PART 1 – ELASTICITY, STRESS AND STRAIN

1.1 Learning Objectives

After studying this part, you will be able to:

- Understand the meaning of elasticity and plasticity.
- Explain deformation of a solid under an external force.
- Define stress and strain.
- Distinguish between longitudinal, volumetric and tangential deformation.
- Derive and use the expressions for different types of stress and strain.
- Understand Hooke's law.
- Interpret the physical meaning of stress–strain relationships.
- Apply these concepts to Board, NEET and JEE-level problems.

1.2 What Are Mechanical Properties of Solids?

A solid body has a definite shape and volume because its constituent particles are held together by strong intermolecular forces.

When an external force is applied to a solid, its shape, size or volume may change.

This change is called **deformation**.

Examples:

- A wire becomes slightly longer when stretched.
- A rubber band changes its length considerably.
- A metal rod may become shorter when compressed.
- A solid block may change its shape when a tangential force is applied.

The study of how solids respond to external forces is called the study of their **mechanical properties**.

1.3 Deforming Force

An external force that changes the shape, size or volume of a body is called a **deforming force**.

For example, if a force F is applied to a wire along its length, the wire may change its length.

Types of deformation

A solid may undergo:

1. Change in length
2. Change in volume
3. Change in shape

These lead to different types of stress and strain.

1.4 Elasticity

Elasticity is the property of a material by virtue of which it tends to regain its original shape and size when the deforming force is removed, provided the elastic limit has not been exceeded.

For example:

- Steel is highly elastic.
- Rubber is also elastic, but its behaviour is different from that of steel.

Important Point

A material is considered perfectly elastic if it completely recovers its original configuration after removal of the deforming force.

In real materials, complete recovery is possible only within a certain range of deformation.

1.5 Elastic Deformation

If a body returns to its original shape and size after the external force is removed, the deformation is called **elastic deformation**.
Example:
A steel wire stretched slightly and released returns approximately to its original length.

1.6 Plastic Deformation

If a body does not completely regain its original shape and size after the deforming force is removed, the deformation is called **plastic deformation**.
The material retains a permanent deformation.
Examples include:

- Bending a metal wire permanently.
- Pressing clay.
- Permanently deforming a metal beyond its elastic range.

Elastic vs Plastic Deformation

Elastic Deformation	Plastic Deformation
Temporary deformation	Permanent deformation
Original shape is recovered	Original shape is not completely recovered
Occurs within elastic limit	Occurs beyond elastic limit
No permanent strain	Permanent strain remains

1.7 Why Do Solids Show Elasticity?

Consider the atoms or molecules of a solid.
They are held near their equilibrium positions by intermolecular forces.
When an external force is applied, the particles are displaced slightly from their equilibrium positions.
Internal restoring forces develop inside the material.
These restoring forces oppose the deformation.
When the external force is removed, the internal forces tend to bring the particles back toward their original positions.
Thus, elasticity is associated with the **interatomic or intermolecular forces** within the material.

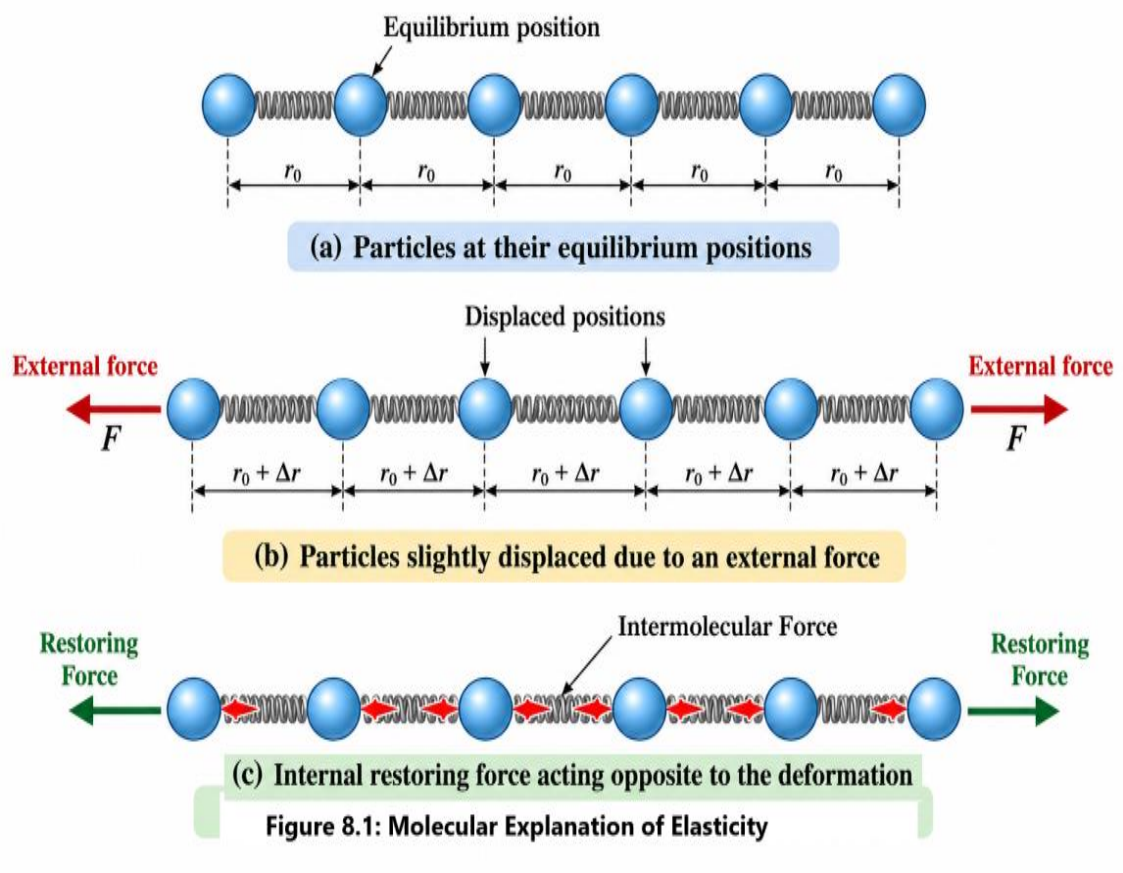


Figure 8.1: Molecular Explanation of Elasticity

1.8 Restoring Force

When a solid is deformed, internal forces develop within the material.
These forces tend to restore the original configuration.
They are called **restoring forces**.
The restoring force acts in the direction opposite to the deformation.
For example, if a wire is stretched:

- External force tends to increase its length.
- Internal elastic force tends to decrease its length.

1.9 Elasticity and Plasticity: Important Concept

The terms **elastic** and **flexible** should not be confused.
A material that undergoes large deformation is not necessarily more elastic.
In physics, elasticity refers to the ability to **restore the original shape and size** after removal of the deforming force.
This distinction is important in conceptual questions.

1.10 Stress

When an external force acts on a body, internal restoring forces develop.
The internal restoring force per unit area is called **stress**.
Therefore,
Stress = Restoring force / Area

If the magnitude of the restoring force is F and cross-sectional area is A:
Stress = F/A
SI unit: **N/m^2**
This is also called the **pascal (Pa)**.
Therefore: **1 Pa = 1 N/m^2**
Dimensional Formula: [Stress] = $ML^{-1}T^{-2}$
Stress has the same dimensions and SI unit as pressure.

1.11 Important Note About Stress

Stress is not simply the external force.
It is the **internal restoring force per unit area** developed inside the material.
In static equilibrium, the magnitudes of the external deforming force and internal restoring force are equal.

1.12 Types of Stress

Depending upon the type of deformation, stress can be classified into three main types:

1. Longitudinal stress
2. Bulk or volumetric stress
3. Tangential or shear stress

1.13 Longitudinal Stress

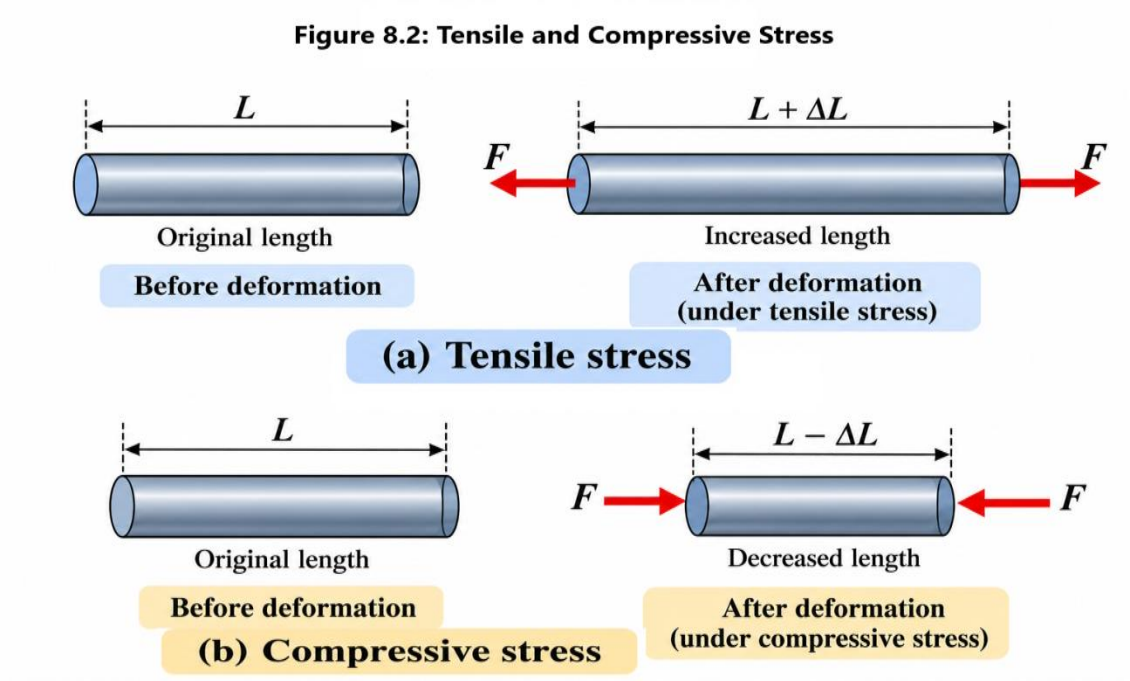
When the deforming force acts perpendicular to the cross-sectional area and produces a change in length, the resulting stress is called **longitudinal stress**.
There are two types:

(a) Tensile Stress

When a body is stretched, tensile stress is produced.
Example:
A wire being pulled from both ends.
Tensile stress = F/A

(b) Compressive Stress

When a body is compressed, compressive stress is produced.
Example:
A rod is being pushed from both ends.
Compressive stress = F/A
The magnitude is the same expression, but the nature of deformation is different.



1.14 Longitudinal Strain

When a body is subjected to longitudinal stress, its length changes.
Let:

- Original length = L
- Change in length = ΔL

Then longitudinal strain is defined as:
Longitudinal strain = ΔL/L
It is a ratio of two lengths.
Therefore, it has **no unit** and **no dimensions**.
For tensile deformation:
Longitudinal strain = ΔL/L
For compression, the change in length may be considered negative if sign convention is used.

1.15 Strain

Strain is a measure of the fractional deformation produced in a body.
In general:
Strain = Change in dimension / Original dimension
Since strain is a ratio of two quantities of the same dimension:
Strain is dimensionless.
It has no SI unit.

1.16 Types of Strain

There are three principal types:

1. Longitudinal strain
2. Volumetric strain
3. Shear strain

1.17 Longitudinal Strain

When the length of a body changes:

Longitudinal strain = ΔL/L

Where:

- L = original length
- ΔL = change in length

It may be:

- Tensile strain
- Compressive strain

1.18 Volumetric Strain

When the volume of a body changes due to external pressure, the resulting strain is called **volumetric strain**.

Let:

- Original volume = V
- Change in volume = ΔV

Then:

Volumetric strain = ΔV/V

Like all strain quantities, it is dimensionless.

1.19 Shear or Tangential Strain

When a tangential force changes the shape of a body without appreciably changing its volume, the resulting strain is called **shear strain** or **tangential strain**.

Suppose a rectangular block has height L.

Its upper surface is displaced horizontally through Δx.

Then:

Shear strain = Δx/L

Where:

- Δx is horizontal displacement in upper surface
- L is the height of the block.

If the angular deformation is θ and θ is small:

Shear strain = tan θ ≈ θ

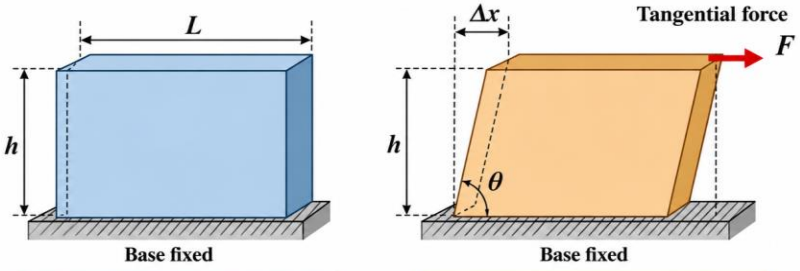
Where: θ is measured in radians.

Therefore:

Shear strain ≈ θ

Since an angle in radians is dimensionless, shear strain is also dimensionless.

Figure 8.3: Shear Deformation of a Block



$$\text{Shear strain} = \frac{\Delta x}{L} \approx \theta$$

1.20 Summary of Stress and Strain

Type	Stress	Strain
Longitudinal	F/A	ΔL/L
Volumetric	F/A or pressure p	ΔV/V
Shear	F/A	Δx/L ≈ θ

The exact type of stress depends on how the external force is applied.

Note: For bulk modulus, the negative sign is included in the definition because volumetric strain is negative during compression. Thus:

$$K = -\frac{p}{\Delta V/V}$$

Where, p is the increase in pressure.

1.21 Hooke's Law

Within the elastic limit of a material:

Stress is directly proportional to strain.

Therefore,

Stress ∝ Strain

or

Stress = Elastic modulus × Strain

Thus,

Elastic modulus = Stress/Strain

Different types of deformation correspond to different elastic moduli.

1.22 Elastic Moduli

The three important elastic moduli are:

1. Young's Modulus

For longitudinal deformation: **Y = Longitudinal stress / Longitudinal strain**

2. Bulk Modulus

For volumetric deformation:

K = -Bulk stress / Volumetric strain

For uniform increase in pressure

$$\boxed{K = -\frac{p}{\Delta V/V}}$$

The negative sign ensures that the bulk modulus is positive during compression, for which $\Delta V < 0$.

3. Shear Modulus

For shear deformation: **G = Shear stress / Shear strain**

These will be studied in detail in the next parts.

1.23 Physical Meaning of Elastic Modulus

The elastic modulus measures the resistance offered by a material to deformation.

A **larger elastic modulus** means:

- Greater resistance to deformation.
- Smaller strain for a given stress.
- Material is mechanically stiffer.

A **smaller elastic modulus** means:

- Material deforms more easily under the same stress.

1.24 Important Concept: Steel vs Rubber

A common conceptual question is:

Which is more elastic: steel or rubber?

In physics, **steel is more elastic than rubber** because steel has a much larger Young's modulus.
For the same stress: **Strain ∝ 1/Y**
Therefore, the material with larger Y undergoes smaller deformation.
Thus, steel resists deformation more strongly than rubber.

Important:

Elasticity ≠ ability to undergo large deformation
Rubber can undergo large deformation, but steel is more elastic in the physics sense.

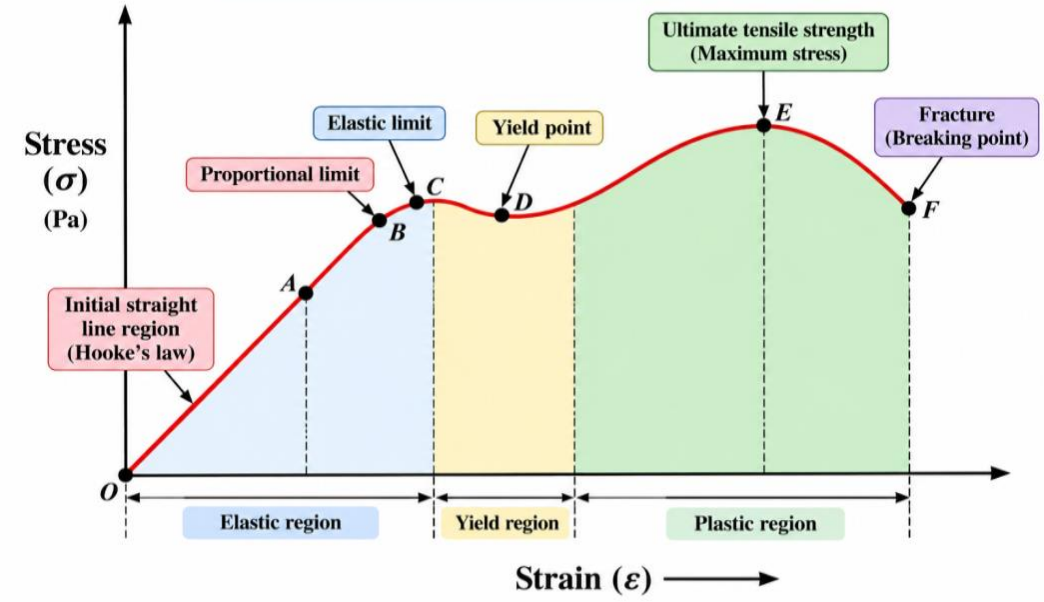
1.25 Stress–Strain Relationship

The relationship between stress and strain is extremely important for understanding the mechanical behaviour of solids.
For a typical ductile material such as mild steel:

- Initially, stress is proportional to strain.
- Then the material enters a region where the relationship is no longer perfectly linear.
- Beyond the elastic region, permanent deformation begins.
- At sufficiently high stress, the material eventually breaks.

The detailed stress–strain curve will be studied in Part 2.

Figure 8.4: Basic Stress–Strain Curve



Note: The exact shape of stress-strain curve and the positions of its characteristic points depend on the material. The curve shown here represents a typical ductile material.

1.26 Elastic Limit

The **elastic limit** is the maximum value of stress up to which a body can completely regain its original shape and size when the deforming force is removed.
If the stress exceeds the elastic limit:

- Permanent deformation may occur.
- The body does not completely return to its original state.

1.27 Proportional Limit

The **proportional limit** is the maximum stress up to which stress remains directly proportional to strain.
Thus, within the proportional limit: **Stress ∝ Strain** and the stress–strain graph is approximately a straight line.

Important distinction

- **Proportional limit:** stress is proportional to strain.
- **Elastic limit:** body completely regains original shape.

For many materials, these limits are close but conceptually different.

1.28 Why Is Hooke's Law Important?

Hooke's law provides the basic relationship between force and deformation within the elastic range.
For a wire: **Stress ∝ Strain**
Therefore: **$F/A \propto \Delta L/L$**
For a particular wire: **$F \propto \Delta L$**
This is the basis of the spring-like behaviour of an elastic wire within the applicable range.

1.29 Factors Affecting Elastic Behaviour

The elastic behaviour of a material depends on several factors, including:

- Nature of the material
- Temperature
- Impurities
- Heat treatment
- Mechanical treatment

For example, heating can change the elastic properties of many materials.
These effects are important in practical applications and will be discussed further in later parts.

1.30 Worked Example 1 – Longitudinal Strain

Question

A wire of original length 2 m is stretched by 1 mm. Calculate its longitudinal strain.
Given: $L = 2$ m; $\Delta L = 1$ mm $= 1 \times 10^{-3}$ m;
Therefore: Longitudinal strain $= \Delta L/L = (1 \times 10^{-3})/2$
Hence: **Longitudinal strain $= 5 \times 10^{-4}$**
Answer: 5×10^{-4}

1.31 Worked Example 2 – Stress

Question

A force of 100 N acts normally on a wire having cross-sectional area 2×10^{-6} m². Calculate the stress.

Solution

Stress = F/A = $100/(2 \times 10^{-6})$
Therefore: **Stress = 5×10^{7} Pa**

Answer: 5×10^{7} Pa

1.32 Worked Example 3 – Shear Strain

Question

The upper surface of a block of height 0.5 m is displaced horizontally by 1 mm. Calculate the shear strain.

Solution

$\Delta x = 1 \times 10^{-3}$ m, L = 0.5 m
Shear strain = $\Delta x/L = 10^{-3}/0.5$
Therefore: **Shear strain = 2×10^{-3}**

1.33 NEET/JEE Quick Insights

Insight 1

Stress has dimensions of pressure: **$[\text{Stress}] = ML^{-1}T^{-2}$**

Insight 2

Strain is dimensionless.

Insight 3

Elastic modulus has the same dimensions as stress: **$[\text{Elastic modulus}] = ML^{-1}T^{-2}$**

Insight 4

For the same stress: **Strain $\propto$ 1/Elastic modulus**
Therefore, greater modulus means greater stiffness.

Insight 5

For small angular deformation: **Shear strain $\approx \theta$**
Where, θ is in radians.

The complete conceptual chain is:
External Force → Deformation → Internal Restoring Force → Stress → Strain → Elastic Modulus → Mechanical Response of Material
The central relation to remember is: **Stress = Elastic Modulus × Strain**
and, within the proportional region: **Stress $\propto$ Strain**

PART 2 – YOUNG'S MODULUS, STRESS–STRAIN CURVE AND ELASTIC BEHAVIOUR

2.1 Learning Objectives

After studying this part, you will be able to:

- Define Young's modulus.
- Derive the expression for extension of a wire.
- Understand the stress–strain curve for a ductile material.
- Identify proportional limit, elastic limit, yield point, ultimate tensile stress and breaking point.
- Distinguish between elastic and plastic behaviour.
- Understand elastic fatigue.
- Solve numerical problems based on Young's modulus.
- Apply these concepts to Board, NEET and JEE questions.

2.2 Young's Modulus

Young's modulus is the ratio of **longitudinal stress** to **longitudinal strain**, provided the deformation is within the elastic limit.

Therefore,

Y = Longitudinal stress / Longitudinal strain

Since

Longitudinal stress = F/A and

Longitudinal strain = $\Delta L/L$, we get: $\mathbf{Y = (F/A) / (\Delta L/L)}$

Therefore,

$$\mathbf{Y = FL / (A\Delta L)}$$

Where:

- F = applied force
- L = original length of wire
- A = cross-sectional area
- ΔL = change in length

2.3 SI Unit and Dimensions of Young's Modulus

Since Young's modulus is stress divided by strain, and strain is dimensionless:

SI unit of Y = N/m^2

Therefore,

SI unit = Pa (pascal)

Dimensional formula: $\mathbf{[Y] = ML^{-1}T^{-2}}$

Thus, Young's modulus has the same dimensions and SI unit as pressure and stress.

2.4 Physical Meaning of Young's Modulus

Young's modulus tells us how strongly a material resists a change in its length.

From:

Y = Stress/Strain

We get:

Strain = Stress/Y

For the same applied stress:

- Larger Y → smaller strain
- Smaller Y → larger strain

Therefore, a material with a larger Young's modulus is **more resistant to longitudinal deformation**.

2.5 Extension of a Uniform Wire

Consider a wire of:

- Length = L
- Cross-sectional area = A
- Applied force = F
- Extension = ΔL

Young's modulus: $\mathbf{Y = FL/(A\Delta L)}$

Rearranging: $\mathbf{\Delta L = FL/(AY)}$

This is one of the most important formulas of the chapter.

2.6 Factors Affecting Extension of a Wire

From: $\mathbf{\Delta L = FL/(AY)}$

We can see:

Extension is directly proportional to force: $\Delta L \propto F$

Extension is directly proportional to original length: $\Delta L \propto L$

Extension is inversely proportional to cross-sectional area: $\Delta L \propto 1/A$

Extension is inversely proportional to Young's modulus: $\Delta L \propto 1/Y$

Therefore, a long and thin wire undergoes greater extension than a short and thick wire made of the same material under the same force.

2.7 Effect of Radius of Wire

For a circular wire: $\mathbf{A = \pi r^2}$

Therefore,

$$\mathbf{\Delta L = FL/(\pi r^2 Y)}$$

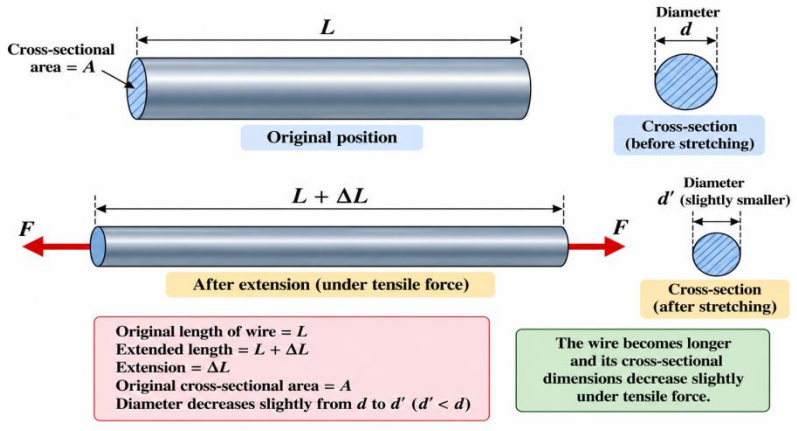
Hence: $\mathbf{\Delta L \propto 1/r^2}$

If the radius of the wire is doubled: $\mathbf{r' = 2r}$

Then: $\mathbf{\Delta L'/\Delta L = 1/4}$

Therefore, the extension becomes one-fourth.

Figure 8.5: Extension of a Wire Under Tensile Force



2.8 Effect of Diameter

Since: $\mathbf{A = \pi d^2/4}$

We get: $\mathbf{\Delta L = 4FL / (\pi d^2 Y)}$

Thus: $\mathbf{\Delta L \propto 1/d^2}$

This relation is frequently useful in numerical problems.

2.9 Equivalent Spring Constant of a Wire

A stretched wire behaves like an elastic element.

For a wire: $\mathbf{F = Y\ A\ \Delta L/L}$

Therefore,

$$F = (YA/L)\,\Delta L$$

Comparing with Hooke's law:

$$F = k\Delta L$$

We obtain:

$$k = YA/L$$

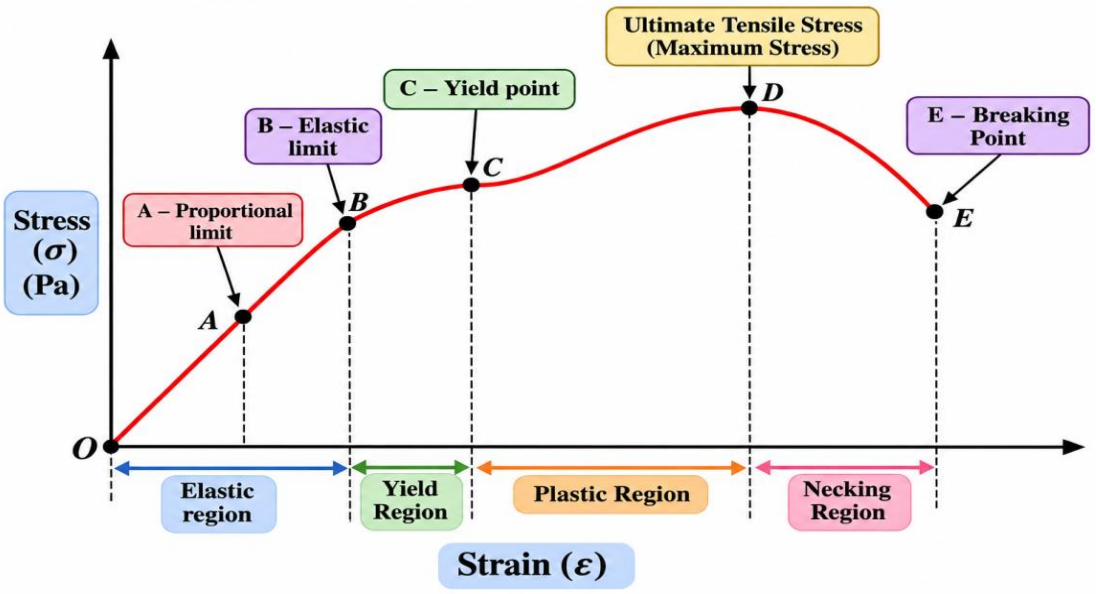
Where k is the effective spring constant of the wire.
Thus: **k ∝ A** and **k ∝ 1/L**
A thicker wire is effectively stiffer, while a longer wire is less stiff.

2.10 Stress–Strain Curve

The stress–strain curve gives the relationship between stress and strain for a material.
For a typical ductile material such as mild steel, the curve shows different regions of elastic and plastic behaviour.
It is generally plotted as:

- Vertical axis → Stress
- Horizontal axis → Strain

Figure 8.6: Stress - Strain Curve for a Ductile Material



2.11 Region OA – Proportional Region

From O to A:
Stress ∝ Strain
Therefore, the graph is approximately a straight line.
This is the region in which Hooke's law is obeyed.
The slope of the stress–strain graph in this region is:
Slope = Stress/Strain
Therefore: **Slope = Young's modulus**
Hence, a steeper initial slope indicates a larger Young's modulus.

2.12 Point A – Proportional Limit

Point A represents the **proportional limit**.
Up to this point: **Stress ∝ Strain**
Therefore, the material follows Hooke's law approximately.
Beyond A, stress is no longer directly proportional to strain.

2.13 Point B – Elastic Limit

Point B represents the **elastic limit**.
Up to B, if the external force is removed, the material can return to its original shape and size.
If the stress exceeds the elastic limit, permanent deformation begins.
Thus:
Within elastic limit: No permanent deformation
Beyond elastic limit: Permanent deformation may remain.

2.14 Region Between A and B

Between A and B:

- Stress is no longer exactly proportional to strain.
- However, the material can still recover its original dimensions when the stress is removed.

Therefore, the material is still behaving elastically, even though Hooke's law is no longer strictly obeyed.

2.15 Yield Point

Beyond the elastic limit, the material enters the **yield region**.
A relatively small increase in stress may produce a large increase in strain.
The material begins to undergo significant permanent deformation.
This phenomenon is called **yielding**.
The corresponding stress is called **yield stress**.

2.16 Plastic Region

Beyond the elastic limit, the material enters the **plastic region**.
In this region:

- Deformation becomes permanent.
- The material does not completely recover its original shape when the force is removed.
- Increasing stress produces substantial deformation.

This behaviour is called **plastic deformation**.

2.17 Ultimate Tensile Stress

As the wire is stretched further, the stress eventually reaches a maximum value.
The **ultimate tensile stress (UTS)** is the maximum engineering stress reached during a tensile test. For a typical ductile material, necking generally begins after the specimen reaches the maximum engineering stress.
It represents the maximum stress that the specimen can withstand during the tensile test before significant failure develops.
On the typical curve, it corresponds to the highest point of the engineering stress curve.

2.18 Necking

After reaching the ultimate tensile stress, the wire may begin to develop a localized reduction in cross-sectional area.
This phenomenon is called **necking**.
The deformation becomes concentrated in a small region of the specimen.
As a result, the specimen becomes progressively thinner at the neck.

2.19 Breaking Point or Fracture Point

After sufficient necking, the material eventually breaks.
The corresponding point is called the **breaking point** or **fracture point**.
At this stage:

- The specimen can no longer sustain the tensile load.
- The material separates into two parts.

2.20 Elastic Region and Plastic Region

Elastic Region	Plastic Region
Deformation is recoverable	Deformation is permanent
Material returns toward original shape	Permanent strain remains
No permanent deformation after unloading	Permanent deformation remains
Ends at elastic limit	Begins beyond elastic limit

2.21 Important Points of Stress–Strain Curve

Point/Region	Meaning
O	Original undeformed state
A	Proportional limit
B	Elastic limit
C	Yield region/point
D	Ultimate tensile stress
E	Breaking/fracture point

Note: Exact shapes and number of yield points can vary between materials. The simplified curve is used for understanding the main concepts.

2.22 Ductile and Brittle Materials

Materials can show different responses to stress.

Ductile Materials

Ductile materials can undergo considerable plastic deformation before breaking.
Examples:

- Mild steel
- Copper
- Aluminium

They generally show a significant plastic region.

Brittle Materials

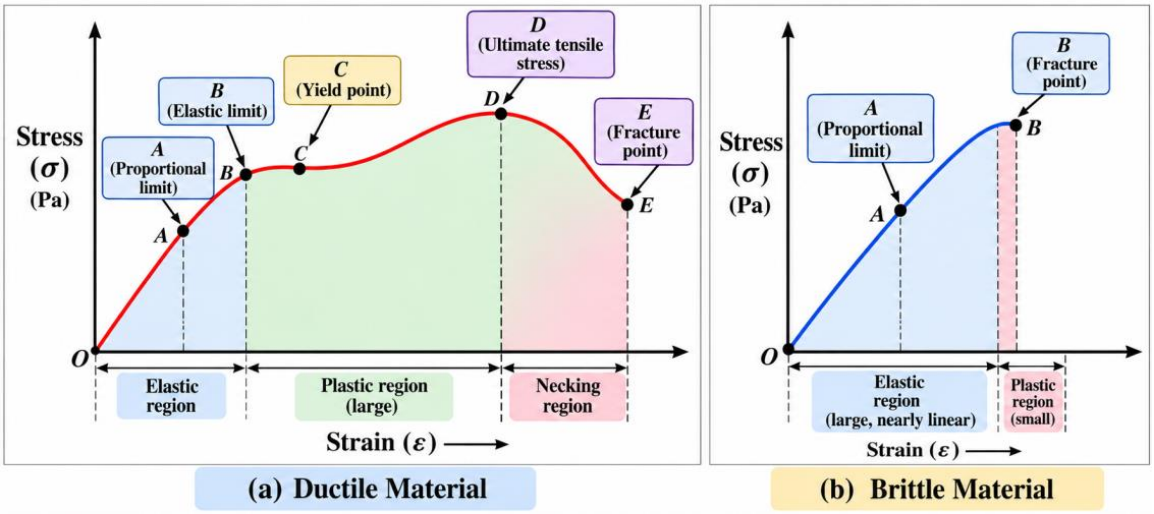
Brittle materials fracture after relatively small plastic deformation.
Examples:

- Glass

- Ceramics
- Cast iron

Their stress–strain curves show little plastic deformation compared with ductile materials.

Figure 8.7: Ductile and Brittle Stress–Strain Curves



2.23 Elastic Fatigue

If a material is subjected to repeated loading and unloading, its elastic behaviour may change.
The phenomenon in which a material becomes weaker or its elastic properties change due to repeated cyclic loading is called **elastic fatigue**.
For example, a metal component subjected to repeated bending may eventually develop cracks even when the applied stress is below its usual breaking stress.
Elastic fatigue is important in:

- Bridges
- Aircraft structures
- Machine components
- Rotating shafts
- Railway tracks

2.24 Why Is Elastic Fatigue Important?

Engineering structures often experience repeated loads rather than a single load.
Therefore, a material must be designed not only to withstand maximum stress but also repeated stress cycles.
This is one reason why engineers consider **fatigue strength** and **safe working stress** while designing structures.

2.25 Working Stress and Factor of Safety

In engineering applications, structures are generally not operated close to their ultimate strength.
A suitable safety margin is maintained.
The **factor of safety** is commonly defined as:
Factor of safety = Limiting stress / Working stress

Depending on the application, the limiting stress may be taken as yield stress or another specified design limit.
Therefore:
Working stress = Limiting stress / Factor of safety
A larger factor of safety means a larger margin between working conditions and the limiting condition.

2.26 Solved Example 1 – Young's Modulus

Question

A wire of length 2 m and cross-sectional area 1×10^{-6} m² is stretched by a force of 100 N. If its extension is 1 mm, calculate Young's modulus.

Given: L = 2 m, A = 1×10^{-6} m², F = 100 N, $\Delta L = 1 \times 10^{-3}$ m
Using: $\mathbf{Y = FL/(A\Delta L)}$
Therefore: $Y = (100 \times 2)/[(1 \times 10^{-6})(1 \times 10^{-3})]$
Hence: $\mathbf{Y = 2 \times 10^{11}\ Pa}$
Answer: $\mathbf{Y = 2 \times 10^{11}\ Pa}$

2.27 Solved Example 2 – Extension of a Wire

Question

A wire has length 1 m, cross-sectional area 2×10^{-6} m² and Young's modulus 2×10^{11} Pa. Find its extension when a force of 100 N is applied.

Solution

Using: $\mathbf{\Delta L = FL/(AY)}$
Substituting:
$\Delta L = (100 \times 1) / [(2 \times 10^{-6})(2 \times 10^{11})]$
Therefore: $\mathbf{\Delta L = 2.5 \times 10^{-4}\ m}$ or $\mathbf{\Delta L = 0.25\ mm}$
Answer: **0.25 mm**

2.28 Solved Example 3 – Effect of Length

Question

Two wires are made of the same material and have the same cross-sectional area. The length of the second wire is twice that of the first. If the same force is applied to both, compare their extensions.

Solution

For a wire: $\mathbf{\Delta L = FL/(AY)}$
For the same F, A and Y: $\mathbf{\Delta L \propto L}$
Therefore: $\mathbf{\Delta L_2/\Delta L_1 = L_2/L_1 = 2}$
Hence: $\mathbf{\Delta L_2 : \Delta L_1 = 2 : 1}$

2.29 Solved Example 4 – Effect of Radius

Question

Two wires of the same material and length are subjected to the same force. Their radii are r and 2r. Find the ratio of their extensions.

Solution

For a wire: $\mathbf{\Delta L \propto 1/r^2}$

Therefore:
$\Delta L_1/\Delta L_2 = (r_2/r_1)^2 = (2r/r)^2 = 4$
Hence: **$\Delta L_1 : \Delta L_2 = 4 : 1$**
The thinner wire stretches four times as much.

2.30 Solved Example 5 – Spring Constant of a Wire

Question

A wire has Young's modulus Y, length L and cross-sectional area A. Find its effective spring constant.

Solution

Young's modulus: **$Y = FL/(A\Delta L)$**
Therefore: $F = (YA/L)\Delta L$
Comparing with: $F = k\Delta L$
We get: **$k = YA/L$**

2.31 NEET/JEE Quick Insights

Insight 1: For a wire: **$\Delta L = FL/(AY)$**
This formula should be memorized.

Insight 2: If length doubles: **ΔL doubles** provided F, A and Y remain unchanged.

Insight 3: If area doubles: **ΔL becomes half**

Insight 4: If radius doubles: **ΔL becomes one-fourth**

Insight 5: If Young's modulus doubles: **ΔL becomes half**

Insight 6: The slope of the initial straight-line portion of the stress–strain curve gives: **Young's modulus**

Insight 7: A larger slope means a larger Young's modulus.

Master Concept – Part 2

The most important mathematical chain is:
Stress = F/A → Strain = $\Delta L/L$ → Young's Modulus = Stress/Strain → $Y = FL/(A\Delta L)$ → $\Delta L = FL/(AY)$
And the most important graphical concept is:
Slope of initial stress–strain curve = Young's modulus
while the stress–strain curve tells us whether the material is behaving **elastically, plastically, yielding, necking or finally fracturing**.

PART 3 – BULK MODULUS, SHEAR MODULUS, ELASTIC POTENTIAL ENERGY AND APPLICATIONS

3.1 Learning Objectives

After studying this part, you will be able to:

- Understand bulk modulus and volumetric deformation.
- Derive the expression for bulk modulus.
- Understand compressibility.
- Understand shear modulus and shear deformation.
- Establish the relation between shear stress and shear strain.
- Calculate elastic energy stored in a deformed body.
- Understand energy density of an elastic material.
- Apply elastic-property concepts to practical situations.
- Solve Board, NEET and JEE-level numerical problems.

3.2 Bulk Modulus

When an external pressure is applied uniformly to all surfaces of a body, its volume changes. The bulk modulus is the negative ratio of the increase in pressure to the corresponding fractional change in volume, for small uniform compression.

It is represented by **K**.

Therefore,

$$\textbf{K = Bulk stress / Volumetric strain}$$

If an increase in pressure p produces a decrease in volume ΔV, then:

$$\mathbf{K = -p / (\Delta V/V)} \quad \text{or} \quad \mathbf{K = -pV/\Delta V}$$

The negative sign is used because an increase in pressure produces a decrease in volume.

3.3 Why Is There a Negative Sign in Bulk Modulus?

Suppose:

- Initial volume = V
- Final volume = $V + \Delta V$

During compression: $\mathbf{\Delta V < 0}$

But pressure p is taken as positive.

Therefore, to make K positive: $\mathbf{K = -p/(\Delta V/V)}$

Hence: $\mathbf{K = -pV/\Delta V}$

3.4 SI Unit and Dimensions of Bulk Modulus

Since:

$$\mathbf{K = Stress/Strain}$$

and strain is dimensionless:

SI unit of K = N/m^2 = Pa

Dimensional formula: $\mathbf{[K] = ML^{-1}T^{-2}}$

Thus, bulk modulus has the same dimensions as pressure and stress.

3.5 Physical Meaning of Bulk Modulus

Bulk modulus measures the resistance of a material to **change in volume**.
A large value of K means:

- The material is difficult to compress.
- A large pressure produces only a small fractional change in volume.

A small value of K means:

- The material is easier to compress.
- A relatively small pressure produces a larger change in volume.

3.6 Compressibility

The reciprocal of bulk modulus is called **compressibility**.
It is represented by **β**.
Therefore,

$$\beta = 1/K$$

Since:

$$K = -p/(\Delta V/V)$$

We can write:

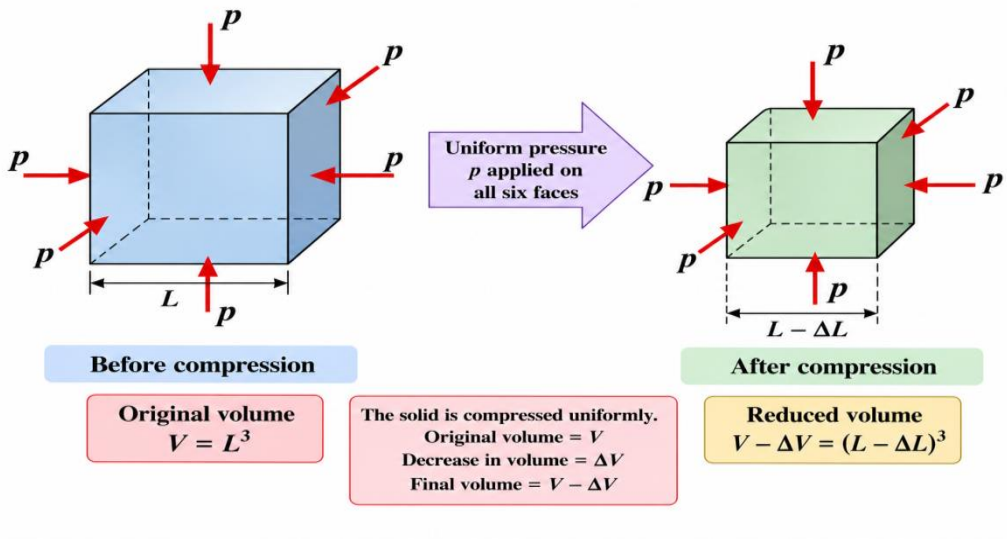
$$\beta = -(\Delta V/V)/p$$

The SI unit of compressibility is:
Pa^{-1}

3.7 Liquids and Bulk Modulus

Liquids have a definite volume but do not have a definite shape.
They can undergo volume changes when subjected to pressure.
Therefore, the bulk modulus is particularly useful for describing the compressibility of liquids.
Liquids generally have a large bulk modulus compared with gases, meaning they are relatively difficult to compress.

Figure 8.8: Uniform Compression of a body



3.8 Shear Modulus

When a tangential force changes the shape of a body without appreciably changing its volume, the deformation is called **shear deformation**.
The ratio of **shear stress** to **shear strain** is called the **shear modulus**.
It is represented by **G**.
Therefore,

G = Shear stress / Shear strain

Since: **Shear stress = F/A** and **Shear strain = Δx/L**
We get: **G = (F/A) / (Δx/L)**
Therefore,

$$G = FL/(A\,\Delta x)$$

3.9 Shear Modulus in Terms of Angular Deformation

Shear modulus is:

$$G = \frac{Shear\ Stress}{Shear\ Strain}$$

For a small angular deformation:
Shear strain $\gamma \approx \theta$
Therefore,

$$G = \frac{Shear\ Stress}{\theta}$$

Or,

$$G = \frac{F}{A\theta}$$

Where, θ is measured in radians.

3.10 SI Unit and Dimensions of Shear Modulus

The SI unit is: **N/m^2 = Pa**
Dimensional formula: **$[G] = ML^{-1}T^{-2}$**
Thus:
Young's modulus, bulk modulus and shear modulus all have the same dimensions and SI unit.
Their physical meanings, however, are different.

3.11 Comparison of Three Elastic Moduli

Modulus	Type of deformation	Expression
Young's modulus Y	Change in length	Y = Longitudinal stress / Longitudinal strain
Bulk modulus K	Change in volume	K = -Bulk stress / Volumetric strain
Shear modulus G	Change in shape	G = Shear stress / Shear strain

3.12 Important Concept: Solids vs Liquids

A solid can resist:

- Change in length
- Change in volume
- Change in shape

Therefore, solids have:

- Young's modulus
- Bulk modulus
- Shear modulus

A liquid cannot sustain a static shear stress.
Therefore, for a liquid: **Shear modulus ≈ 0**
However, liquids do possess a finite bulk modulus.
This is an important conceptual point.

3.13 Elastic Potential Energy

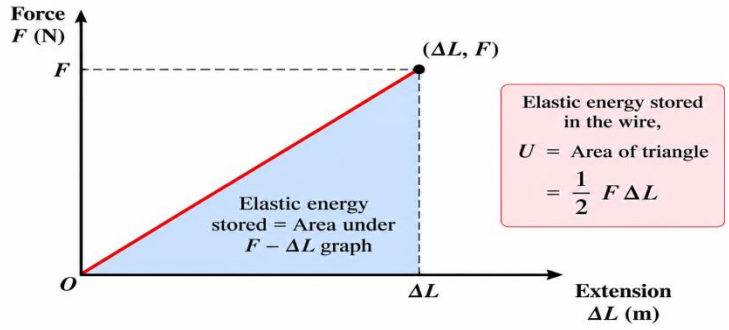
When an elastic body is deformed, work is done against its internal restoring forces.
This work is stored in the body as **elastic potential energy**.
For example:

- A stretched wire stores elastic energy.
- A compressed spring stores elastic energy.
- A deformed elastic body stores mechanical energy.

3.14 Energy Stored in a Stretched Wire

Consider a wire stretched by an amount ΔL.
Suppose the applied force increases gradually from 0 to F.
Within the elastic region: **$F \propto \Delta L$**
Therefore, the force–extension graph is a straight line.
The work done in stretching the wire is equal to the area under the F–ΔL graph.
Since the graph is a triangle:
$W = 1/2 \times F \times \Delta L$
Therefore: **$U = 1/2\ F\Delta L$**
where U is the elastic potential energy stored in the wire.

Figure 8.9: Force- Extension Graph



3.15 Elastic Energy Using Young's Modulus

We know: **$Y = FL/(A\Delta L)$**
Therefore: **$\Delta L = FL/(AY)$**
Substituting in: **$U = 1/2\ F\Delta L$**
We get: **$U = 1/2\ F \times FL/(AY)$**
Hence: **$U = F^2L / (2AY)$**

3.16 Elastic Energy in Terms of Stress and Strain

We know:

$$U = \frac{1}{2} F \Delta L$$

Divide both sides by the volume of the wire.
Volume: **V = AL**
Therefore:

$$\frac{U}{V} = \frac{\frac{1}{2} F \Delta L}{AL}$$

Rearranging:

$$\frac{U}{V} = \frac{1}{2}\left(\frac{F \Delta L}{AL}\right)$$

But: **F/A = Stress** and **ΔL/L = Strain**
Therefore:

Energy density = 1/2 × Stress × Strain

This is an important result.

3.17 Energy Density

Energy stored per unit volume of a deformed elastic body is called **energy density**.
Therefore:

Energy density = U/V

and within the linear elastic region:

Energy density = 1/2 × Stress × Strain

Since: **Stress = Elastic modulus × Strain**
we can also write, for longitudinal deformation:

Energy density = 1/2 Y(Strain)² or **Energy density = Stress²/(2Y)**

These forms are useful in numerical problems.

3.18 Important Energy Formulae

For a stretched wire:

Total elastic energy: U = 1/2 FΔL

Using force and wire properties: U = F²L/(2AY)

Energy density: u = U/V

u = 1/2 × Stress × Strain

In terms of Young's modulus: u = 1/2 Y(Strain)²

In terms of stress: u = Stress²/(2Y)

3.19 Stress–Strain Graph and Elastic Energy

The area under the stress–strain curve represents energy stored per unit volume.
In the linear elastic region, the curve is a straight line.
Therefore, the area is triangular:

Energy density = 1/2 × Stress × Strain

This gives an important graphical interpretation of elastic energy.

3.20 Worked Example 1 – Bulk Modulus

Question

A pressure of 2×10^7 Pa produces a fractional decrease in volume of 0.001. Calculate the bulk modulus.

Solution

Given: $p = 2 \times 10^7$ Pa, $\Delta V/V = -0.001$

Using: $\mathbf{K = -p/(\Delta V/V)}$

Therefore: $K = -(2 \times 10^7)/(-0.001)$

Hence: $\mathbf{K = 2 \times 10^{10}\ Pa}$

3.21 Worked Example 2 – Shear Modulus

Question

A tangential force of 100 N acts on an area of 0.01 m². The upper surface of a block of height 0.5 m is displaced by 1 mm. Find the shear modulus.

Solution

Given: $F = 100$ N, $A = 0.01$ m², $L = 0.5$ m, $\Delta x = 1 \times 10^{-3}$ m

Shear stress: $\mathbf{F/A = 100/0.01 = 10^4\ Pa}$

Shear strain: $\mathbf{\Delta x/L = 10^{-3}/0.5 = 2 \times 10^{-3}}$

Therefore: $G = \text{Stress/Strain} = 10^4/(2 \times 10^{-3})$

Hence: $\mathbf{G = 5 \times 10^6\ Pa}$

3.22 Worked Example 3 – Elastic Energy

Question

A wire is stretched by a force of 200 N and its extension is 2 mm. Calculate the elastic energy stored.

Solution

Using: $\mathbf{U = 1/2\ F\Delta L}$

Given: $F = 200$ N, $\Delta L = 2 \times 10^{-3}$ m

Therefore: $U = 1/2 \times 200 \times 2 \times 10^{-3}$

Hence: $\mathbf{U = 0.2\ J}$

3.23 Worked Example 4 – Effect of Cross-sectional Area

Question

Two wires of the same material and length are stretched by the same force. Their cross-sectional areas are A and 2A. Compare the extensions.

Solution

We know: $\mathbf{\Delta L = FL/(AY)}$

For the first wire: $\Delta L_1 = FL/(AY)$

For the second: $\Delta L_2 = FL/(2AY)$

Therefore: $\mathbf{\Delta L_1 : \Delta L_2 = 2 : 1}$

The wire with twice the area has half the extension.

3.24 Worked Example 5 – Energy Density

Question

A material is subjected to a stress of 2×10^8 Pa and produces a strain of 0.001. Find the elastic energy stored per unit volume in the linear elastic region.

Solution

Energy density: **u = 1/2 × Stress × Strain**

Therefore: $u = 1/2 \times (2 \times 10^8) \times 10^{-3}$

Hence: $\mathbf{u = 1 \times 10^5\ J/m^3}$

3.25 Practical Applications of Elasticity

Knowledge of elasticity is essential in engineering and construction.

1. Bridges

Bridges must be designed so that the stresses produced by loads remain within safe limits.

2. Buildings

Structural materials must withstand:

- Compression
- Tension
- Shear
- Repeated loading

3. Cranes

The cables of cranes must withstand large tensile stresses without excessive deformation.

4. Railway Tracks

Railway tracks experience repeated mechanical loading and therefore require suitable mechanical strength and fatigue resistance.

5. Aircraft Structures

Aircraft components must be strong, sufficiently stiff and able to withstand repeated stresses.

3.26 Why Are Girders Often Made I-Shaped?

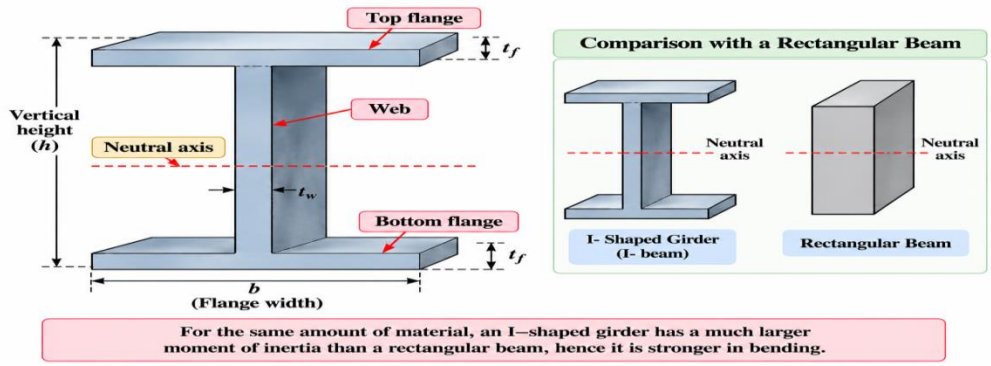
This is an important application of elasticity.

A beam or girder needs to resist bending.

An I-shaped cross-section places more material away from the neutral axis while reducing unnecessary material near the centre.

This provides high bending stiffness for relatively low mass.

Figure 8.10: I- Shaped Girder



3.27 Why Are Structural Materials Designed with a Safety Factor?

A structure may experience:

- Unexpected loads
- Repeated loading
- Temperature changes
- Material imperfections
- Manufacturing variations
- Vibrations

Therefore, structures are designed so that the working stress remains safely below the limiting stress.
This provides a margin of safety.

3.28 Important Conceptual Questions

Q1. Which material is more elastic: steel or rubber?

Answer: Steel, because its Young's modulus is much larger.

Q2. Why is strain dimensionless?

Because strain is the ratio of two quantities having the same dimensions.

Q3. Why is the bulk modulus written with a negative sign?

Because compression produces a decrease in volume: $\Delta V < 0$ while pressure is taken as positive.

Q4. Can a liquid sustain a static shear stress?

No.
A liquid cannot sustain a static shear stress and therefore does not have a finite shear modulus in the usual static sense.

Q5. What does the area under the stress–strain curve represent?

The area under the stress–strain curve up to a given strain represents the **energy stored per unit volume** under the corresponding loading conditions.

3.29 NEET/JEE Quick Shortcuts

Shortcut 1: If: $K_1 = 2K_2$ then for the same pressure, the fractional volume change of material 1 is half that of material 2.

Shortcut 2: For a wire: $\Delta L \propto L$; $\Delta L \propto F$; $\Delta L \propto 1/A$; $\Delta L \propto 1/Y$

Shortcut 3: For a circular wire: $\Delta L \propto 1/r^2$

Shortcut 4: Elastic energy: $U \propto F^2$ for a fixed wire within the elastic range.

Shortcut 5: Energy density: $u = 1/2\ \sigma\varepsilon$ where: σ = stress, ε = strain

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PART 4 – ADVANCED APPLICATIONS, COMPOSITE WIRES, ELASTICITY AND NUMERICAL APPLICATIONS

4.1 Learning Objectives

After studying this part, you will be able to:

- Solve problems involving wires of different lengths and cross-sectional areas.
- Understand wires connected in series and parallel.
- Calculate equivalent spring constants of elastic wires.
- Understand the effect of temperature on the dimensions of solids.
- Relate thermal strain to mechanical stress when expansion is restricted.
- Understand elastic after-effect and elastic hysteresis qualitatively.
- Apply elasticity concepts to practical engineering situations.
- Solve higher-level Board, NEET and JEE numerical problems.

4.2 Elastic Wire as a Spring

In Part 2, we obtained: $\mathbf{F = (YA/L)\,\Delta L}$

Comparing this with Hooke's law: $\mathbf{F = k\Delta L}$

We get: $\mathbf{k = YA/L}$

This means that an elastic wire behaves like a spring with effective spring constant:

$$\mathbf{k = YA/L}$$

Therefore:

- Larger Y → larger k
- Larger A → larger k
- Larger L → smaller k

4.3 Important Comparison

Consider two wires of the same material.

Since: **Y = same**

their stiffness depends mainly on: $\mathbf{k \propto A/L}$

Therefore:

Thick and short wire → more stiff

Thin and long wire → less stiff

This concept is frequently used in numerical problems.

4.4 Wires Connected in Series

Consider two wires connected end-to-end.

Let:

- Spring constants = k_1 and k_2
- Same force F acts through both wires.

Because the wires are in series, the same force acts in each wire.

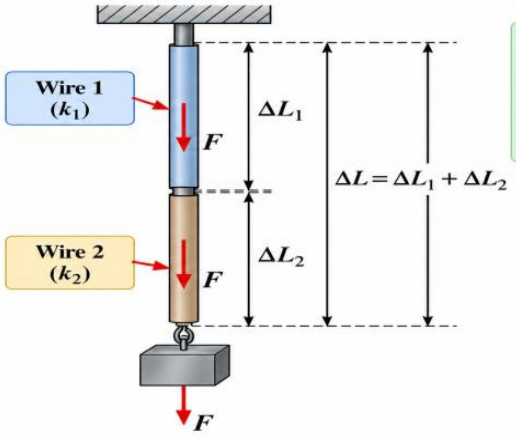
Total extension: $\mathbf{\Delta L = \Delta L_1 + \Delta L_2}$

For each wire: $\mathbf{\Delta L_1 = F/k_1}$ and $\mathbf{\Delta L_2 = F/k_2}$

Therefore: $\Delta L = F/k_1 + F/k_2$

If equivalent spring constant is keq: $\mathbf{\Delta L = F/keq}$

Figure 8.11: Two Elastic Wires in Series



Two elastic wires of spring constants k_1 and k_2 are joined in series and stretched by the same tensile force F.

The total extension is the sum of the extensions of the two wires.

Extension of wire 1:

$$\Delta L_1 = \frac{F}{k_1}$$

Extension of wire 2:

$$\Delta L_2 = \frac{F}{k_2}$$

Total extension:

$$\Delta L = \Delta L_1 + \Delta L_2 = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

Hence: **$1/keq = 1/k_1 + 1/k_2$**

Therefore: **$keq = k_1k_2/(k_1 + k_2)$**

4.5 Wires Connected in Parallel

Consider two elastic wires connected between the same two supports.

The total force is distributed between the wires. Both wires undergo the same extension.

Therefore:

$F = F_1 + F_2$

For each wire:

$F_1 = k_1 \Delta L$

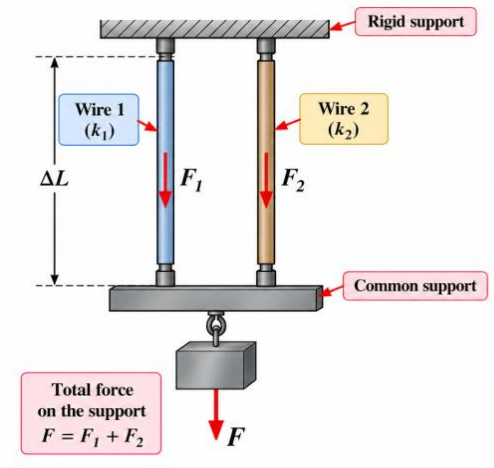
$F_2 = k_2 \Delta L$

Hence: $F = (k_1 + k_2) \Delta L$

Comparing with: **$F = keq \Delta L$**

We obtain: **$keq = k_1 + k_2$**

Figure 8.12: Two Elastic Wires in Parallel



Two elastic wires of spring constants k_1 and k_2 are connected in parallel between a rigid support and a common lower support.

Both wires undergo the same extension ΔL.

Force in wire 1:

$$F_1 = k_1 \Delta L$$

Force in wire 2:

$$F_2 = k_2 \Delta L$$

Total force:

$$F = F_1 + F_2 = (k_1 + k_2)\Delta L$$

Equivalent spring constant:

$$k_{eq} = k_1 + k_2$$

(for parallel combination)

4.6 Equivalent Young's Modulus – Concept

For composite arrangements, an effective Young's modulus can be defined only after specifying the equivalent length, cross-sectional area and the arrangement of the component wires.
The mechanical behaviour can be determined by considering:

- Total force
- Total extension
- Effective cross-sectional area
- Effective length

The basic relation remains:

$$Y = \frac{FL}{A\Delta L}$$

This relation should always be used when dealing with combinations of materials.

4.7 Two Wires of Different Materials in Series

Suppose two wires are connected in series.
Let:

- Lengths = L_1 and L_2
- Areas = A_1 and A_2
- Young's moduli = Y_1 and Y_2

Under the same force F:
For wire 1:

$$\Delta L_1 = FL_1/(A_1Y_1)$$

For wire 2:

$$\Delta L_2 = FL_2/(A_2Y_2)$$

Therefore, total extension is:

$$\Delta L = F[L_1/(A_1Y_1) + L_2/(A_2Y_2)]$$

This expression is very useful for composite-wire problems.

4.8 Two Wires of Different Materials in Parallel

When wires are arranged in parallel:

- Their extensions are the same.
- Their forces may be different.

For each wire:

$$F_1 = Y_1A_1\Delta L/L_1$$
$$F_2 = Y_2A_2\Delta L/L_2$$

Therefore:

$$F(total) = F_1 + F_2 \text{ or}$$
$$F(total) = \Delta L(Y_1A_1/L_1 + Y_2A_2/L_2)$$

This is useful for advanced numerical problems.

4.9 Thermal Expansion of Solids

The dimensions of a solid generally change when its temperature changes.
When temperature increases, most solids expand.
For a rod of original length L:

Change in length: **$\Delta L = \alpha L \Delta T$**
Where:

- α = coefficient of linear expansion
- ΔT = change in temperature

Therefore: **Linear strain = $\Delta L/L$**
Hence: **Linear thermal strain = $\alpha \Delta T$**

4.10 Thermal Strain

Thermal strain is the fractional change in length produced due to a change in temperature.
Therefore: **Thermal strain = $\Delta L/L$**
Using: $\Delta L = \alpha L \Delta T$
We get: **Thermal strain = $\alpha \Delta T$**
This is dimensionless.

4.11 Thermal Stress When Expansion Is Prevented

Suppose a rod is heated but its expansion is completely prevented.
Normally, it would expand by: **$\Delta L = \alpha L \Delta T$**
But because expansion is prevented, an internal compressive stress develops.
The corresponding strain that would have occurred is: **$\alpha \Delta T$**
Using Young's modulus: **Stress = Y × Strain**
Therefore: **Thermal stress = $Y\alpha \Delta T$**
This is an important result.
Important Note: The expression $\sigma = Y\alpha\Delta T$ gives the magnitude of thermal stress when expansion is completely prevented. For heating, the stress is compressive; for cooling, it is tensile, provided the rod is fully constrained and the material remains in the linear elastic range.

4.12 Thermal Stress: Physical Interpretation

If a rod is heated freely:

- It expands.
- No thermal stress develops due solely to the free expansion.

If the rod is completely prevented from expanding:

- Thermal strain cannot occur.
- Internal stress develops.

Thus:

Free expansion → No thermal stress
Restricted expansion → Thermal stress

4.13 Applications of Thermal Stress

Thermal stresses are important in:

- Railway tracks
- Bridges
- Concrete structures
- Pipelines
- Machine components
- Long metal structures

For example, railway tracks are provided with suitable arrangements to accommodate thermal expansion.

4.14 Elastic After-Effect

When a deforming force is removed, a material may not immediately return to its original configuration.
The gradual recovery of the original shape after removal of the deforming force is called **elastic after-effect**.
The time required for complete recovery depends on the material.
This phenomenon is important when studying materials subjected to changing loads.

4.15 Elastic Hysteresis

When a material is loaded and then unloaded, the strain during unloading may not follow exactly the same path as during loading.
The resulting difference between the loading and unloading curves is called **elastic hysteresis**.
The area enclosed by the loading and unloading curves represents energy lost per unit volume during one complete loading cycle.
This energy is converted mainly into heat.

4.16 Elasticity and Temperature

Temperature can affect the elastic properties of a material.
For many materials, increasing temperature changes:

- Young's modulus
- Yield behaviour
- Elastic limit
- Strength

The exact effect depends on the material.
Therefore, engineering structures must be designed considering the operating temperature range.

4.17 Worked Example 1 – Wires in Series

Question

Two wires have spring constants 100 N/m and 200 N/m and are connected in series. Find the equivalent spring constant.

Solution

For series connection: $\mathbf{1/k_{eq} = 1/k_1 + 1/k_2}$
Therefore: $1/k_{eq} = 1/100 + 1/200 = 3/200$
Hence: $\mathbf{k_{eq} = 200/3\ N/m}$ or approximately: $\mathbf{k_{eq} \approx 66.7\ N/m}$

4.18 Worked Example 2 – Wires in Parallel

Question

Two elastic wires have spring constants 100 N/m and 200 N/m and are connected in parallel. Find the equivalent spring constant.

Solution

For parallel connection: $\mathbf{k_{eq} = k_1 + k_2}$
Therefore: $k_{eq} = 100 + 200$

Hence: $k_{eq} = 300$ N/m

4.19 Worked Example 3 – Thermal Stress

Question

A metal rod has Young's modulus 2×10^{11} Pa and coefficient of linear expansion $1 \times 10^{-5}\ K^{-1}$. If its expansion is completely prevented and its temperature is increased by 50 K, calculate the thermal stress.

Solution

Given:

- $Y = 2 \times 10^{11}\ Pa$
- $\alpha = 1 \times 10^{-5}\ K^{-1}$
- $\Delta T = 50\ K$

Thermal stress:

$$\sigma = Y\alpha\Delta T$$

Therefore:

$$\sigma = (2 \times 10^{11})(1 \times 10^{-5})(50) = 1 \times 10^{8}\ Pa$$

Answer:

$$\boxed{Thermal\ Stress, \sigma = 1 \times 10^{8}\ Pa}$$

The stress is compressive because the rod is prevented from expanding.

4.20 Worked Example 4 – Thermal Expansion

Question

A rod of length 2 m has coefficient of linear expansion 1.5×10^{-5} K^{-1}. Find its increase in length when its temperature rises by 100 K.

Solution

Using: $\mathbf{\Delta L = \alpha L \Delta T}$

Therefore: $\Delta L = (1.5 \times 10^{-5})(2)(100)$

Hence: $\mathbf{\Delta L = 3 \times 10^{-3}\ m}$ or $\mathbf{\Delta L = 3\ mm}$

4.21 Worked Example 5 – Extension of Two Different Wires

Two wires are connected in series.

For wire 1:

- $L_1 = 1$ m
- $A_1 = 1 \times 10^{-6}$ m^2
- $Y_1 = 2 \times 10^{11}$ Pa

For wire 2:

- $L_2 = 2$ m
- $A_2 = 2 \times 10^{-6}$ m^2
- $Y_2 = 1 \times 10^{11}$ Pa

A force F = 100 N is applied.

Find the total extension.

Solution

For wire 1:

$\mathbf{\Delta L_1 = FL_1/(A_1Y_1)} = 100 \times 1 / [(1 \times 10^{-6})(2 \times 10^{11})]$

Therefore: $\mathbf{\Delta L_1 = 5 \times 10^{-4}\ m}$

For wire 2:
$\Delta L_2 = FL_2/(A_2Y_2) = 100 \times 2 / [(2 \times 10^{-6})(1 \times 10^{11})]$
Therefore: $\Delta L_2 = 1 \times 10^{-3}$ m
Total extension: $\Delta L = \Delta L_1 + \Delta L_2 = 0.5 \times 10^{-3} + 1 \times 10^{-3}$
Hence: $\Delta L = 1.5 \times 10^{-3}$ m or $\Delta L = 1.5$ mm

---

4.22 NEET/JEE Concept: Same Material Wires

If two wires are made of the same material: $Y_1 = Y_2$
Then: $\Delta L = FL/(AY)$
This means comparisons can often be made directly using: $\Delta L \propto L/A$
This saves time in multiple-choice questions.

4.23 Important Ratio-Based Results

For the same material and same applied force:
If length doubles: ΔL doubles
If area doubles: ΔL becomes half
If radius doubles: ΔL becomes one-fourth
If diameter doubles: ΔL becomes one-fourth
If Young's modulus doubles: ΔL becomes half

4.24 Important Concept: Two Wires of Same Material and Volume

Suppose two wires have the same material and same volume.
Since: **Volume = AL**
We have: $A_1L_1 = A_2L_2$
For the same applied force:

$$\Delta L = FL/(AY)$$

Since Y is same: $\Delta L \propto L/A$
Using $A \propto 1/L$ for constant volume:

$$\Delta L \propto L^2$$

Therefore, for wires of the same material and same volume under the same force:
Extension is proportional to the square of length.
This result applies when both wires are made of the same material, have equal volumes, are stretched to the same force, and remain within the linear elastic range.
This is a useful higher-level result.

4.25 Safety and Engineering Design

Mechanical properties of solids are essential for designing safe structures.
Engineers consider:

- Young's modulus
- Yield stress
- Ultimate tensile stress
- Breaking stress
- Fatigue strength
- Thermal expansion
- Factor of safety

A structure must remain within its safe operating range under expected loading conditions.

4.26 Important Conceptual Questions

Q1. Why are gaps provided between railway tracks?

Because metals expand when their temperature increases. Gaps or suitable expansion arrangements allow thermal expansion and reduce excessive thermal stress.

Q2. Why is expansion joints used in bridges?

It is used o accommodate changes in dimensions due to temperature variations and prevent excessive thermal stress.

Q3. Why is a thick wire stiffer than a thin wire of the same material and length?

Because:

k = YA/L

Therefore, increasing A increases the effective spring constant.

Q4. Which arrangement has a smaller equivalent spring constant: series or parallel?

For positive spring constants:

Series combination → smaller equivalent stiffness

Parallel combination → larger equivalent stiffness

Q5. Why does a restrained heated rod develop stress?

Because, it cannot undergo its natural thermal expansion.

The prevented thermal strain produces internal stress.

CHAPTER 8 – MASTER CONCEPT

The entire chapter can be remembered through this sequence:

External Force → Deformation → Stress → Strain → Elastic Modulus → Young's Modulus / Bulk Modulus / Shear Modulus → Elastic Energy → Practical Elastic Behaviour → Composite Structures & Thermal Stress

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