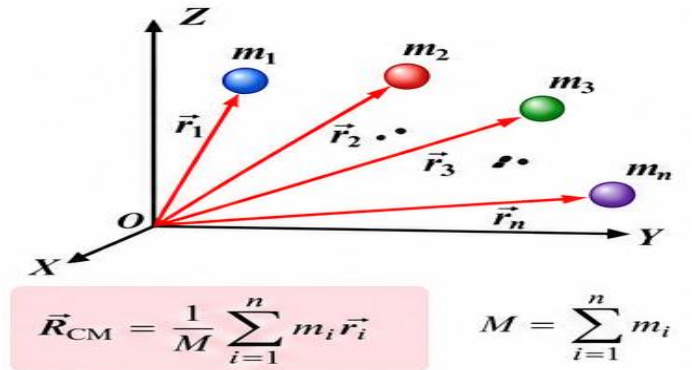
$$\boxed{r_{cm} = \frac{\sum_{i=1}^{n} m_i r_r}{\sum_{i=1}^{n} m_i}}$$

Where

$$\sum_{i=1}^{n} m_i = M$$

is the total mass of the system.

**Figure 6.8: Centre of Mass of Many Particles**




$$\vec{R}_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i \qquad M = \sum_{i=1}^{n} m_i$$

Centre of mass of a system of $n$ particles having masses $m_1, m_2, \ldots, m_n$ at position vectors $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_n$.

## 2.2 Cartesian Form

For coordinates

$$(x_i, y_i, z_i)$$

the coordinates of the centre of mass are

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + - - - - -}{M}$$

$$y_{com} = \frac{m_1 y_1 + m_2 t_2 + m_3 y_3 + - - - - -}{M}$$

$$z_{com} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3 + - - - - -}{M}$$

## 2.3 Physical Meaning of COM Formula

The centre of mass is the **weighted average position** of all the particles.

- Larger mass → Greater influence on COM.
- Smaller mass → Lesser influence on COM.

Thus, the COM always shifts towards the heavier side.

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## Solved Example 1

Three particles of masses **2 kg**, **3 kg**, and **5 kg** are placed on the x-axis at **1 m**, **4 m**, and **8 m**, respectively. Find the centre of mass.

### Solution:

Given: $m_1 = 2\ kg; x_1 = 1\ m, m_2 = 3\ kg; x_2 = 4\ m, m_3 = 5\ kg; x_3 = 8\ m$

Using,

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \ - - - - -}{M}$$

Or

$$x_{com} = \frac{2 \times 1 + 3 \times 4 + 5 \times 8}{2 + 3 + 5} = \frac{54}{10} = 5.4\ m$$

**Answer:** $\boldsymbol{x_{com}}$ = 5.4 m

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## 2.4 Centre of Mass in Two Dimensions

For particles placed in the x–y plane,

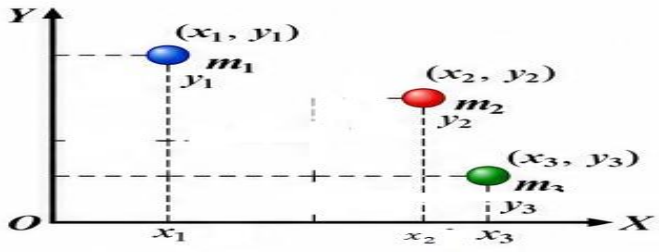
Coordinates are

$$x_{cm} = \frac{\sum m_i x_i}{M}$$

$$y_{cm} = \frac{\sum m_i y_i}{M}$$

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**Figure 6.9: Centre of Mass in Two Dimensions**




$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i}, \quad y_{CM} = \frac{\sum m_i y_i}{\sum m_i}$$

Centre of mass of a system of particles in a plane (two dimensions).

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### Solved Example 2

Masses: 2 kg at (0 m, 0 m), 3 kg at (4 m, 0 m), 5 kg at (2 m, 6 m). Find the centre of mass.

### Solution

Given:

- $m_1 = 2\ kg,\ x_1 = 0\ m, y_1 = 0\ m$
- $m_2 = 3\ kg, x_2 = 4\ m, y_2 = 0\ m$
- $m_3 = 5\ kg, x_3 = 2\ m, y_3 = 6\ m$

Total mass: $M = m_1 + m_2 + m_3 = 2 + 3 + 5 = 10\ kg$

For x-coordinate,

$$x_{cm} = \frac{m_x x_1 + m_2 x_2 + m_3 x_3}{M} = \frac{2(0) + 3(4) + 5(2)}{10} = \frac{22}{10} = 2.2\ m$$

For y-coordinate,

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{M} = \frac{2(0) + 3(0) + 5(6)}{10} = \frac{30}{10} = 3\ m$$

Therefore, the centre of mass is:

Answer: $(x_{cm}, y_{cm}) = (2.2\ m, 3\ m)$

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## 2.5 Centre of Mass of Continuous Bodies

When mass is continuously distributed, summation is replaced by integration.

$$r_{com} = \frac{1}{M}\int r dm$$

This is important for **JEE Main and JEE Advanced**.

### Linear Mass Density

For a thin rod,

$$\boxed{\lambda = \frac{M}{L}}$$

Where:

- M = Mass
- L = Length

And, local density formula along the rod in $x$ −direction:

$$\lambda = \frac{dm}{dx}$$

**SI Unit**: $\boxed{kg\ m^{-1}}$

### Surface Mass Density

For a thin sheet,

$$\sigma = \frac{dm}{dA}$$

Where

- A = Area

**SI Unit:** $\boxed{kg\ m^{-2}}$

### Volume Mass Density

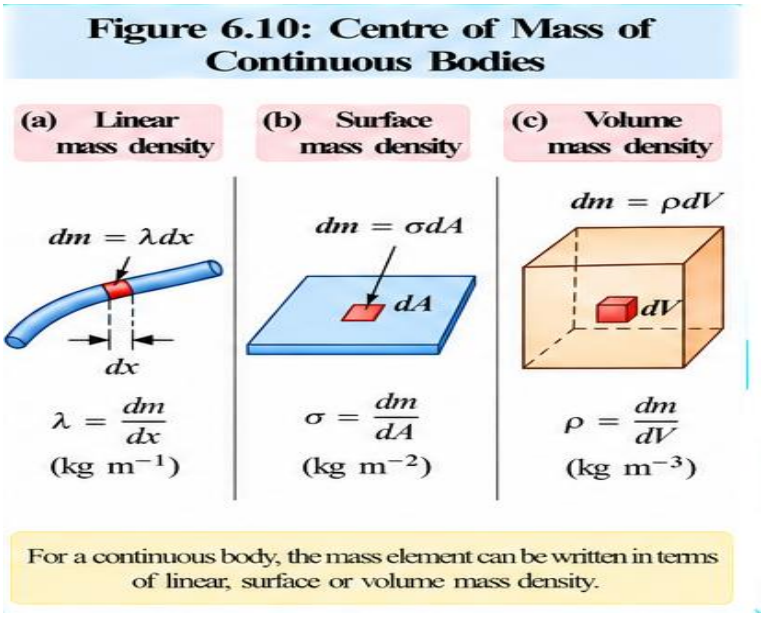
For a three-dimensional body,

$$\rho = \frac{dm}{dV}$$

Where: V = Volume

**SI Unit**: $\boxed{kg\ m^{-3}}$

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## 2.6 Standard Centres of Mass (Must Remember)

| Body | Centre of Mass |
|---|---|
| Uniform rod | Midpoint |
| Rectangular lamina | Geometric centre |
| Square lamina | Geometric centre |
| Circular disc | Centre |
| Solid sphere | Centre |
| Hollow sphere | Centre |
| Circular ring | Centre (outside the material) |
| Uniform cylinder | Midpoint on its axis |

**Exam Tip:** In a ring, the COM is at the centre even though there is **no material** at that point.

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For a uniform body with appropriate symmetry, the centre of mass lies at its geometric centre. For a uniform rod or cylinder, it lies at the midpoint along its length.

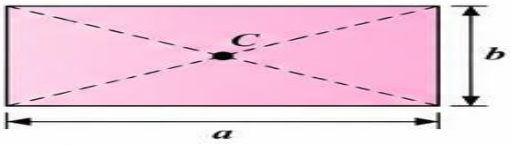
**Figure 6.11: Standard Centre of Mass (1) Uniform Rod and (2) Rectangular Lamina**

(a) Uniform rod (length $L$)




Centre of mass at the mid-point ($L/2$).

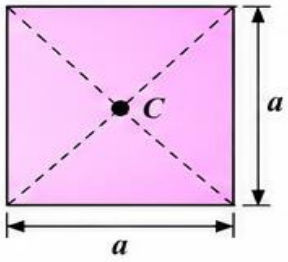
(b) Rectangular lamina (length $a$, breadth $b$)




Centre of mass at the centre ($a/2$, $b/2$).

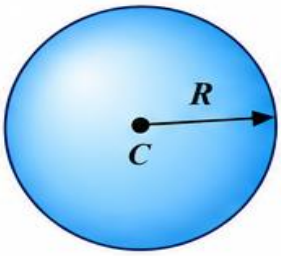
**Figure 6.12: Standard Centre of Mass (3) Square Lamina and (4) Circular Disc**

(a) Square lamina (side $a$)




Centre of mass at the centre ($a/2$, $a/2$).

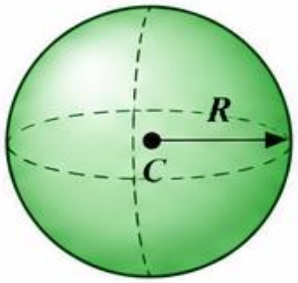
(b) Circular disc (radius $R$)




Centre of mass at the centre.

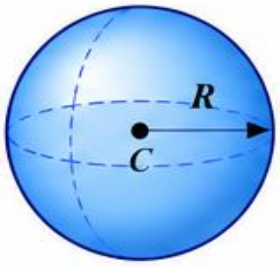
**Figure 6.13: Standard Centre of Mass (5) Solid Sphere and (6) Hollow Sphere**

(a) Solid sphere (radius $R$)




Centre of mass at the centre.

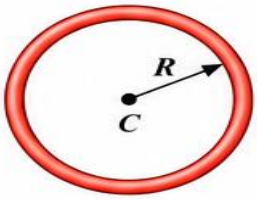
(b) Hollow sphere (radius $R$)




Centre of mass at the centre.

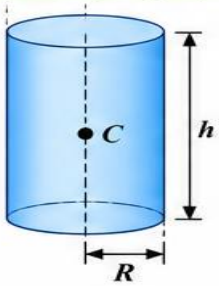
**Figure 6.14: Standard Centre of Mass (7) Circular Ring and (8) Uniform Cylinder**

(a) Circular ring (radius $R$)




Centre of mass at the centre.

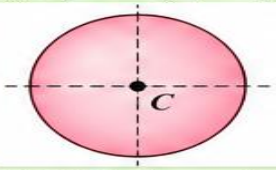
(b) Uniform cylinder (radius $R$, height $h$)




Centre of mass at the centre (on the axis, at $h/2$).

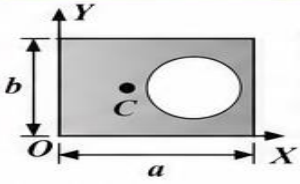
**Figure 6.15: Symmetry Principle and Effect of Removing a Portion**

(a) Symmetry principle




If a body has a plane, line or point of symmetry, its centre of mass lies at the symmetry point.

(b) Effect of removing a portion




Centre of mass of a body with a removed portion can be found by treating the removed part as a negative mass.

## 2.7 Symmetry Principle

**Rule:**

If a body has uniform mass distribution and geometric symmetry, then its centre of mass will lie on every axis (or plane) of symmetry.

Examples:

- Square plate → Centre
- Cube → Centre
- Sphere → Centre
- Circular disc → Centre

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## 2.8 Effect of Removing a Portion

If a part of a body is removed:

- Treat the removed part as having **negative mass**.
- Apply the centre of mass formula.

This technique is frequently used in **JEE Advanced**.

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## Solved Example 3

A uniform rod of length **2 m** has mass **4 kg**. Find the centre of mass.

### Solution

For a uniform rod, COM lies at the midpoint.

$$x_{com} = 1\ m$$

**Answer:** 1 m

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## Solved Example 4 (Conceptual)

Can the centre of mass of a ring lie in empty space?

**Answer: Yes.**

The centre of mass of a uniform circular ring lies at its geometric centre, even though no material exists there.

## NEET/JEE Concept Box

### Is the Centre of Mass Always Inside the Body?

**No.**

| Body | COM Position |
|---|---|
| Solid sphere | Inside |
| Disc | Inside |
| Rod | Inside |
| Ring | Outside the material |
| Hollow sphere | At the centre (empty region) |

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## Important Properties of Centre of Mass

1. Depends only on mass distribution.
2. Independent of external forces.
3. May lie inside or outside the material.
4. Coincides with the geometric centre for uniform symmetric bodies.
5. Used to analyse translational motion of complex systems.

## Chapter Summary (Part–2)

- The centre of mass of many particles is obtained by taking the mass-weighted average of their position vectors.
- For continuous bodies, integration replaces summation.
- Linear, surface and volume mass densities simplify COM calculations.
- Symmetry is a powerful tool for locating the COM.
- In problems involving removed portions, treat the removed part as **negative mass**.
- The COM may lie in an empty region, as in the case of a circular ring.

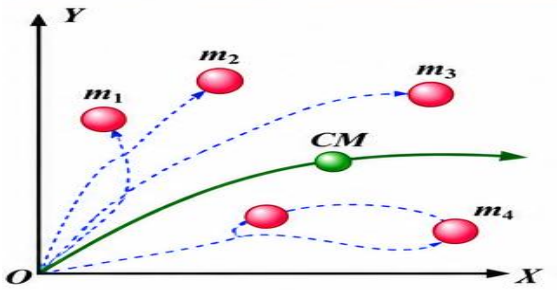
# Part – 3: Motion of Centre of Mass, Linear Momentum & Conservation of Momentum

## 3.1 Motion of the Centre of Mass

In Part–2, we studied how to locate the Centre of Mass (COM). Now we shall study **how the COM moves** when the particles of a system are in motion.

The motion of the COM represents the **overall translational motion** of the system.

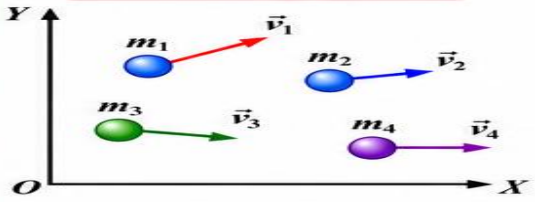
**Figure 6.16: Motion of the Centre of Mass**




The centre of mass of a system of particles moves as if the entire mass $M$ were concentrated at the point CM.

**Figure 6.17: Velocity of the Centre of Mass**

$$\vec{V}_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{v}_i$$




Velocity of the centre of mass is the mass weighted average of velocities of all the particles.

## 3.2 Velocity of the Centre of Mass

Suppose a system contains n particles with masses

$$m_1, m_2, m_3, - - - - - - m_n$$

and velocities

$$\vec{v}_1, \vec{v}_2, \vec{v}_3, - - - - \vec{v}_n$$

The velocity of the Centre of Mass is

$$\boxed{\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}}$$

Where: $\sum m_i = M$
is the total mass.

## 3.3 Physical Meaning

The velocity of the COM is the **mass-weighted average velocity** of all particles.

Hence,

- heavier particles influence the COM velocity more than lighter particles.

## 3.4 Acceleration of the Centre of Mass

Differentiating the velocity,

$$\boxed{\vec{a}_{cm} = \frac{\sum m_i \vec{a}_i}{\sum a_i}}$$

This equation is valid for any system of particles.

**Figure 6.18: Acceleration of the Centre of Mass**

$$\vec{a}_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{a}_i$$

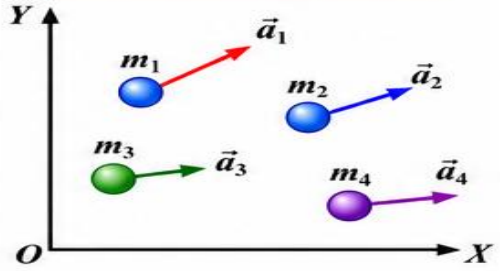
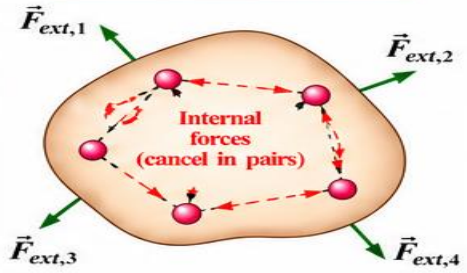



**Figure 6.19: Newton's Second Law for a System of Particles**

$$M\vec{a}_{CM} = \sum \vec{F}_{ext}$$




## 3.5 Newton's Second Law for a System of Particles

The total force acting on a system is

$$\sum \vec{F} = M\vec{a}_{cm}$$

Hence,

$$\boxed{\vec{F}_{ext} = M\vec{a}_{cm}}$$

**Important:** Only **external forces** determine the motion of the Centre of Mass. Internal forces cancel in pairs according to Newton's Third Law.

## 3.6 Internal and External Forces

### Internal Forces

Forces acting **between particles of the same system** are internal forces.
Examples:

- Tension in a string joining two blocks
- Force between two magnets of the same system

These do **not** change the motion of the COM.
For a system of particles, internal forces cancel in equal and opposite pair under the usual Newtonian assumptions.
Therefore, the acceleration of the centre of mass is determined by the net external force.

### External Forces

Forces acting on the system from outside are external forces.
Examples:

- Gravity
- Applied force
- Friction exerted by the ground

These **can change** the motion of the COM.

## 3.7 Linear Momentum

**Definition:** The linear momentum of a particle is the product of its mass and velocity.

$$\boxed{\vec{p} = m\vec{v}}$$

**SI Unit:** $\boxed{kg\ m\ s^{-1}}$

**Dimensions:** $\boxed{M\ L\ T^{-1}}$

## Total Linear Momentum of a System

For many particles,

$$\boxed{\vec{p} = \sum m_i \vec{v}_i}$$

Since

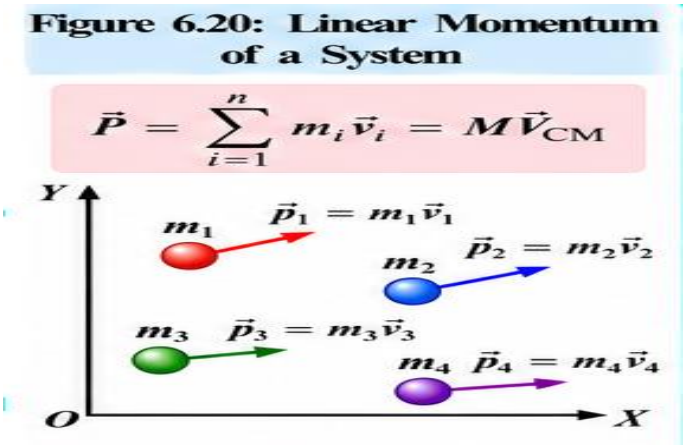
$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

Therefore,

$$\boxed{\vec{p} = M\vec{v}_{cm}}$$

As, $\sum m_i$ =M

This is an extremely important relation.




## 3.8 Newton's Second Law in Terms of Momentum

Since

$$\vec{F} = m\vec{a}$$

And

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Therefore,

$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$$

The net force acting on a particle is equal to the time rate of change of its linear momentum.

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

For a particle of constant mass, this reduces to:

$$\vec{F} = m\vec{a}$$

This momentum form is useful in problems solving involving impulse and changes in momentum. For variable-mass systems, the transfer of mass must be treated separately.

---

## Solved Example 1

A **2 kg** body moves with a velocity of **6 m s⁻¹**. Find its momentum.

**Solution:** $p = mv = 2 \times 6 = 12\ kg\ m/s$

Answer: $\boxed{12\ kg\ m\ s^{-1}}$

## Solved Example 2

Two bodies of masses **2 kg** and **3 kg** move with velocities **4 m s⁻¹** and **6 m s⁻¹**, respectively, in the same direction. Find the velocity of the Centre of Mass.

**Solution**

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{2 \times 4 + 3 \times 6}{2 + 3} = \frac{26}{5} = 5.2\ m/s$$

**Answer:** $\boxed{\boldsymbol{v_{cm} = 5.2\ m/s}}$

## Solved Example 3

A **5 kg** body at rest explodes into two fragments of masses **2 kg** and **3 kg**. The **2 kg** fragment moves with **12 m s⁻¹**. Find the velocity of the second fragment.

**Solution:** Initial momentum = 0

Let the velocity of second fragment be v, Therefore, using

$$\sum p_i = \sum p_f$$

Or

$$0 = 2 \times 12 + 3v_2$$

Or

$$v_2 = -8\ m/s$$

Negative sign indicates motion in the opposite direction.

**Answer:** $\boxed{v_2 = -8\ m/s}$ opposite to the first fragment

---

## 3.9 NEET/JEE Concept Box

### Can Internal Forces Change the Total Momentum?

**No,** internal forces occur in equal and opposite pairs and cancel each other.

Only **external forces** can change the total momentum of a system.

---

## Important Results

### If External Force = 0

$$\boxed{\vec{p} = Constant}$$

---

**If External Force ≠ 0**

Momentum changes according to: $\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$

## Chapter Summary (Part–3)

- The velocity and acceleration of the Centre of Mass are mass-weighted averages of the velocities and accelerations of all particles.
- Only **external forces** influence the motion of the Centre of Mass.
- Linear momentum is a vector quantity given by $\vec{p}=m\vec{v}$.
- The total momentum of a system is $\vec{P}= M\vec{v}_{cm}$.
- In the absence of external forces, the total linear momentum of a system remains constant.

# Part – 4: Rotational Motion, Angular Variables and Relation between Linear & Angular Quantities

## Learning Objectives

After studying this part, students should be able to:

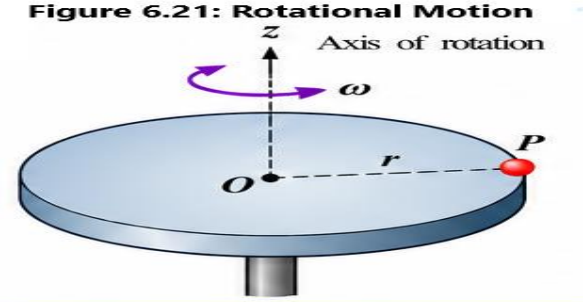
- Define rotational motion.
- Distinguish between translational and rotational motion.
- Understand angular displacement, angular velocity and angular acceleration.
- Derive relations between linear and angular quantities.
- Solve basic NEET/JEE numerical problems.

## 4.1 Rotational Motion

### Definition

A rigid body is in **rotational motion** about a fixed axis when its points move in circular paths centred on that axis. Points on the axis remain stationary.

Figure 6.21: Rotational Motion




A rigid body (disc) rotates about a fixed axis ($z$-axis). All points of the body move in circular paths around the axis with the same angular velocity $\omega$.

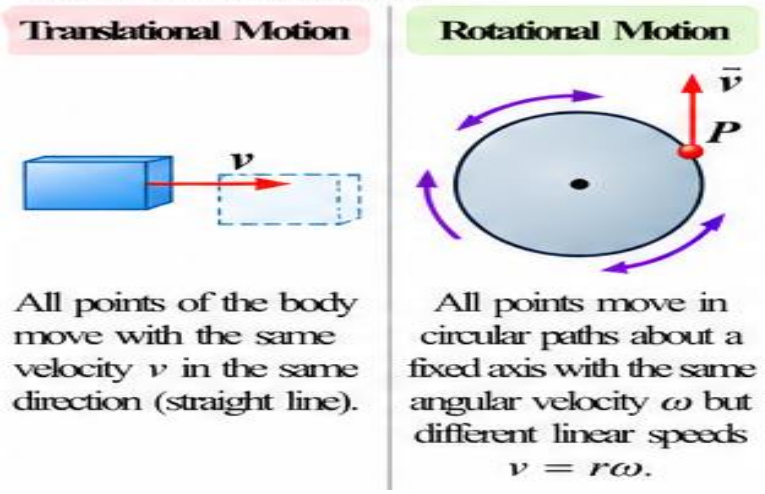
## Examples

- Ceiling fan
- Bicycle wheel
- Earth rotating about its axis
- Electric motor
- Potter's wheel

## 4.2 Characteristics of Rotational Motion

1. Every particle moves in a circular path.
2. All particles have the **same angular displacement**.
3. Linear speed depends on the distance from the axis.
4. Angular quantities are the same for every point of a rigid body.

**Figure 6.22: Translational versus Rotational Motion**




## 4.3 Translational vs Rotational Motion

| Translational Motion | Rotational Motion |
|---|---|
| Entire body moves from one place to another<br>All particles have the same velocity<br>Described by displacement and velocity | Body rotates about an axis<br><br>Linear velocity differs at different points<br>Described by angular displacement and angular velocity |

## 4.4 Angular Displacement (θ)

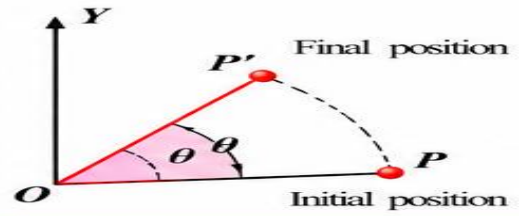
### Definition

Angular displacement is the angle through which a body rotates about a fixed axis.

**Unit:** $\boxed{radian\ (rad)}$

**SI Unit:** Radian

**Figure 6.23: Angular Displacement**




Angular displacement θ is the angle through which the radius vector of a point rotates about a fixed axis.

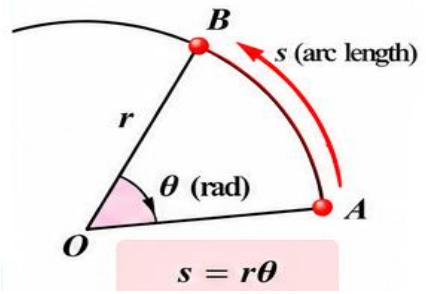
## 4.5 Relation between Arc Length and Angular Displacement

Consider a particle moving along a circular path of radius r.

$$\boxed{s = r\theta}$$

Where, θ is in radians.

**Figure 6.24: Relation between Arc Length and Angular Displacement**




### Radian

**Definition:** One radian is the angle subtended at the centre of a circle by an arc equal in length to the radius.

**Important Conversion**

$$\boxed{2\pi\ rad = 360^0}$$

## 4.6 Angular Velocity (ω)

### Definition

Angular velocity is the rate of change of angular displacement.
Average angular velocity,

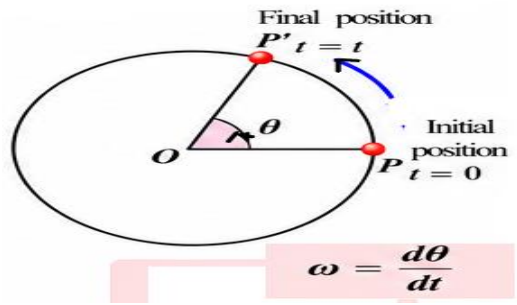
$$\boxed{\omega = \frac{\Delta\theta}{\Delta t}}$$

Instantaneous angular velocity,

$$\boxed{\omega = \frac{d\theta}{dt}}$$

**SI Unit:** $\boxed{rad/s}$

**Figure 6.25: Angular Velocity**



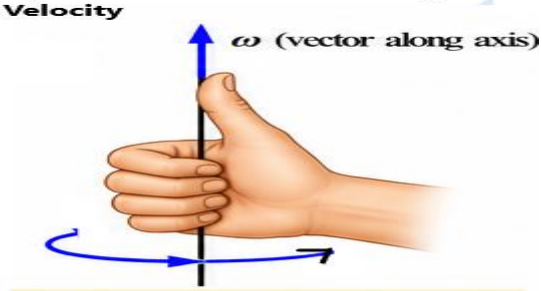

## 4.7 Direction of Angular Velocity

Angular velocity is a **vector quantity**.
Its direction is determined by the **Right-Hand Thumb Rule**.

### Right-Hand Thumb Rule

Curl the fingers of your right hand in the direction of rotation.
The extended thumb gives the direction of the angular velocity vector.

**Figure 6.26: Direction of Angular Velocity**




The direction of angular velocity is along the axis of rotation. Its sense is given by the right hand thumb rule (anticlockwise seen from top).

## 4.8 Angular Acceleration (α)

### Definition

Angular acceleration is the rate of change of angular velocity.
Average angular acceleration,

$$\boxed{\alpha = \frac{\Delta\omega}{\Delta t}}$$

Instantaneous angular acceleration,

$$\boxed{\alpha = \frac{d\omega}{dt}}$$

---

**SI Unit:** $\boxed{rad\ s^{-2}}$

---

## 4.9 Uniform and Non-Uniform Rotational Motion

### Uniform Rotational Motion

Angular velocity remains constant.
Since,

$$\omega = Constant$$

Therefore,

$$\alpha = \frac{d\omega}{dt} = \frac{d(constant)}{dt} = 0$$

Example: A ceiling fan rotating at constant speed

---

### Non-Uniform Rotational Motion

Angular velocity changes with time.

$$\alpha \neq 0$$

Example: A fan starting from rest

## 4.10 Linear and Angular Quantities

| Linear Quantity | Angular Quantity |
|---|---|
| **Displacement s** | Angular displacement θ |
| **Velocity v** | Angular velocity ω |
| **Acceleration a** | Angular acceleration α |
| **Mass m** | Moment of inertia I (analogy) |
| **Force F** | Torque τ |

---

## 4.11 Relation between Linear and Angular Velocity

Since,

$$s = r\theta$$

Differentiating with respect to time,

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

Hence: $\boxed{v = r\omega}$

## Physical Meaning

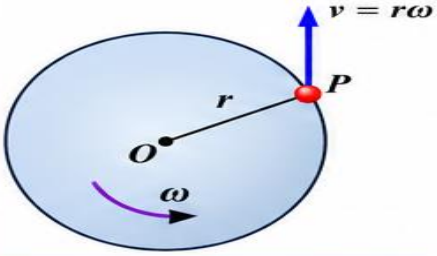
For the same angular velocity,

- Larger radius → Larger linear speed.
- Smaller radius → Smaller linear speed.

Thus, points farther from the axis move faster.

---

**Figure 6.27: Relation between Linear and Angular Velocity**




The linear speed $v$ of a point at a distance $r$ from the axis is related to its angular velocity $\omega$ by $v = r\omega$.

---

## 4.12 Relation between Linear and Angular Acceleration

Differentiating $v = r\omega$ with respect to time,

$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

Or

$$\boxed{a_t = r\alpha}$$

Where: $a_t$ is the **tangential acceleration**.

## 4.13 Centripetal Acceleration

Even if angular velocity is constant, a particle moving in a circle has radial acceleration,

$$\boxed{a_c = \frac{v^2}{r}}$$

Using: $v = r\omega$

We obtain: $\boxed{a_c = \frac{v^2}{r}}$

This acceleration is always directed **towards the centre** of the circle.

---

## Total Acceleration

In non-uniform rotational motion, both tangential and centripetal accelerations are present.

The magnitude of total acceleration is: $\boxed{a = \sqrt{{a_c}^2 + {a_t}^2}}$

Since these two components are perpendicular.

---

### Solved Example 1

A wheel rotates through **8 radians** in **4 seconds**. Find its average angular velocity.

**Solution:** $\omega = \frac{\theta}{t} = \frac{8}{4} = 2 rad/s$

**Answer:** $\boxed{2\ rad/s}$

---

### Solved Example 2

A wheel of radius **0.50 m** rotates with an angular velocity of **12 rad s⁻¹**. Find the linear speed of a point on its rim.

**Solution:** $v = r\omega = 0.5 \times 12 = 6.0\ m/s$

**Answer:** $\boxed{6\ m/s}$

---

### Solved Example 3

A wheel has radius **0.20 m** and angular acceleration **15 rad s⁻²**. Find the tangential acceleration.

**Solution:** $a_t = r\alpha = 0.2 \times 15 = 3\ m/s^2$

**Answer:** $\boxed{3\ ms^{-2}}$

---

### Solved Example 4 (NEET Level)

A particle moves in a circle of radius **2 m** with angular velocity **5 rad s⁻¹**. Find its centripetal acceleration.

**Solution:** $a_c = r\omega^2 = 2 \times 25 = 50\ ms^{-2}$

**Answer:** $\boxed{50\ ms^{-2}}$

---

### NEET/JEE Concept Box

**Do all points on a rotating rigid body have the same linear speed?**

**No.**

They have the **same angular velocity**, but their **linear speed depends on the radius**.

$$v = r\omega$$

Hence, the outer edge of a rotating disc moves faster than points near the centre.

---

## Chapter Summary (Part–4)

- Rotational motion occurs when a body rotates about a fixed axis.
- Angular displacement is measured in **radians**.
- Angular velocity and angular acceleration describe the rate of rotation.
- Linear and angular quantities are related by:
  - $v = r\omega$
  - $a_c = r\omega^2$
- In non-uniform rotational motion, both tangential and centripetal accelerations act simultaneously.

# Part – 5: Torque (Moment of Force), Couple, Equilibrium of Rigid Bodies & Moment of Inertia

## Learning Objectives

After studying this part, students should be able to:

- Define torque (moment of force).
- Calculate torque using vector and scalar forms.
- Understand the concept of a couple.
- Explain conditions for equilibrium of a rigid body.
- Define and calculate the moment of inertia.
- Understand the radius of gyration.
- Apply the Parallel Axis and Perpendicular Axis Theorems.

## 5.1 Torque (Moment of Force)

### Definition

**Torque** is the turning effect (or rotational effect) of a force about a fixed point or axis.
It is the rotational analogue of force in translational motion.

### Everyday Examples

- Opening a door.
- Tightening a nut with a spanner.
- Turning a steering wheel.
- Rotating a screwdriver.
- Pedaling a bicycle.

A force applied farther from the pivot produces a greater turning effect.

### Mathematical Definition

If a force **F** acts at a perpendicular distance **r** from the axis,

$$\tau = Fr_{\perp}$$

More generally,

$$\tau = rF \sin\theta$$

Where:

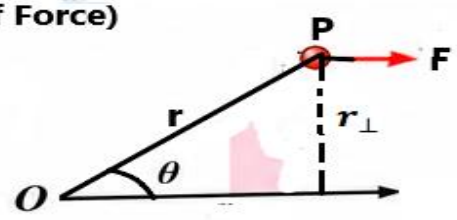
- r = Position vector from the axis to the point P of application.
- F = Applied force.
- θ = Angle between $\vec{r}$ and $\vec{F}$.

**Vector Form:** $\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$

**SI Unit:** $\boxed{\text{Nm}}$

**Dimensions:** $\boxed{[ML^2T^{-2}]}$

Figure 6.28: Torque (Moment of Force)




$$\tau = rF\sin\theta = Fr_\perp$$

Torque (moment of a force) about point $O$ is equal to the product of the force and the perpendicular distance from $O$ to the line of action of the force.

## Direction of Torque

Torque is a **vector quantity**.
Its direction is determined by the **Right-Hand Thumb Rule**.

## Sign Convention

| Rotation | Sign |
|---|---|
| **Anticlockwise** | Positive |
| **Clockwise** | Negative |

## Maximum and Minimum Torque

### Maximum Torque

When $\theta = 90^0$ then

$$\boxed{\tau_{max} = rF}$$

### Minimum Torque

When $\theta = 0^0$

$$\boxed{\tau = 0}$$

## Principle of Moments

For a body in rotational equilibrium,

$$\boxed{\tau_{clockwise} = \tau_{anticlockwise}}$$

This principle is widely used in balancing problems.

## 5.2 Couple

### Definition

A **couple** consists of two equal and opposite parallel forces acting at different points of a body.

### Characteristics

- Net force = Zero
- Net torque ≠ Zero
- The resultant force of a couple is zero, while its net torque is non-zero. Therefore, a couple produces no translational acceleration of the centre of mass, although it can produce angular acceleration.

### Examples

- Turning a steering wheel.
- Rotating a water tap.
- Opening a bottle cap.

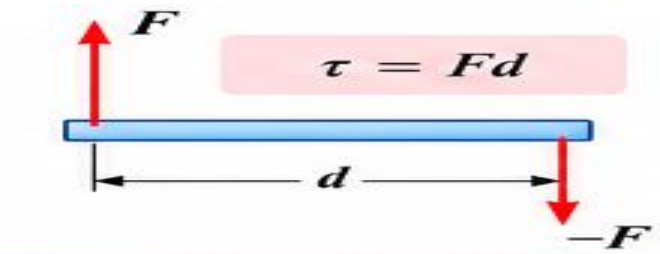
### Moment of a Couple

If the perpendicular distance between the two forces is d,

$$\boxed{\tau = Fd}$$

Where, d is called the **arm of the couple**.

**Figure 6.29: Couple**




A couple consists of two equal and opposite parallel forces separated by a distance $d$. The torque of a couple is $\tau = Fd$ (independent of the point).

---

## 5.3 Equilibrium of a Rigid Body

A rigid body is in equilibrium when:

### (i) Translational Equilibrium

$$\boxed{\sum \vec{F} = 0}$$

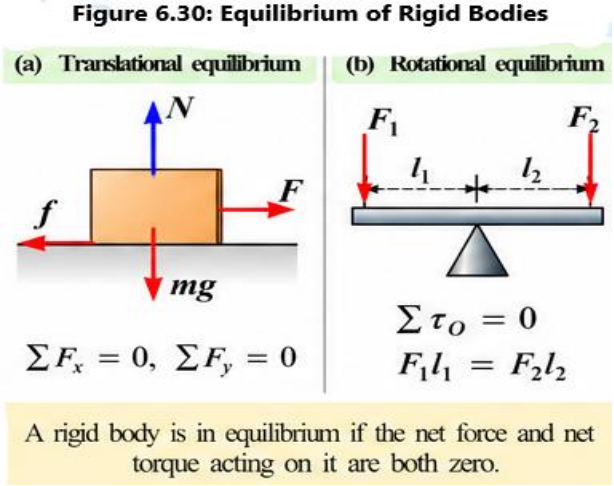
### (ii) Rotational Equilibrium

$$\boxed{\sum \vec{\tau} = 0}$$

Both conditions must be satisfied simultaneously.

---

Figure 6.30: Equilibrium of Rigid Bodies




A rigid body is in equilibrium if the net force and net torque acting on it are both zero.

## 5.4 Moment of Inertia (MOI)

### Definition

The **Moment of Inertia** of a body about a given axis is the rotational analogue of mass.
It measures the body's **resistance to angular acceleration**.

### Mathematical Definition

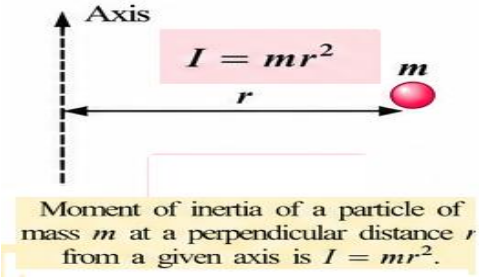
For a particle,

$$\boxed{I = mr^2}$$

Where

- m = Mass
- r = Perpendicular distance from the axis

Figure 6.31: Moment of Inertia (Single Particle)




Moment of inertia of a particle of mass $m$ at a perpendicular distance $r$ from a given axis is $I = mr^2$.

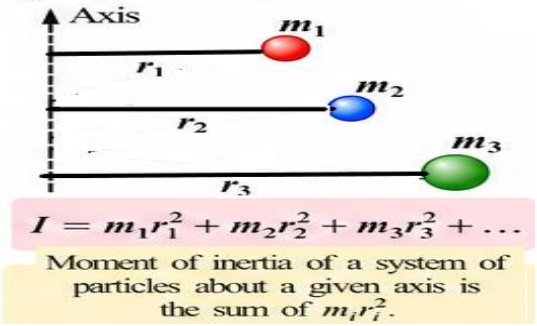
### Moment of Inertia of More Than Two Particles about a Given Axis

For a system of particles,
If mass of the particles is $m_1, m_2, m_3, ----$ are located at perpendicular distances $r_1, r_2, r_3, ---------$, then total moment of inertia is given by

$$I_{axis} = m_1 {r_1}^2 + m_2 {r_2}^2 + m_3 {r_3}^2 + \ -----$$

$$I = \sum_{i=1}^{n} m_i {r_i}^2$$

Figure 6.32: Moment of Inertia (Many Particles)




## 5.5 (a) Parallel Axis Theorem

### Statement

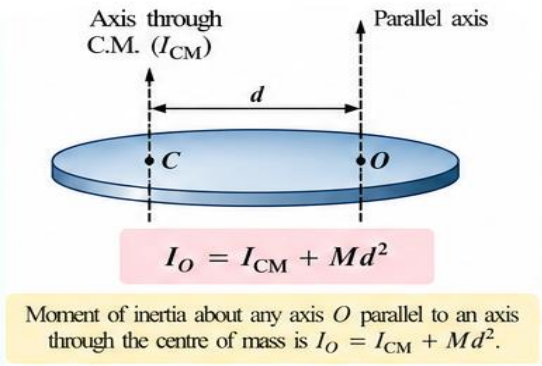
The moment of inertia of a body about any axis parallel to an axis through its centre of mass is

$$I = I_{cm} + Md^2$$

Where,

- $I_{cm}$ = MOI about the centre of mass
- d = Distance between the two parallel axes

Figure 6.32 (a): Parallel Axis Theorem

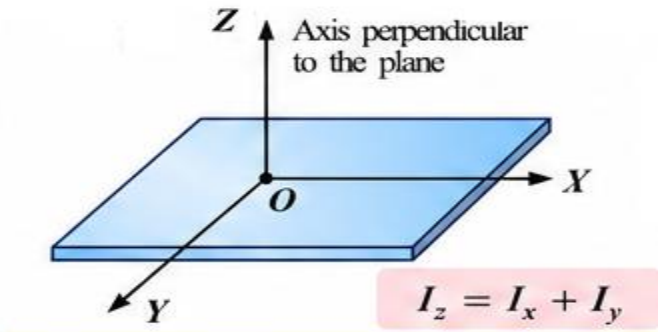



## 5.6 (b) Perpendicular Axis Theorem

Applicable only to **plane lamina**

$$\boxed{I_z = I_x + I_y}$$

**Figure 6.32 (b): Perpendicular Axis Theorem (For a Plane Lamina)**




For a thin plane lamina, the moment of inertia about an axis perpendicular to the plane (Z-axis) passing through $O$ is equal to the sum of moments of inertia about two mutually perpendicular axes ($X$ and $Y$) in the plane: $I_z = I_x + I_y$.

## 5.7 Moment of Inertia of a Continuous Body

**For a continuous body or rigid body,**

$$\boxed{I = \int r^2 dm}$$

**To derive moment of inertia of this rod about an axis passing through its COM and perpendicular to the rod:**

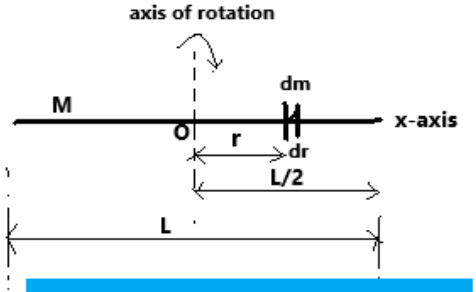



**Figure 6.33: Moment of Inertia of a Thin Rod about its COM O**

Suppose a thin rod of length L and uniformly distributed mass M is placed in the $x - axis$. Since the rod has uniform mass distribution, its linear mass density is constant. Therefore,

$$\lambda = \frac{dm}{dr} = \frac{M}{L} - - - -(1)$$

Or,

$$dm = \frac{M}{L} dr - - - - - (2)$$

Using:

$$dI_{com} = r^2 dm - - - (3)$$

Hence,

Using equations (2) and (3) and integrating equation (3); dr varies from –L/2 to +L/2, we get:

$$I_{com} = \int_{-L/2}^{+L/2} \frac{M}{L} r^2 dr$$

Or,

$$I_{com} = \frac{M}{L}\left(\frac{r^3}{3}\right)_{-L/2}^{L/2} = \frac{M}{L}\left[\frac{L^3}{24} - \left(-\frac{L^3}{24}\right)\right] = \frac{ML^2}{12}$$

Therefore,

$$\boxed{I_{com} = \frac{ML^2}{12}}$$

## Special Cases:

1. Moment of inertia of this rod about an axis passing through one of its end and perpendicular to the rod:

$$I_{rod\ end} = I_{com} + M\frac{L^2}{4} = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2}{3}$$

Hence,

$$\boxed{I_{rod\ end} = \frac{ML^2}{3}}$$

2. Moment of inertia of this rod about an axis passing along the rod:

$$\boxed{I_{along\ rod} = 0}$$

---

**SI Unit:** $\boxed{kg\ m^2}$

**Dimensions:** $\boxed{[ML^2]}$

## Factors Affecting MOI

1. Mass of the body.
2. Shape of the body.
3. Distribution of mass.
4. Position of the axis of rotation.

---

## 5.8 Radius of Gyration

### Definition

The radius of gyration is the distance from the axis at which the entire mass of the body may be assumed to be concentrated without changing its moment of inertia.

### Formula

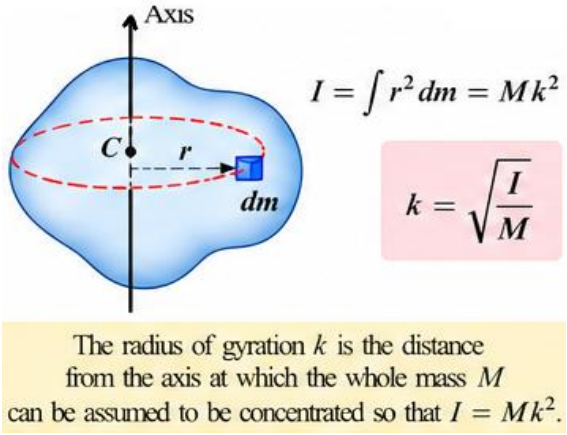
$$\boxed{I = mk^2}$$

$$k = \sqrt{\frac{I}{m}}$$

Where,

- k = Radius of gyration

**Figure 6.34: Radius of Gyration**




## 5.9 Standard Moments of Inertia (Must Memorize)

| Body | Axis | Moment of Inertia |
|---|---|---|
| Thin Rod | Through centre, perpendicular to length | $I = ML^2/12$ |
| Thin Rod | Through one end | $I = ML^2/3$ |
| Ring | Through centre | $MR^2$ |
| Disc | Through centre | $MR^2/2$ |
| Solid Cylinder | Along axis | $MR^2/2$ |
| Solid Sphere | Diameter | $I = 2MR^2/5$ |
| Hollow Sphere | Diameter | $I = 2MR^2/3$ |

**Where R = radius**

**Exam Tip:** Students should remember these standard results. Their derivations are generally beyond the CBSE syllabus but are useful for JEE preparation.

**Figure 6.35 (a): Moment of Inertia of a Thin Rod**

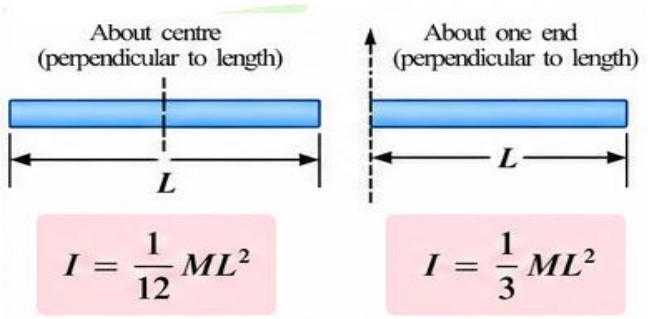



**Figure 6.35 (b): Moment of Inertia of a Thin Ring**

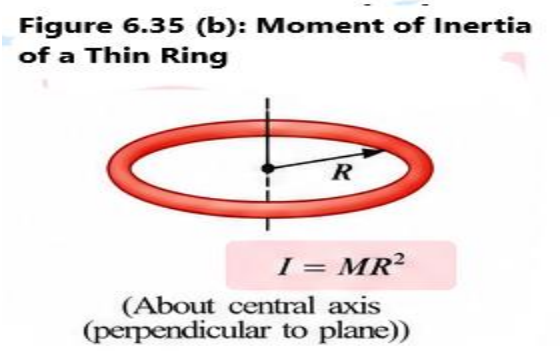
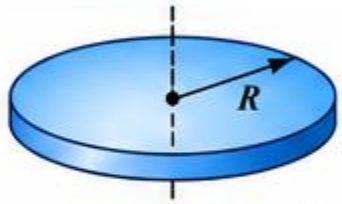


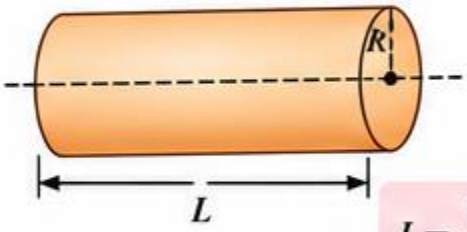
**Figure 6.35 (c): Moment of Inertia of a Disc**




$$I = \frac{1}{2} MR^2$$

(About central axis (perpendicular to plane))

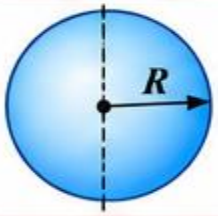
**Figure 6.35 (d): Moment of Inertia of a Solid Cylinder**




$$I = \frac{1}{2} MR^2$$

(About its own axis (along the axis of cylinder))

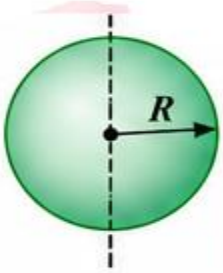
**Figure 6.35 (e): Moment of Inertia of a Solid Sphere**




$$I = \frac{2}{5} MR^2$$

(About a diameter)

**Figure 6.35 (f): Moment of Inertia of a Hollow Sphere**




$$I = \frac{2}{3} MR^2$$

(About a diameter)

## Solved Example 1

A force of **20 N** acts perpendicular to a rod at a distance of **0.5 m** from the pivot. Find the torque.

**Solution**

$$\tau = rF = 0.5 \times 20 = 10\ Nm$$

**Answer**: $\boxed{10\ Nm}$

## Solved Example 2

Find the moment of inertia of a **4 kg** particle situated **0.50 m** from the axis.

**Solution**

$$I = mr^2 = 4 \times 0.25 = 1\ kg\ m^2$$

### Solved Example 3

A disc has Mass = **2 kg,** Radius = **0.30 m,** Find its MOI about its central axis.

**Solution**

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 2 \times 0.09 = 0.09\ kg\ m^2$$

**Answer:** $\boxed{0.09\ kg\ m^2}$

---

### NEET/JEE Concept Box

**Why is it easier to open a door by pushing at the handle instead of near the hinges?**

Since,

$$\tau = rF$$

Therefore, a larger perpendicular distance (r) produces a larger torque for the same applied force. Hence, less effort is required when pushing at the handle.

---

## Chapter Summary (Part–5)

- **Torque** is the turning effect of a force and is given by $\vec{\tau} = \vec{r} \times \vec{F}$
- A **couple** has zero resultant force but produces a non-zero torque.
- A rigid body is in equilibrium when both the net force and the net torque are zero.
- The **Moment of Inertia** is the rotational analogue of mass and depends on the mass distribution about the axis.
- The **Parallel Axis** and **Perpendicular Axis Theorems** simplify the calculation of the moment of inertia for different axes.

# Part–6: Angular Momentum, Conservation of Angular Momentum & Rolling Motion

## Learning Objectives

After studying this part, students should be able to:

- Define angular momentum.
- Derive the relation between torque and angular momentum.
- State and apply the law of conservation of angular momentum.
- Understand pure rolling motion.
- Calculate the kinetic energy of a rolling body.
- Explain common applications such as a cyclist leaning while turning.

## 6.1 Angular Momentum

### Definition

The **angular momentum** of a particle about a point is the **moment of its linear momentum** about that point.
It is the rotational analogue of linear momentum.

### Mathematical Expression

For a particle,

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

Since,

$$\vec{p} = m\vec{v}$$

Therefore,

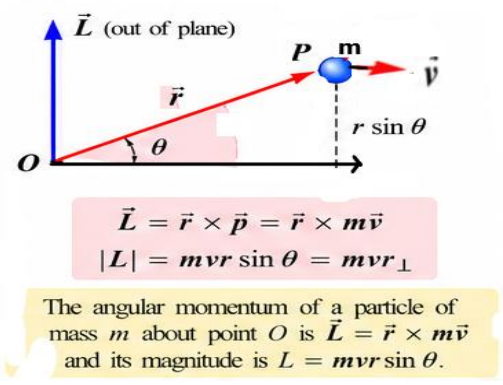
$$\boxed{\vec{L} = \vec{r} \times m\vec{v}}$$

### Magnitude

$$\boxed{L = mvr \sin\theta}$$

Where: r = Position vector; v = Velocity; θ = Angle between $\vec{r}$ and $\vec{v}$

**Figure 6.36: Angular Momentum of a Particle**



## Special Case: Circular Motion

In circular motion, $\theta = 90^0$
Hence,

$$\boxed{L = mvr}$$

Since,

$$v = r\omega$$

We get

$$\boxed{L = mr^2\omega}$$

As

$$I = mr^2$$

therefore,

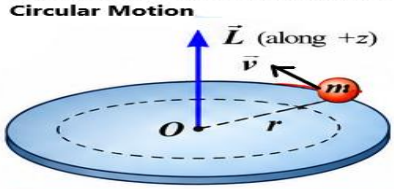
$$\boxed{L = I\omega}$$

This is one of the most important equations in rotational dynamics.

**SI Unit:** $\boxed{kg\ m^2 s^{-1}}$

---

**Figure 6.37: Angular Momentum in Circular Motion**




$\vec{L} = m\vec{r} \times \vec{v}$ , $|L| = mrv$ (for $\theta = 90°$)

For a particle moving in a circle of radius $r$ with speed $v$, the angular momentum about the centre is $L = mrv$ (along the axis).

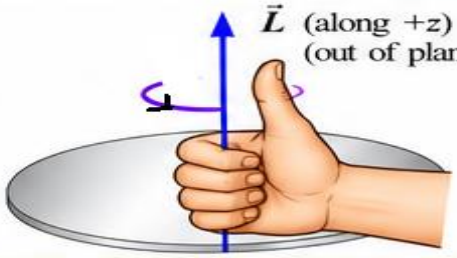
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## Direction of Angular Momentum

The direction of angular momentum is obtained using the **Right-Hand Thumb Rule**.
It is perpendicular to the plane containing $\vec{r}$ and $\vec{v}$.

---

**Figure 6.37-1: Direction of Angular Momentum (Right-Hand Rule)**




If the rotation is anticlockwise as seen from the top, the angular momentum is along +z direction (right-hand thumb rule).

## Relation between Torque and Angular Momentum

From Newton's Second Law,

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Taking the moment about a point,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Substituting,

$$\vec{\tau} = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \frac{d(m\vec{v})}{dt} = \frac{d(\vec{r} \times \vec{p})}{dt} = \frac{d\vec{L}}{dt}$$

Therefore,

$$\boxed{\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}}$$

The net external torque acting on a system equals the time rate of change of its total angular momentum, under the usual Newtonian assumptions.

---

## Interpretation

- Torque changes angular momentum.
- If no external torque acts, angular momentum remains constant.

This relation is the rotational analogue of

$$\vec{F} = \frac{d\vec{p}}{dt}$$

## 6.2 Law of Conservation of Angular Momentum

### Statement

If the net external torque acting on a system is zero, the total angular momentum of the system remains constant.

Mathematically,

$$\boxed{\vec{L}_i = \vec{L}_f}$$

---

**Figure 6.38: Law of Conservation of Angular Momentum**

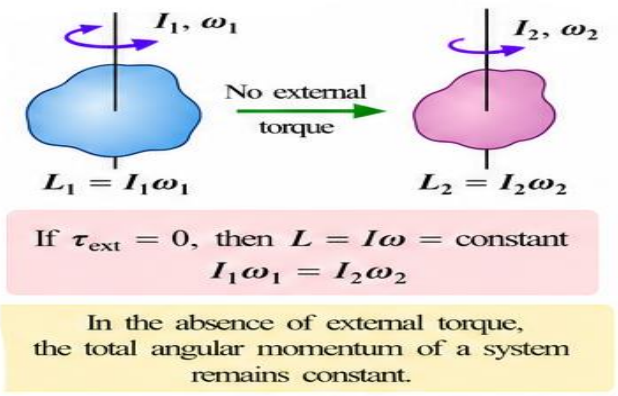
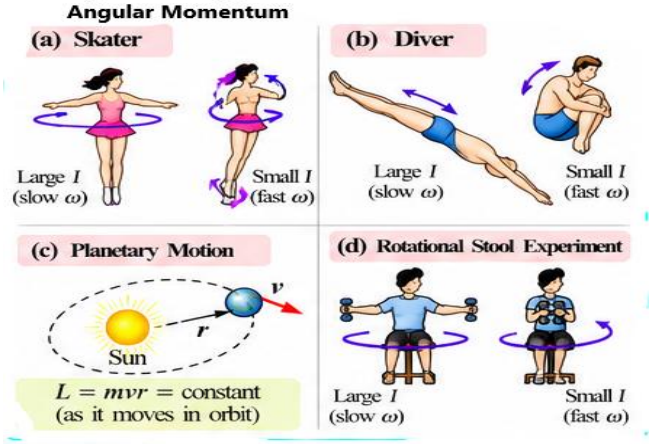



**Figure 6.39: Applications of Conservation of Angular Momentum**




---

## Applications

### 1. Skater

When a skater pulls in the arms, net external torques should be negligible for angular momentum to be conserved.

- Moment of inertia decreases.
- Angular velocity increases.

Since L= Iω and L remains constant,
If net external torque is negligible,

$$I_i \omega_i = I_f \omega_f$$

Therefore,

$$\boxed{\omega_f = \frac{I_i}{I_f} \omega_f}$$

$$I \downarrow \ \omega \uparrow$$

---

### 2. Diver

A diver folds the body while rotating to increase angular speed before entering the water.

---

### 3. Planetary Motion

The revolution of planets around the Sun obeys conservation of angular momentum.

---

### 4. Rotating Stool Experiment

A person holding dumbbells spins faster when the arms are pulled inward because the moment of inertia decreases.

---

## 6.3 Rolling Motion

### Definition

Rolling motion is a combination of:

- Translational motion of the centre of mass.
- Rotational motion about the centre of mass.

## 6.4 Pure Rolling

In pure rolling, there is **no slipping** between the body and the surface.

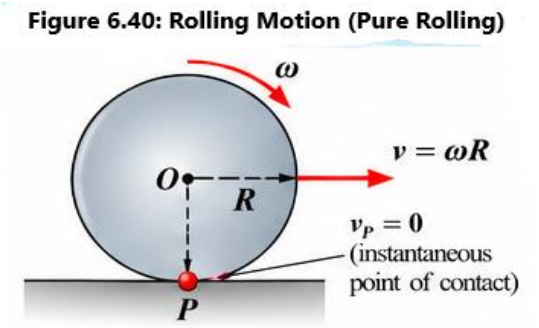



Figure 6.40: Rolling Motion (Pure Rolling)

In pure rolling, the point of contact with the ground has zero velocity. Hence, $v = \omega R$.

## Condition for Pure Rolling

$$\boxed{v_{CM} = R\omega}$$

Where,

- v = Linear velocity of the centre of mass
- R = Radius
- ω = Angular velocity

---

## Kinetic Energy of Rolling

A rolling body possesses two types of kinetic energy.

### Translational

$$K_T = \frac{1}{2} M {v_{CM}}^2$$

### Rotational

$$K_R = \frac{1}{2} I_{CM} \omega^2$$

### Total Kinetic Energy

$$K = K_T + K_R$$

Hence,

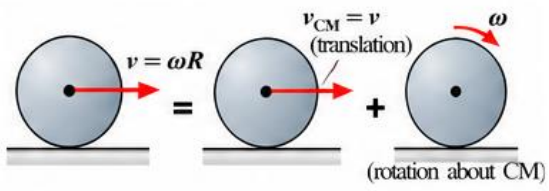
$$\boxed{K = \frac{1}{2} M {v_{CM}}^2 + \frac{1}{2} I_{CM} \omega^2}$$

Using,

$$v = R\omega$$

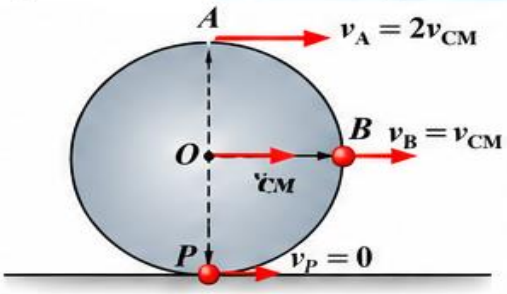
The total kinetic energy can be expressed entirely in terms of either $v$ or ω, using the pure-rolling condition $v = R\omega$

**Figure 6.41: Pure Rolling as Combination of Translation and Rotation**



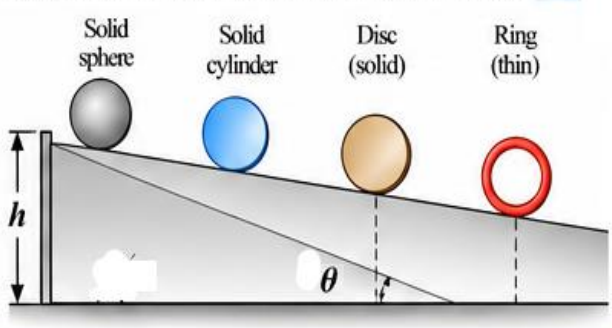

Pure rolling can be considered as the combination of translation of the centre of mass with speed $v$ and rotation about the centre with angular speed $\omega$, where $v = \omega R$.

**Figure 6.42: Velocities in Rolling Wheel**




In pure rolling, the velocity of the top point is $2v_{CM}$, the centre has velocity $v_{CM}$ and the point of contact has zero velocity.

**Figure 6.43 (a): Rolling Down an Incline Plane**




For the same height $h$ and without slipping, the body with smaller moment of inertia (smaller $I/(mR^2)$) reaches the ground first.

## 6.5 Rolling Down an Inclined Plane

For a body rolling without slipping,

- Both translational and rotational kinetic energies increase.
- Gravitational potential energy decreases.

Applying conservation of mechanical energy: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

This is a very important result for NEET/JEE preparation.

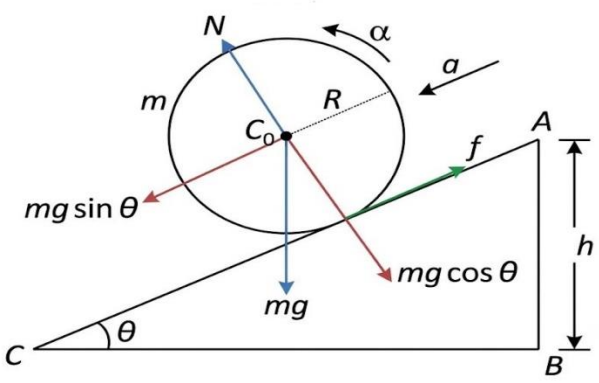
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## Derivation: Acceleration of a Cylinder Rolling Down an Inclined Plane

**Given:**

- Mass of cylinder = m
- Radius of cylinder = R
- Moment of inertia about its axis = I
- Angle of inclination = θ
- Vertical height of inclined plane = h
- Acceleration of the cylinder = a
- In triangle ABC, let AC = s

**Figure 6.43 (b): Forces on a Rolling Body in an Inclined Plane**




**Assumption:** The cylinder rolls down the inclined plane without slipping.

### Step 1: Forces Acting on the Cylinder

The forces acting on the cylinder are:

1. Weight mg, acting vertically downward.
2. Normal reaction N, acting perpendicular to the inclined plane.
3. Static friction f, acting up the inclined plane.

Resolving the weight along the inclined plane:

$$mg\sin\theta - f = ma \text{ ----- } (1)$$

### Step 2: Rotational Equation of Motion

The torque due to friction about the centre of the cylinder is

$$fR = I\alpha$$

For pure rolling,

$$a = R\alpha \Rightarrow \alpha = \frac{a}{R}$$

Therefore,

$$f = \frac{Ia}{R^2} \text{ ------ } (2)$$

### Step 3: Derivation of Acceleration

Substituting Equation (2) into Equation (1),

$$mg\sin\theta - \frac{Ia}{R^2} = ma$$

Or,

$$mg\sin\theta = a\left(m + \frac{I}{R^2}\right)$$

Hence, the acceleration of the rolling body is

$$\boxed{a = \frac{g \sin\theta}{1 + \frac{I}{mR^2}}}$$

This is the required expression for the acceleration of a body rolling down an inclined plane without slipping.

### Step 4: Velocity at the Bottom

Using the relation

$$v^2 = u^2 + 2as$$

For a body starting from rest, u = 0. If the distance travelled along the incline is s, then:

$$v^2 = 2as$$

Since $h = s \sin\theta$, we obtain

$$\boxed{v = \sqrt{\frac{2gh}{1 + \frac{I}{mR^2}}}}$$

### Step 5: Special Case — Solid Cylinder

For a uniform solid cylinder: $I = \frac{mR^2}{2}$

Substituting into the acceleration formula,

$$a = \frac{g \sin\theta}{1 + \frac{1}{2}}$$

Therefore,

$$a = \frac{2}{3} g \sin\theta$$

The velocity at the bottom becomes

$$\boxed{v = \sqrt{\frac{4}{3} gh}}$$

**Conclusion:** The acceleration of a rolling body depends on its moment of inertia, mass, radius and the angle of inclination. For a solid cylinder rolling without slipping, its acceleration is

$$\boxed{a = \frac{2}{3} g \sin\theta}$$

**Important note:** These results assume sufficient static friction to prevent slipping and neglect air resistance.

### Which Body Reaches First?

For bodies of the same mass and radius rolling without slipping:

1. Solid Sphere
2. Solid Cylinder / Disc
3. Hollow Sphere
4. Ring

The **solid sphere** reaches first because it has the smallest ratio of moment of inertia among all the above mentioned.

This order assumes ideal rolling without slipping and compares bodies with the stated standard mass distributions.

---

**Figure 6.44: Comparison of Rolling Bodies**

| Body | Moment of inertia ($I$) | $\frac{I}{(mR^2)}$ | Acceleration $a=\frac{a=\sin\theta}{1+I/(mR^2)}$ | Order of arrival |
|---|---|---|---|---|
| Solid sphere | $\frac{2}{5}mR^2$ | 2/5 | $\frac{5}{7}g\sin\theta$ | 1 (first) |
| Solid cylinder | $\frac{1}{2}mR^2$ | 1/2 | $\frac{2}{3}g\sin\theta$ | 2 |
| Disc (solid) | $\frac{1}{2}mR^2$ | 1/2 | $\frac{2}{3}g\sin\theta$ | 2 |
| Ring (thin) | $mR^2$ | 1 | $\frac{1}{2}g\sin\theta$ | 3 (last) |

---

## Solved Example 1

A particle of mass **2 kg** moves in a circle of radius **3 m** with a speed of **4 m s⁻¹**. Find its angular momentum.

**Solution:** $L = mvr = 2 \times 4 \times 3 = 24\ kg\ m^2 s^{-1}$

---

## Solved Example 2

A wheel of radius **0.40 m** rolls without slipping at **8 m s⁻¹**. Find its angular velocity.

**Solution:** $v = R\omega$

Or

$$\omega = \frac{8}{0.4} = 20\ rad/s$$

**Answer:** $\boxed{20\ rad/s}$

---

## Solved Example 3 (NEET Level)

A cyclist moves along a circular path of radius **25 m** with a speed of **10 m s⁻¹**. Find the angle of lean.

**Solution:** $\tan\theta = \frac{v^2}{rg} = \frac{100}{25\times9.8} = 20/49 = 0.4082$

Or

$$\theta = tan^{-1}(0.4082)$$

**Answer**: $\boxed{\theta \approx 22^0}$

## NEET/JEE Concept Box

**Why does a rolling wheel possess more kinetic energy than a sliding block moving with the same speed?**

A sliding block has only **translational kinetic energy**, whereas a rolling wheel possesses **both translational and rotational kinetic energies**.

## Chapter Summary (Part–6)

- Angular momentum is the rotational analogue of linear momentum and is given by: $\boxed{\vec{L} = \vec{r} \times \vec{p}}$
- For a rigid body rotating about a fixed axis, L=Iω.
- External torque changes angular momentum according to: $\boxed{\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}}$
- In the absence of external torque, angular momentum remains conserved.
- Pure rolling combines translational and rotational motion with the condition: $\boxed{v_{CM} = R\omega}$
- A rolling body possesses both translational and rotational kinetic energies.
- A cyclist leans inward while turning, and banked roads help vehicles negotiate curves safely.

# Appendix – A: Complete Formula Sheet & Standard Results

## 1. Important SI Units

| Quantity | SI Unit |
|---|---|
| Centre of Mass | metre (m) |
| Momentum | kg m $s^{-1}$ |
| Impulse | N s |
| Angular Velocity | rad $s^{-1}$ |
| Angular Acceleration | rad $s^{-2}$ |
| Torque | N m |
| Moment of Inertia | kg $m^2$ |
| Angular Momentum | kg $m^2$ $s^{-1}$ |

## 2. NEET/JEE Quick Revision Table

| Translational Motion | Rotational Motion |
|---|---|
| Mass m | Moment of Inertia I |
| Force F | Torque τ |
| Displacement x | Angular Displacement θ |
| Velocity v | Angular Velocity ω |
| Acceleration a | Angular Acceleration α |
| Momentum p = mv | Angular Momentum L= Iω |
| Newton's Law: F= ma | Rotational Form: τ = Iα |
| Work W= Fs | Work W= τθ |

**Note:** These rotational relations are written for rotation about a fixed axis. The expression $W = \tau\theta$ assumes constant torque; for variable torque, rotational work is calculated by integration.

## 3. Frequently Asked Viva Questions

Q1. Can the centre of mass lie outside a body?
Ans. Yes. Example: A ring.

Q2. Why is the handle of a door fixed away from the hinges?
Ans. To produce a larger torque with the same applied force.

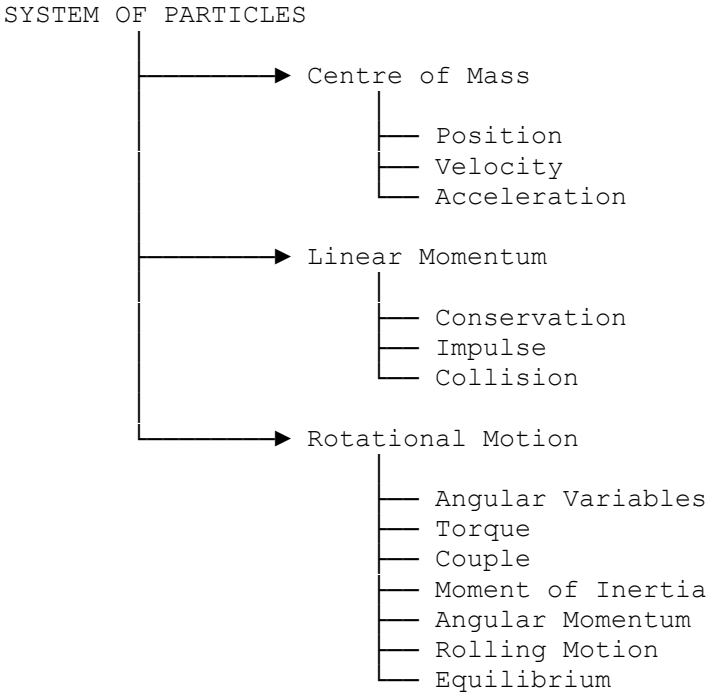
Q3. Why does a skater rotate faster after pulling in the arms?
Ans. Because the moment of inertia decreases, so angular velocity increases to conserve angular momentum.

Q4. Why does a cyclist lean while turning?
Ans. To ensure that the resultant of weight and the required centripetal force passes through the centre of mass, preventing toppling.

Q5. Why does a solid sphere roll faster than a ring?
Ans. For bodies rolling without slipping down the same incline, the solid sphere has a smaller ratio ($I/MR^2$) than a ring. Therefore, its translational acceleration is greater.

## Appendix – B: Concept Booster, Memory Tricks & Frequently Confused Concepts

# Concept Map of the Chapter



## Memory Trick–1
## Translational Motion vs Rotational Motion

| Translation | Rotation |
|---|---|
| Mass | Moment of Inertia |
| Force | Torque |
| Momentum | Angular Momentum |
| Velocity | Angular Velocity |
| Acceleration | Angular Acceleration |
| Newton's Law | Rotational Dynamics |

### Easy Memory Line
**Mass → Force → Motion**
**Moment of Inertia → Torque → Rotation**

---

## Memory Trick–2
## Which Quantity Depends on Radius?
The following comparisons assume a rigid body rotating about a fixed axis. For a given angular velocity, linear speed and centripetal acceleration depend on the distance from the axis.
Because,

$$v = r\omega$$

And,

$$a_c = r\omega^2$$

| Quantity | Depends on Radius? |
|---|---|
| Angular Velocity | No |
| Angular Displacement | No |
| Angular Acceleration | No |
| Linear Velocity | Yes |
| Tangential Acceleration | Yes |
| Centripetal Acceleration | Yes |
| Torque | Yes |
| Moment of Inertia | Yes |

**Shortcut:**
**Angular quantities are the same for every point of a rigid body, while most linear quantities depend on the distance from the axis.**

---

## Memory Trick–3
## Momentum vs Angular Momentum
Linear Momentum: $p = mv$
Angular Momentum: $L = I\omega$
Remember
**Linear → m**
**Rotational → I**

---

## Frequently Confused Concept–1
## Centre of Mass vs Centre of Gravity

| Centre of Mass | Centre of Gravity |
|---|---|
| Depends only on mass distribution | Depends on gravity |
| Independent of gravitational field | Depends on gravitational field |
| Exists even in space | Meaningful when gravity acts |

The centre of mass is defined by the mass distribution and exists independently of whether a gravitational field is present or not.

**Note**

In a **uniform gravitational field**, the Centre of Mass and Centre of Gravity coincide.

---

## Frequently Confused Concept–2
## Force vs. Torque

| Force | Torque |
|---|---|
| Changes translation motion | Changes rotational motion |
| SI Unit: Newton | SI Unit: Newton metre |
| Vector | Vector |
| Depends only on force | Depends on force **and** perpendicular distance |

---

## Frequently Confused Concept–3
## Mass vs. Moment of Inertia

| Mass | Moment of Inertia |
|---|---|
| Resistance to linear acceleration | Resistance to angular acceleration |
| Constant for a body | Depends on the axis of rotation |
| Unit: kg | Unit: kg $m^2$ |

---

## Frequently Confused Concept–4
## Linear Velocity vs. Angular Velocity

| Linear Velocity | Angular Velocity |
|---|---|
| Different for different points | Same for all points of a rigid body |
| Depends on radius | Independent of radius |
| Unit: m $s^{-1}$ | Unit: rad $s^{-1}$ |

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## Frequently Confused Concept–5
## Tangential vs. Centripetal Acceleration

| Tangential | Centripetal |
|---|---|
| Changes speed | Changes direction |
| Along the tangent | Towards the centre |
| $a_t = r\alpha$ | $a_c = r\omega^2$ |

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## Frequently Confused Concept–6
## Elastic vs. Inelastic Collision

| Elastic | Inelastic |
|---|---|
| Momentum conserved | Momentum conserved |
| Kinetic energy conserved | Kinetic energy not conserved |
| $e = 1$ | $0 \leq e < 1$ |

## Frequently Confused Concept–7
## Sliding vs. Rolling

| Sliding | Rolling |
|---|---|
| Only translational motion | Translation + Rotation |
| Friction opposes relative slipping or the tendency of relative slipping between surfaces in contact | Static friction enables pure rolling |
| $v \neq R\omega$ | $v_{cm} = R\omega$ |

## Quick Concept Box
## Which Quantities are conserved?

| Quantity | Conserved When |
|---|---|
| Linear Momentum | Net external force = 0 |
| Angular Momentum | Net external torque = 0 |
| Mechanical Energy | Only when non-conservative work is zero |
| Kinetic Energy | Only in elastic collisions |

## Signs to Remember

| Rotation | Sign |
|---|---|
| Anticlockwise | Positive |
| Clockwise | Negative |

## Right-Hand Thumb Rule

The right-hand rule gives the direction of a cross product such as $\vec{r} \times \vec{p}$ or $\vec{r} \times \vec{F}$
Used for:

- Angular Velocity
- Angular Momentum
- Torque

## Method

1. Curl the fingers in the direction of rotation.
2. The thumb gives the direction of the vector.

## High-Yield NEET/JEE Facts

1. The **centre of mass** of a uniform ring lies at its geometric centre, even though there is no material there.
2. **Moment of inertia** changes when the axis changes.
3. **Torque** is maximum when the force is perpendicular to the lever arm.

4. For bodies of the same mass and radius rolling without slipping down the same incline, a **solid sphere** reaches the bottom before a ring because it has a smaller ratio $I/mR^2$.
5. During pure rolling, the point of contact with the ground is **instantaneously at rest** relative to the ground. *(Very important for JEE Advanced.)*
6. Pulling the arms inward increases angular velocity due to the conservation of angular momentum.

## Chapter-6 Mind Map (One-Page Revision)

| | |
|---|---|
| System of Particles | Centre of Mass |
| | Momentum<br>-Impulse<br>-Collision<br>-Conservation |
| | Rotation<br>-Angular Variables<br>-Torque<br>-Moment of Inertia<br>-Rolling Motion<br>-Equilibrium |