PART–1: Reflection of Light and Spherical Mirrors

1.1 Introduction to Ray Optics

Ray optics or **geometrical optics** deals with the propagation of light in terms of straight-line paths called **rays**.

A ray represents the direction of propagation of light.

Ray optics is particularly useful when the wavelength of light is much smaller than the dimensions of the optical system.

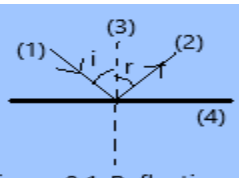


Figure 9.1-Reflection of light through a plane surface

Here, (1) Incident ray, (2) Reflected ray, (3) Normal, (4) Plane surface

Important ideas

Light can undergo:

1. Reflection
2. Refraction
3. Total internal reflection
4. Dispersion
5. Image formation by mirrors and lenses

1.2 Reflection of Light

Reflection of light is the phenomenon in which light, after striking a surface, returns into the same medium.

Laws of Reflection

First Law

The incident ray, reflected ray and normal at the point of incidence all lie in the same plane.

Second Law

The angle of reflection is equal to the angle of incidence.

$$\boxed{i = r}$$

Where:

- i = angle of incidence
- r = angle of reflection

1.3 Plane Mirror — Important Results

For a plane mirror:

Image distance

$$\boxed{v = -u}$$

under the Cartesian sign convention

Magnification

$$\boxed{m = +1}$$

Therefore, the image is:

- virtual
- erect
- same size as the object
- laterally inverted
- formed behind the mirror at the same distance as the object is in front.

1.4 Spherical Mirrors

A **spherical mirror** is a part of a hollow sphere whose one surface is polished.
There are two types:

1. Concave mirror

The reflecting surface is on the **inner side** of the spherical surface.

2. Convex mirror

The reflecting surface is on the **outer side** of the spherical surface.

1.5 Important Terms Related to Spherical Mirrors

1. Pole P

The geometrical centre of the reflecting surface of a spherical mirror is called its **pole**.

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2. Centre of Curvature C

The centre of the sphere of which the mirror is a part is called the **centre of curvature**.

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3. Radius of Curvature R

The distance between the pole and centre of curvature is called the radius of curvature.

$$\boxed{R = PC}$$

---

4. Principal Axis

The straight line passing through the pole and centre of curvature is called the **principal axis**.

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5. Principal Focus F

When rays parallel to the principal axis fall on a concave mirror, the reflected rays meet at a point on the principal axis.
This point is called the **principal focus**.
For a convex mirror, the reflected rays appear to diverge from the principal focus behind the mirror.

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6. Focal Length f

The distance between the pole P and principal focus F is called the focal length.

$$\boxed{f = PF}$$

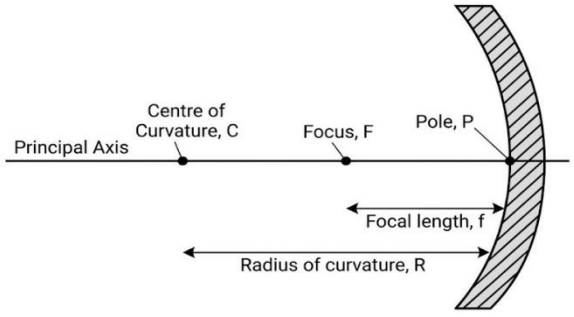


Figure 9.2: Schematic Diagram of Concave Mirror

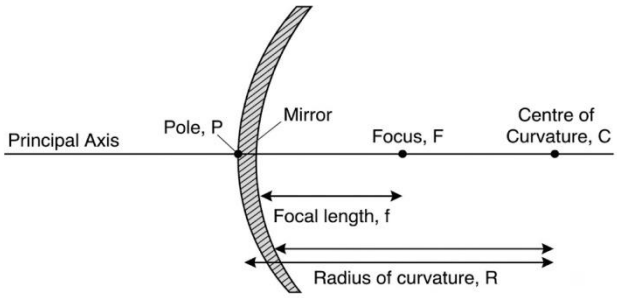


Figure 9.3: Schematic Diagram of Convex Mirror

1.6 Relation between Radius of Curvature and Focal Length

For a spherical mirror, under the paraxial approximation:

$$\boxed{R = 2f}$$

or

$$\boxed{f = \frac{R}{2}}$$

This is one of the most important relations in this chapter.

1.7 Paraxial Rays

Rays that make small angles with the principal axis and strike the mirror close to the pole are called **paraxial rays**.

The relation

$$\boxed{f = \frac{R}{2}}$$

is applicable under the paraxial approximation.

Important

For large-aperture mirrors and non-paraxial rays, spherical aberration can occur.

1.8 Cartesian Sign Convention

For numerical problems involving mirrors, use the **Cartesian sign convention**.

Rules

1. The pole of the mirror is taken as the origin.
2. The principal axis is taken as the x-axis.
3. All distances are measured from the pole.
4. Distances measured in the direction of incident light are positive.
5. Distances measured opposite to the direction of incident light are negative.
6. Heights measured upward from the principal axis are positive.
7. Heights measured downward are negative.

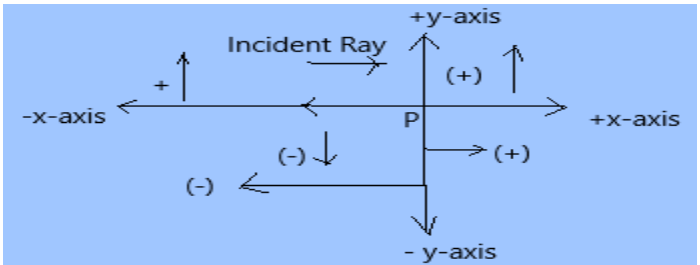


Figure 9.4-Cartesian Sign Convention for Spherical Mirrors

1.9 Mirror Formula

For a spherical mirror:

$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

Where

- f = focal length
- u = object distance
- v = image distance

This is the **mirror formula.**

It is valid for both concave and convex spherical mirrors when the Cartesian sign convention is used consistently.

1.10 Magnification Produced by a Spherical Mirror

Linear magnification is defined as:

$$m = \frac{h_I}{h_0}$$

Where

- hi = image height
- ho = object height

For a spherical mirror:

$$\boxed{m = -\frac{v}{u}}$$

Therefore,

$$\boxed{\frac{h_i}{h_0} = -\frac{v}{u}}$$

Interpretation

- If $m > 0$, the image is **erect**.
- If $m < 0$, the image is **inverted**.
- If |m|>1, the image is magnified.
- If |m|<1, the image is diminished.
- If |m|=1, the image has the same size as the object.

1.11 Ray Diagrams for a Concave Mirror

For a concave mirror, remember these principal rays:

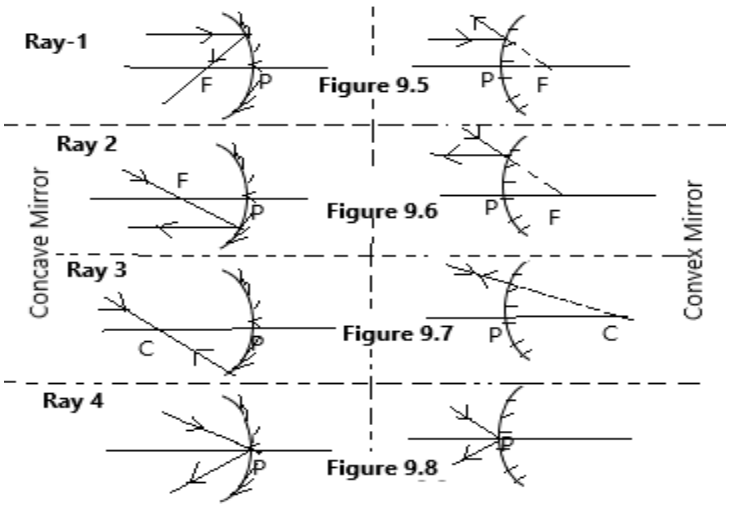
Ray 1

A ray parallel to the principal axis is reflected through the principal focus F.

Ray 2

A ray passing through F is reflected parallel to the principal axis.

Ray 3
A ray passing through C is reflected back along the same path.
Ray 4
A ray incident at the pole obeys the law of reflection.



1.12 Image Formation by Concave Mirror

Object position	Image position	Nature	Size
(a) At infinity	At F	Real, inverted	Highly diminished
(b) Beyond C	Between C and F	Real, inverted	Diminished
(c) At C	At C	Real, inverted	Same size
(d) Between C and F	Beyond C	Real, inverted	Enlarged
(e) At F	At infinity	Real, inverted	Highly enlarged
(f) Between F and P	Behind mirror	Virtual, erect	Enlarged

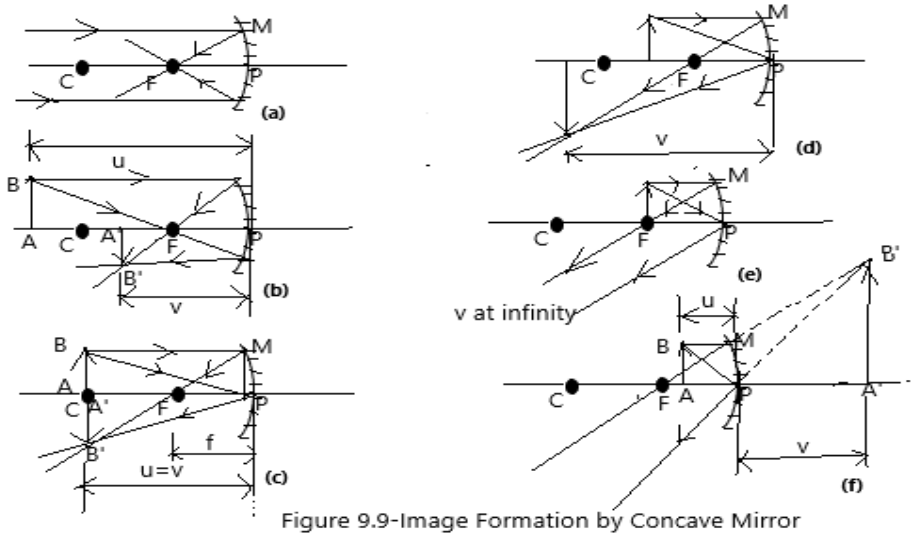


Figure 9.9-Image Formation by Concave Mirror

1.13-1 Convex Mirror

A convex mirror always produces an image that is:

- virtual
- erect
- diminished
- formed behind the mirror
- located between P and F

for a real object.

Important application

Convex mirrors are used as **rear-view mirrors in vehicles** because they provide a wider field of view.

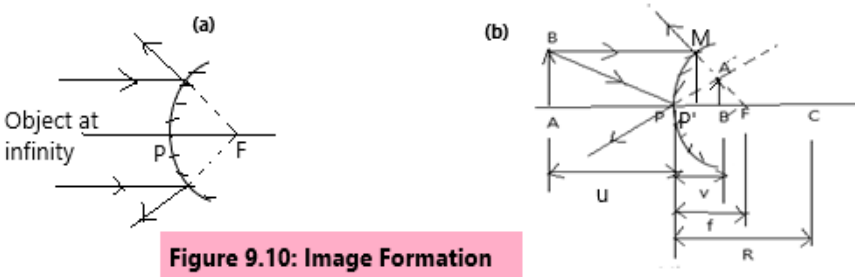


Figure 9.10: Image Formation By Convex Mirror

1.13-2 Image Formation by Convex Mirror

Object position	Image position	Nature	Size
(a) At infinity	At F	Virtual, erect	Highly diminished
(b) At any position	Between P and F	Virtual, erect	Diminished

1.14 Quick Concept Check

Q1. Which mirror is used as a rear-view mirror?

Answer: Convex mirror.

Q2. Which mirror can produce both real and virtual images?

Answer: Concave mirror.

Q3. Which mirror always produces a virtual, erect and diminished image for a real object?

Answer: Convex mirror.

Q4. What is the relation between focal length and radius of curvature?

Answer: R=2f

Both R and f must be assigned signs according to the Cartesian sign convention.

Q5. Write the mirror formula.

Answer: $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

1.15 One Board-Level Numerical

Question

An object is placed 30 cm in front of a concave mirror having focal length 15 cm. Find the position of the image.

Given: u = −30 cm f = −15 cm
Using the mirror formula:

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

Therefore,

$$\frac{1}{-15}=\frac{1}{v}+\frac{1}{-30}\Rightarrow\frac{1}{v}=\frac{1}{30}-\frac{1}{15}\Rightarrow\frac{1}{v}=-\frac{1}{30}$$

Therefore: $\boldsymbol{v=-30\ cm}$
Answer
The image is formed **30 cm in front of the mirror**.
Since v < 0, the image is real.

1.16 NEET/JEE Concept

A concave mirror has: R=40 cm
Therefore:
f=20 cm
For a concave mirror, according to Cartesian convention: f < 0
Hence, f = −20 cm
This sign-convention point is frequently useful in numerical MCQs.

Derivation-1: Mirror formula for Concave Mirror

Derivation of the Mirror formula for a concave mirror:

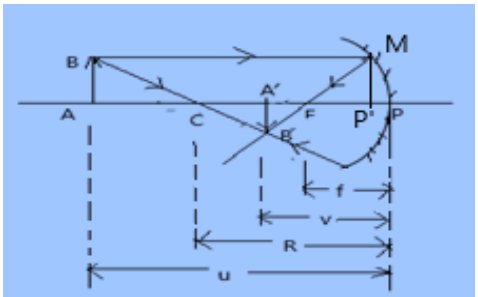


Figure 9.11: Image formation by Concave Mirror

Let an extended object AB be placed beyond centre of curvature and its image is formed between F and C. As shown in the figure,
Triangle ABC and triangle A'B'C are similar triangle.
Therefore, (using sign convention)

$$\frac{A'B'}{AB}=\frac{A'C}{AC}$$

Or

$$\frac{A'B'}{AB}=\frac{-(R-v)}{-(u-R)}--(1)$$

Also, triangle A'B'F and triangle PMF are similar triangles.
(P' and P are very close and PM = AB)

Therefore,

$$\frac{A'B'}{PM = AB} = \frac{A'F}{PF}$$

Or

$$\frac{A'B'}{AB} = \frac{-(v-f)}{-f} - - - (2)$$

From equation (1) and (2), we get

$$\frac{R-v}{u-R} = \frac{v-f}{f}$$

Or

$$\frac{2f-v}{u-2f} = \frac{v-f}{f} \Rightarrow 2f^2 - vf = uv - uf - 2vf + 2f^2$$

Or

$$vf + uf = uv \Rightarrow f(u+v) = uv$$

Or

$$\frac{u+v}{uv} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Or

$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

Derivation-2: Mirror formula for Convex Mirror

Derivation of the mirror formula for a convex mirror:

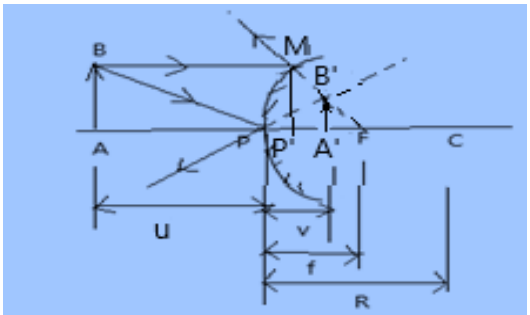


Figure 9.12-Image Formation by Convex Mirror

Let an extended object AB be placed at a point on the principal axis. Its image is formed beyond the mirror between P and F. As shown in the figure,
Triangle ABP and triangle A'B'P are similar triangle.
Therefore, (using sign convention)

$$\frac{A'B'}{AB} = \frac{A'P}{AP}$$

Or

$$\frac{A'B'}{AB} = \frac{v}{-u} - - - (1)$$

Also, triangle PMF and triangle A'B'F are similar triangle.

So,

$$\frac{A'B'}{PM = AB} = \frac{A'F}{PF}$$

Or

$$\frac{A'B'}{AB} = \frac{f - v}{f} - - - (2)$$

From equation (1) and (2), we get

$$\frac{-v}{u} = \frac{f - v}{f} \Rightarrow -vf = uf - uv \Rightarrow f(u + v) = uv$$

Or

$$\frac{u + v}{uv} = \frac{1}{f} \Rightarrow \frac{u}{uv} + \frac{v}{uv} = \frac{1}{f}$$

Or

$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

Questions on Mirror

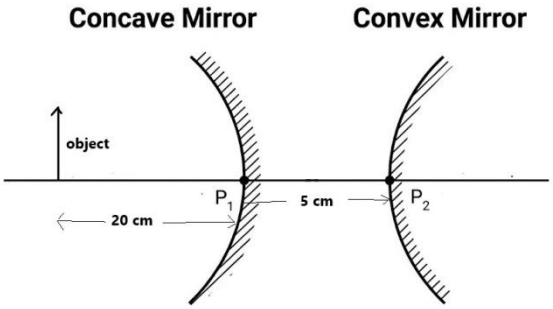
Try Yourself

Q.1 The size of image is half that of the object placed in-front of a

- Case (a): Concave Mirror, f = -20 cm
- Case (b): Convex mirror, f = + 20 cm.

Determine the image position in each case if it is possible. Also draw ray diagram(s).

Q.2 Two mirrors, one concave mirror and other convex mirror are coaxially placed. They are separated by 5 cm distance from their pole to pole. The focal lengths of each mirror are 15 cm and 20 cm respectively. An object is placed at distance 20 cm from pole of concave mirror. Find the position of final image formed by convex mirror.



Q.3 Using mirror formula (without ray diagram), find the position of image formed when an object is placed

(a) Between F and C in concave mirror

(b) At any position except at infinity in convex mirror

PART–2: Refraction, Total Internal Reflection & Optical Fibre

2.1 Refraction of Light

Refraction of light is the change in the direction of propagation of light when it passes obliquely from one transparent medium to another.

The change occurs because the **speed of light changes** when it enters a different medium.

Important point

The **frequency of light remains unchanged** during refraction.

However:

- speed changes,
- wavelength changes,
- direction may change.

2.2 Laws of Refraction — Snell's Law

The laws of refraction are:

First Law

The incident ray, refracted ray and normal at the point of incidence lie in the same plane.

Second Law — Snell's Law

For two given media,

$$\boxed{\frac{\sin i}{\sin r} = constant}$$

More generally,

$$\boxed{n_1 \sin i = n_2 \sin r}$$

Where:

- n_1 = refractive index of first medium
- n_2 = refractive index of second medium.

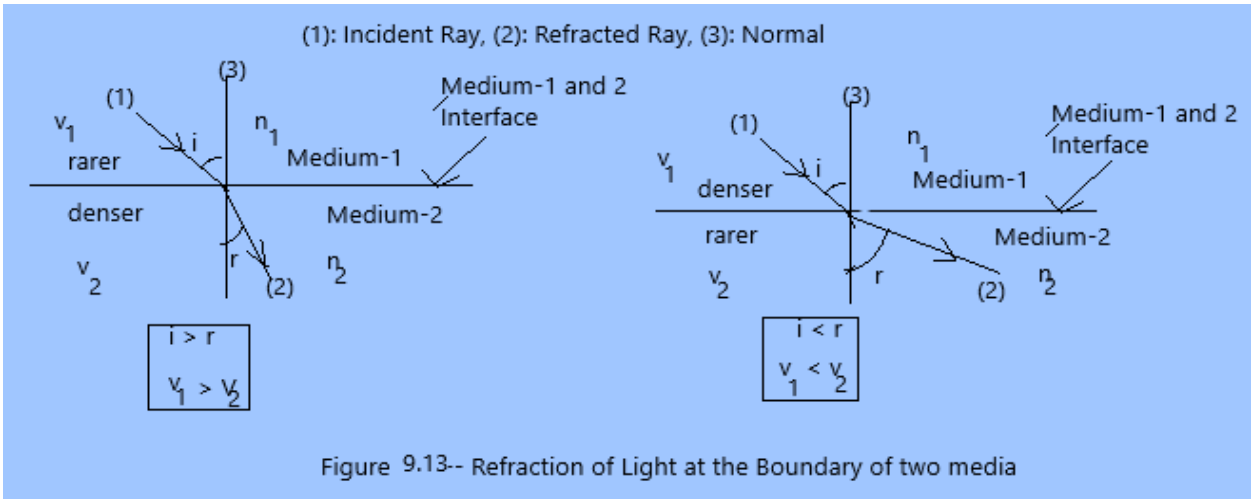


Figure 9.13-- Refraction of Light at the Boundary of two media

2.3 Refractive Index

The refractive index of a medium is related to the speed of light in that medium.
For a medium of refractive index n,

$$\boxed{n = \frac{c}{v}}$$

Where:

- c = speed of light in vacuum
- v = speed of light in the medium.

Since $c > v$, for an ordinary transparent medium,

$$\boxed{n > 1}$$

2.4 Relative Refractive Index

The refractive index of medium 2 with respect to medium 1 is

$$\boxed{n_{21} = \frac{n_2}{n_1}}$$

Using speeds:

$$\boxed{n_{21} = \frac{v_1}{v_2}}$$

Therefore, if light travels from medium 1 to medium 2,

$$\boxed{n_{21} = \frac{\sin i}{\sin r}}$$

2.5 Bending of Light

Case 1: Rarer → denser medium

When light travels from a rarer medium to a denser medium, it bends **towards the normal**.
Therefore:

$$\boxed{r < i}$$

Case 2: Denser → rarer medium

When light travels from a denser medium to a rarer medium, it bends **away from the normal**.
Therefore:

$$\boxed{r > i}$$

Important exception

If light is incident normally, $i = 0^0$, then the direction does not change, although its speed and wavelength change.

2.6 Frequency and Wavelength During Refraction

The frequency of light is determined by the source and **does not change** when light enters another medium.
Therefore:

$$\boxed{\nu_1 = \nu_2}$$

$$\boxed{frequency = constant\ during\ refraction}$$

Since, $v = \nu\lambda$, and frequency remains constant,

$$\boxed{\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}}$$

Hence, if light enters a medium where its speed decreases, its wavelength also decreases.

Very Important NEET Point

Frequency remains constant; speed and wavelength change.

2.7 Apparent Depth

When an object is viewed through a transparent medium such as water or glass, it may appear to be at a different depth from its actual position.

This is called **apparent depth**.

For an object inside a denser medium viewed normally from air:

$$\boxed{\mu = \frac{Real\ depth}{Apparent\ depth}}$$

Therefore,

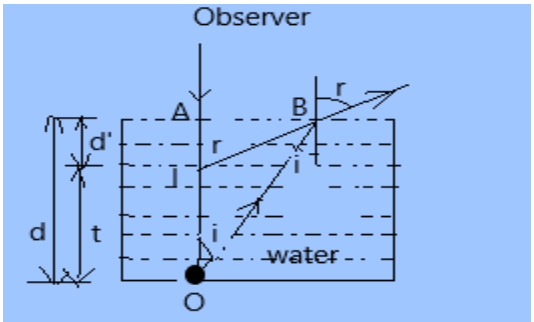
$$\boxed{Apparent\ depth = \frac{Real depth}{\mu}}$$

Since $\mu > 1$,

$$\boxed{Apparent\ depth < Real\ depth}$$

Thus, a coin at the bottom of water appears **raised**.

Derivation-3: Obtaining the relation between apparent depth and real depth



Here, d= real depth, d' = apparent depth, t = normal shift

Figure 9.14: Refraction through water

From the Figure 9.14, as shown, observer is looking at the coin (object O) in water from air (refractive index = 1). Its image is formed at I.

Let apparent depth = d', real depth = d, refractive index of liquid (here, water) μ. For very small angles, points A and B are very close to each other, then:

$$\sin i = \tan i$$

And

$$\sin r = \tan r$$

In Δ OAB,

$$\sin i = \tan i = \frac{AB}{OA} - - - (1)$$

In Δ IAB,

$$\sin r = \tan r = \frac{AB}{AI} - - - (2)$$

Dividing equation (1) by (2), we obtain

$$\frac{\sin i}{\sin r} = \frac{AI}{AO} \Rightarrow \frac{\sin i}{\sin r} = \frac{d'}{d} - - - (3)$$

Using Snell's law,

$$\mu \sin i = (1) \sin r$$

Or

$$\frac{\sin i}{\sin r} = \frac{1}{\mu} - - - - (4)$$

From equation (3) and (4), we have

$$\frac{d'}{d} = \frac{1}{\mu} \Rightarrow d' = \frac{d}{\mu} - - - (5)$$

Hence,

$$\boxed{Apparent\ depth = \frac{Real\ depth}{Refractive\ index\ of\ liquid}}$$

Normal Shift (t)

It is the perpendicular distance upto what extent the object appears to be raised.

Here, Normal shift, t is

$$t = OA - OI = d - d' = d - \frac{d}{\mu}$$

Or,

$$\boxed{t = d\left(1 - \frac{1}{\mu}\right)}$$

2.8 Refraction through a Parallel-Sided Glass Slab

When a ray passes through a rectangular glass slab:

1. It bends towards the normal on entering glass.
2. It bends away from the normal on emerging into air.
3. The emergent ray is **parallel to the incident ray**.
4. The ray suffers a **lateral displacement.**

2.9 Lateral Displacement

The perpendicular distance between the original direction of the incident ray and the emergent ray is called **lateral displacement**.

For a glass slab of thickness t:

$$\boxed{d = t\frac{\sin(i-r)}{\cos r}}$$

Where:

- d = lateral displacement
- t = thickness of slab
- i = angle of incidence
- r = angle of refraction.

Important

Lateral displacement increases with:

- thickness of slab,
- angle of incidence,
- refractive index of the slab.

Derivation-4: Obtaining the relation between lateral displacement and thickness of slab:

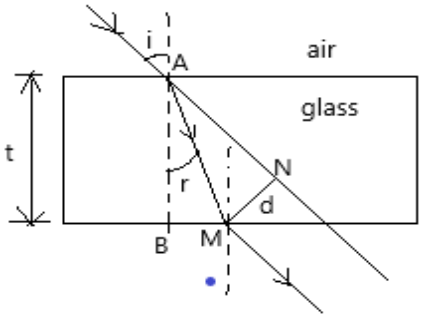


Figure 9.15: Refraction Through a Rectangular Glass Slab

From the Figure 9.15,
In right angle triangle ABM,

$$\cos r = \frac{AB}{AM} = \frac{t}{AM} - - - (1)$$

Also, in right angle triangle ANM, ∡ NAM = i – r, then

$$\sin(i-r) = \frac{NM}{AM} = \frac{d}{AM} - - - (2)$$

From equation (1) and (2), we obtain

$$\frac{t}{d} = \frac{\cos r}{\sin(i-r)} - - - - (3)$$

Or

$$\boxed{d = t\frac{\sin(i-r)}{\cos r}}$$

2.10 Total Internal Reflection

Total internal reflection (TIR) is the phenomenon in which light travelling from a denser medium to a rarer medium is completely reflected back into the denser medium when the angle of incidence exceeds the critical angle.

There are **two necessary conditions**.
Condition 1: Light must travel from an optically denser medium to an optically rarer medium.
Condition 2: The angle of incidence must be greater than the critical angle:

$$\boxed{i > i_C}$$

2.11 Critical Angle

The **critical angle** is the angle of incidence in the denser medium for which the angle of refraction in the rarer medium becomes 90^0.
At critical angle: r = 90^0.

Derivation-5: Finding the relation between refractive Indices and critical angle-

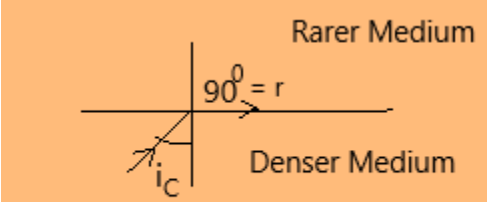


Figure 9.16-For Criticalangle

Let refractive index of rarer medium be μ_R and that of denser Medium, it is μ_D, then
Using Snell's law:

$$\mu_D \sin i_C = \mu_R \sin r$$

Or

$$\sin i_C = \frac{\mu_R}{\mu_D}(\sin 90^0)$$

Therefore,

$$\boxed{\sin i_C = \frac{\mu_R}{\mu_D}}$$

For light travelling from a denser medium to air:$\mu_R = 1$ and let $\mu_D = \mu$

$$\boxed{\sin i_C = \frac{1}{\mu}}$$

Where, μ is the refractive index of the denser medium with respect to air.

2.12 Three Cases at a Denser–Rarer Boundary

Case 1

$$i < i_C$$

Normal refraction occurs.

Case 2

$$i = i_C$$

The refracted ray travels along the interface: r = 90^0, TIR just occurs.

Case 3

$$i > i_C$$

Total internal reflection occurs.

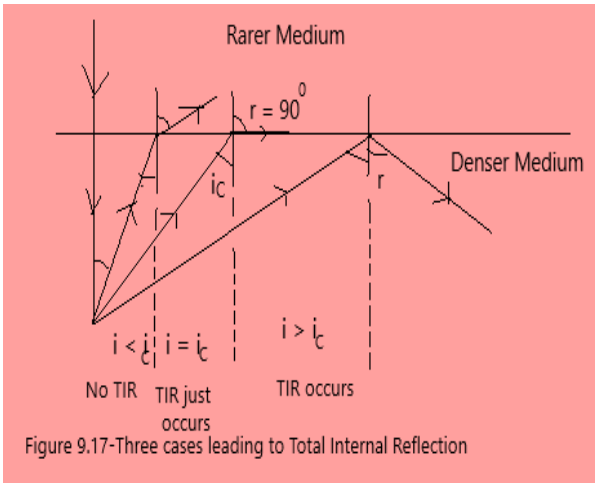


Figure 9.17-Three cases leading to Total Internal Reflection

This is an excellent NEET/JEE diagram.

2.13 Applications of Total Internal Reflection

Total internal reflection has important applications.

1. Optical Fibre

Optical fibres transmit light signals over long distances using repeated total internal reflection.

2. Prisms

Prisms can use TIR to deviate or turn light.

3. Brilliance of Diamonds

The high refractive index and small critical angle of diamond allow light to undergo repeated total internal reflection, contributing to its brilliance.

4. Mirage

Total internal reflection contributes to the formation of certain types of mirages under suitable atmospheric conditions.

2.14 Optical Fibre

An optical fibre is a thin, flexible transparent fibre that transmits light from one location to another.

It generally consists of:

- **Core**
- **Cladding**
- Protective outer covering.

The core has a higher refractive index than the cladding:

$$\boxed{\mu_{core} > \mu_{cladding}}$$

Light entering the core within the appropriate acceptance conditions undergoes repeated total internal reflection.

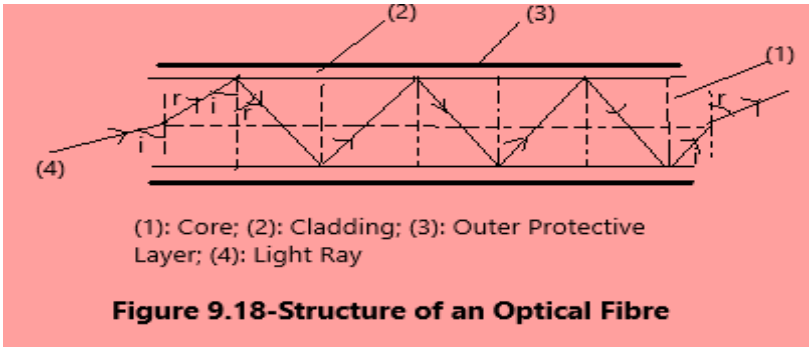


Figure 9.18-Structure of an Optical Fibre

2.15 Principle of Optical Fibre

The working principle is:
Total Internal Reflection
The light remains confined mainly within the core because the core–cladding interface satisfies the conditions required for TIR.

Applications

- Internet communication
- Telephone communication
- Cable television
- Medical endoscopy
- Sensors
- Data transmission.

2.16 Optical Fibre — NEET/JEE Concept

For efficient guidance of light:

$$\boxed{\mu_1 > \mu_2}$$

Where:

- μ_1 = refractive index of core
- μ_2 = refractive index of cladding.

The critical-angle condition at the core–cladding interface is

$$\boxed{\sin i_c = \frac{\mu_2}{\mu_1}}$$

2.17 Numerical Example — Refractive Index

Question

The speed of light in a transparent medium is 2×10^8 ms^{-1}. Find its refractive index.

Solution

$$\mu = \frac{c}{v}$$

Taking, c = 3×10^8 m s^{-1}, and v = 2×10^8 m s^{-1}
We get

$$\mu = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

Therefore,
Answer: $\mu = 1.5$

2.18 Numerical Example — Critical Angle

Question

The refractive index of a transparent medium with respect to air is 1.5. Find its critical angle.

Solution

For a medium–air interface:

$$\sin i_C = \frac{1}{\mu}$$

Therefore,

$$\sin i_C = \frac{1}{3/2} = \frac{2}{3}$$

Hence, C ≈ 41.8^0

2.19 Quick Concept Questions

Q1. What happens to frequency when light enters glass from air?

Answer: It remains unchanged.

Q2. What happens to wavelength?

Answer: It decreases.

Q3. In which direction does light bend when it enters a denser medium?

Answer: Towards the normal.

Q4. What are the conditions for total internal reflection?

Answer:

1. Light must travel from denser to rarer medium.
2. Angle of incidence must be greater than critical angle. $i > i_C$.

Q5. What is the principle of optical fibre?

Answer: Total internal reflection.

Q6. Which has the greater refractive index in an optical fibre?

Answer: The core: $\boxed{\mu_{core} > \mu_{cladding}}$

PART–3: Refraction at Spherical Surfaces & Thin Lenses

3.1 Refraction at a Spherical Surface

When light passes from one transparent medium to another transparent medium through a **spherical refracting surface**, the direction of the ray will change according to the laws of refraction.

Consider a spherical surface separating two media having refractive indices μ_1 and μ_2.

For a paraxial ray, the relation between object distance, image distance and radius of curvature is: (light passes from rarer to denser)

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Where:

- μ_1 = refractive index of object-side medium
- μ_2 = refractive index of image-side medium
- u = object distance
- v = image distance
- R = radius of curvature of spherical surface.

This is called the **refraction formula for a spherical surface**.

3.2 Important Sign Convention

The Cartesian sign convention used earlier continues to apply.

For spherical refracting surfaces:

- distances are measured from the pole of the surface;
- the direction of incident light is taken as positive;
- opposite direction is negative;
- heights above the principal axis are positive;
- heights below the principal axis are negative.

Derivation-6: Refraction formula at a convex spherical surface (rarer-denser)

Derivation of the refraction formula through a convex spherical surface when a ray of light is travelling from rarer medium, refractive index (μ_1) to denser medium, refractive index (μ_2):

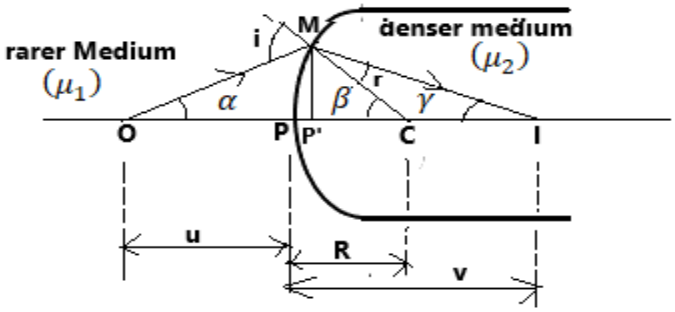


Figure 9.19 (a): Refraction Through a Convex Spherical Surface When Light Passes from Rarer to Denser Medium

Answer:

The ray diagram through a convex spherical surface is shown in the Figure 9.19 (a). Real image I is formed for real object O.

Considering the paraxial rays (rays which are nearer to principal axis), point object and its point image, the angles are very small.

Hence,

$$\sin i = i, \sin r = r ------ (1)$$

Also,

$$\tan \alpha = \alpha, \tan \beta = \beta, \tan \gamma = \gamma ----(2)$$

Also considering that the surface is separating two media; rarer and denser.

Using Snell's law,

$$\mu_1 \sin i = \mu_2 \sin r$$

Or,

$$\frac{\sin i}{\sin r} = \frac{i}{r} = \frac{\mu_2}{\mu_1} ------ (3)$$

From the ray diagram, In ΔCMO,

$$i = \alpha + \beta ------ (4)$$

In ΔCMI,

$$\beta = r + \gamma; or\ r = \beta - \gamma ----(5)$$

Putting the value of i and r in equation (3), we get

$$\frac{\mu_2}{\mu_1} = \frac{\alpha + \beta}{\beta - \gamma} ----(6)$$

Using equation (2) and (6), we have

$$\frac{\mu_2}{\mu_1} = \frac{\alpha + \beta}{\beta - \gamma} = \frac{\tan \alpha + \tan \beta}{\tan \beta - \tan \gamma}$$

Since, points P and P' are very close to each other, therefore MP = MP'

So,

$$\frac{\mu_2}{\mu_1} = \frac{\tan \alpha + \tan \beta}{\tan \beta - \tan \gamma} = \frac{\frac{MP}{OP} + \frac{MP}{PC}}{\frac{MP}{PC} - \frac{MP}{PI}}$$

Substituting the value and using proper sign, we have

$$\frac{\mu_2}{\mu_1} = \frac{\frac{1}{-u} + \frac{1}{R}}{\frac{1}{R} - \frac{1}{v}}$$

Or

$$\frac{\mu_2}{R} - \frac{\mu_2}{v} = \frac{-\mu_1}{u} + \frac{\mu_1}{R}$$

Thus, we obtain

$$\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}}$$

Derivation-7: Refraction formula at a convex spherical surface (denser-rarer)
Derivation of the refraction formula through a convex spherical surface when a ray of light is travelling from denser medium, refractive index (μ_2) to rarer medium, refractive index (μ_1):

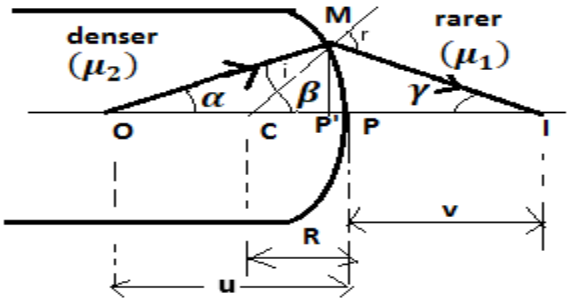


Figure 9.19 (b): Refraction Through a Convex Surface When Light Passes from Denser to Rarer Medium

---

The ray diagram through a convex spherical surface is shown in the Figure 9.19 (b).
The nature and position of the image depend on the object position, the refractive indices of the two media, and the radius of curvature of refracting surfaces.
Considering the paraxial rays (rays which are nearer to principal axis), point object and its point image, the angles are very small.
Hence,

$$\sin i = i, \sin r = r - - - - - (1)$$

Also,

$$\tan\alpha = \alpha, \tan\beta = \beta, \tan\gamma = \gamma - - - - (2)$$

Also considering that the surface separates the denser medium from a rarer medium.
Using Snell's law,

$$\mu_2 \sin i = \mu_1 \sin r$$

Or,

$$\frac{\sin i}{\sin r} = \frac{i}{r} = \frac{\mu_1}{\mu_2} - - - - - (3)$$

From the ray diagram, In ΔCMO,

$$\beta = \alpha + i \; or, i = \beta - \alpha - - - - - (4)$$

In ΔCMI,

$$r = \beta + \gamma - - - - (5)$$

Putting the value of i and r in equation (3), we get

$$\frac{\mu_1}{\mu_2} = \frac{\beta - \alpha}{\beta + \gamma} - - - - (6)$$

Using equation (2) and (6), we have

$$\frac{\mu_1}{\mu_2} = \frac{\beta - \alpha}{\beta + \gamma} = \frac{\tan\beta - \tan\alpha}{\tan\beta + \tan\gamma}$$

Since, points P and P' are very close to each other, therefore MP = MP'
So,

$$\frac{\mu_1}{\mu_2} = \frac{\frac{MP}{PC} - \frac{MP}{OP}}{\frac{MP}{PC} + \frac{MP}{PI}}$$

Substituting the value and using proper sign, we have

$$\frac{\mu_1}{\mu_2} = \frac{\frac{1}{-R} + \frac{1}{u}}{-\frac{1}{R} + \frac{1}{v}}$$

Or

$$-\frac{\mu_1}{R} + \frac{\mu_1}{v} = \frac{-\mu_2}{R} + \frac{\mu_2}{u}$$

Thus, we obtain

$$\boxed{\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}}$$

3.3 Special Case — Plane Refracting Surface

For a plane surface:

$$R \to \infty$$

Therefore,

$$\mu_2 - \mu_1 \to 0$$

Hence:

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = 0$$

Or

$$\boxed{\frac{v}{u} = \frac{\mu_2}{\mu_1}}$$

This result is useful for apparent-depth problems.

3.4 Introduction to Lenses

A **lens** is a transparent optical element bounded by two refracting surfaces, at least one of which is generally spherical.

The two basic types are:

1. Convex lens

It is thicker at the centre than at the edges.

It is generally a **converging lens** when placed in air.

2. Concave lens

It is thinner at the centre than at the edges.

It is generally a **diverging lens** when placed in air.

3.5 Types of Thin Lenses

Convex lenses

1. Double convex / biconvex
2. Plano-convex
3. Positive meniscus or concavo-convex: A meniscus lens that is converging.

Concave lenses

1. Double concave / biconcave
2. Plano-concave
3. Negative meniscus Or convexo-concave: A meniscus lens that is diverging.

For your main notes, a simple classification diagram is sufficient.

Different Types of Convex Lenses

It is shown below in Figure 9.20-1

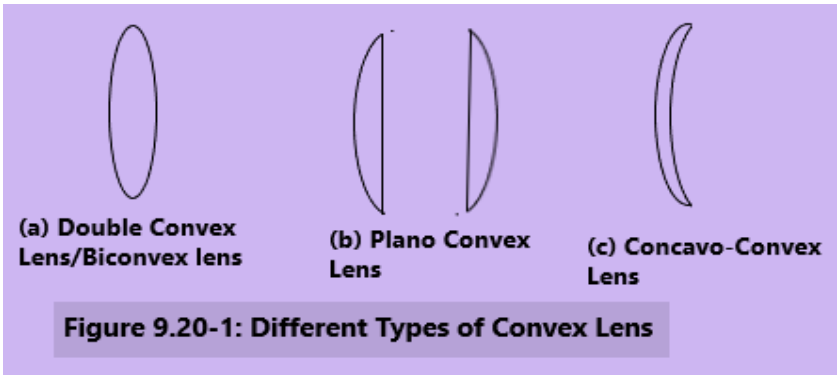


Figure 9.20-1: Different Types of Convex Lens

Different Types of Concave Lenses

It is shown below in Figure 9.20-2

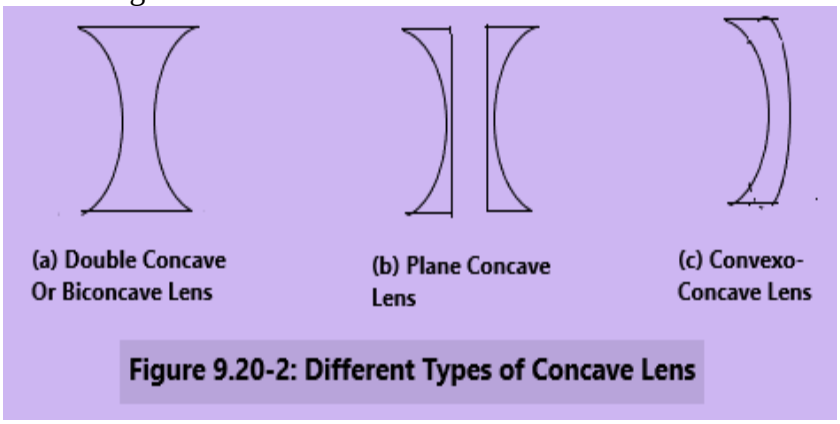


Figure 9.20-2: Different Types of Concave Lens

3.6 Principal Axis of a Lens

The straight line passing through the optical centre and the centres of curvature of the two spherical surfaces of a lens is called the **principal axis**.

3.7 Optical Centre

The point on the principal axis of a thin lens through which a ray passes approximately without undergoing angular deviation is called the **optical centre**.

It is represented by O.

Important ray

A ray passing through the optical centre of a thin lens is considered to pass undeviated.

3.8 Principal Focus of a Convex Lens

When rays parallel to the principal axis are incident on a convex lens, they converge at a point on the principal axis.
This point is called the **principal focus**.
A convex lens has two principal foci: F_1, F_2.

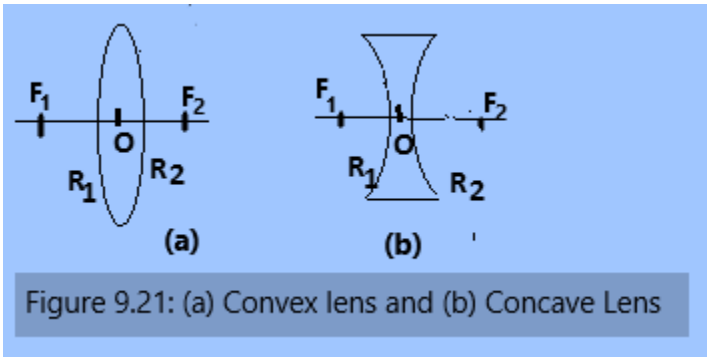


Figure 9.21: (a) Convex lens and (b) Concave Lens

3.9 Principal Focus of a Concave Lens

When rays parallel to the principal axis pass through a concave lens, they diverge.
Their backward extensions appear to meet at a point on the principal axis.
This point is the principal focus.

3.10 Focal Length of a Lens

The distance between the optical centre O and principal focus F is called the focal length. f=OF
A lens has two focal lengths, one corresponding to each principal focus.
For a lens in the same medium on both sides, their magnitudes are equal.

3.11 Thin Lens

A lens whose thickness is small compared with its focal length and other relevant dimensions of the optical system is treated as a **thin lens**.
For thin-lens calculations, the optical centre is taken as the reference point for measuring u and v.

3.12 Important Ray Rules for a Convex Lens

Ray 1
A ray parallel to the principal axis passes through the principal focus after refraction.
Ray 2
A ray passing through the principal focus emerges parallel to the principal axis.
Ray 3
A ray passing through the optical centre passes approximately undeviated.

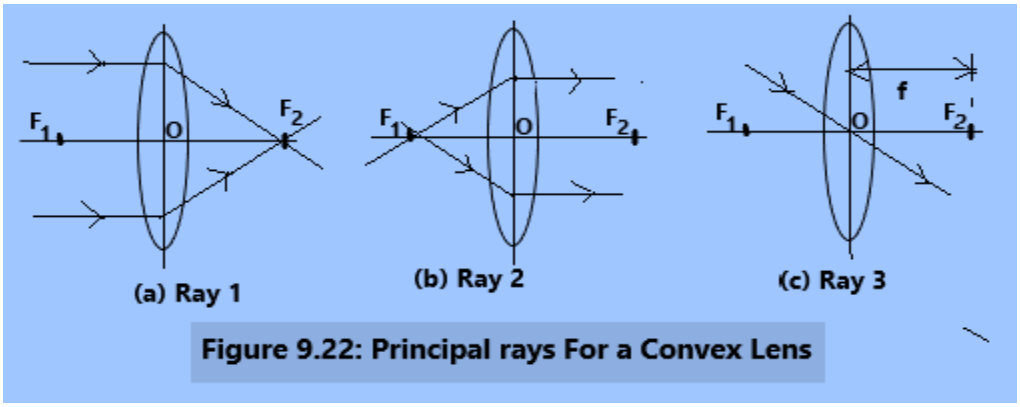


Figure 9.22: Principal rays For a Convex Lens

3.13 Important Ray Rules for a Concave Lens

Ray 1

A ray parallel to the principal axis appears to diverge from the principal focus.

Ray 2

A ray directed towards the principal focus on the opposite side emerges parallel to the principal axis.

Ray 3

A ray passing through the optical centre passes approximately undeviated.

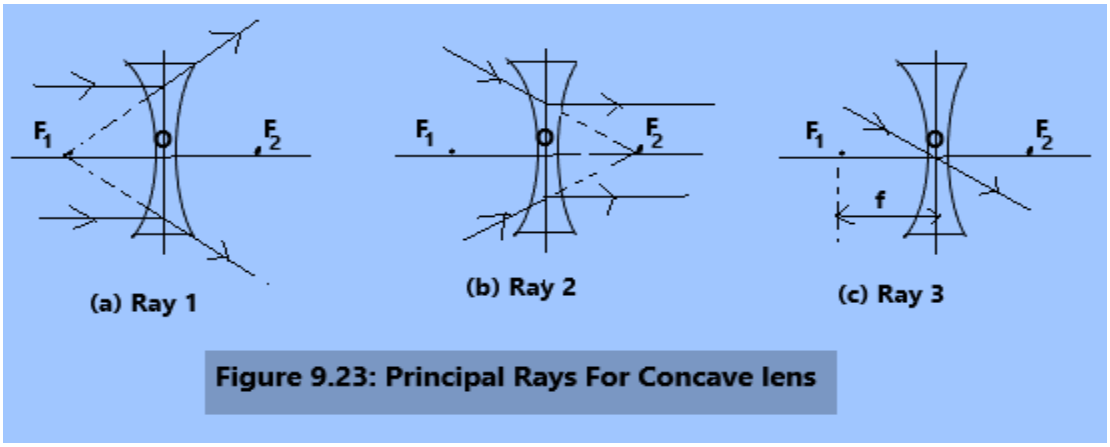


Figure 9.23: Principal Rays For Concave lens

3.14 Thin Lens Formula

For a thin lens:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

This is the **thin lens formula** under the Cartesian sign convention.

Compare it carefully with the mirror formula:

Mirror:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Lens:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

This difference is extremely important in numerical problems.

3.15 Magnification Produced by a Thin Lens

Linear magnification is:

$$m=\frac{h_I}{h_0}$$

For a thin lens:

$$m=\frac{v}{u}$$

Where:

- h_o = object height
- h_I = image height.

Interpretation

If m > 0, image is erect.
If m < 0, image is inverted.
If |m| > 1, image is enlarged.
If |m| < 1, image is diminished.

3.16 Lens Maker's Formula

For a thin lens of refractive index μ_g placed in a surrounding medium of refractive index μ_m:

$$\boxed{\frac{1}{f}=\left(\frac{\mu_g}{\mu_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

For a lens in air, approximately $\mu_m \approx 1$, so:

$$\boxed{\frac{1}{f}=(\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

This is the **lens maker's formula**.

3.17 Meaning of R_1 and R_2

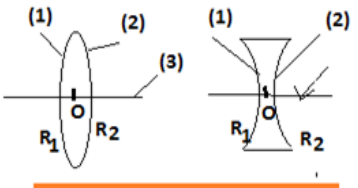


Figure 9.24: Meaning of R_1 and R_2

(1) Surface-1
(2) Surface-2
(3) Principal axis

R_1 and R_2= radius of curvature

In Convex lens:
R_1: positive, R_2: negative

In Concave lens:
R_1: negative, R_2: positive

For a lens:

- R_1 = radius of curvature of first spherical surface
- R_2 = radius of curvature of second spherical surface.

Their signs depend on the adopted Cartesian convention and the direction of incident light.

Important

Do **not** simply use the magnitudes of R_1 and R_2 in numerical problems. Their signs matter.

Derivation-8: Lens Maker's Formula

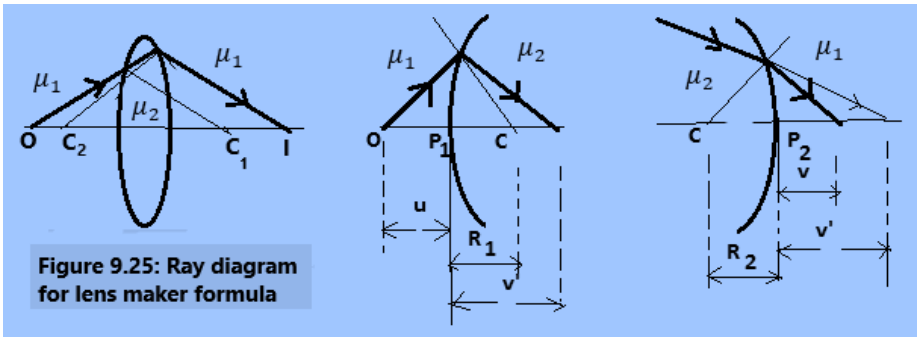


Figure 9.25: Ray diagram for lens maker formula

The derivation is obtained by applying the spherical-surface refraction relation successively at the two refracting surfaces of a thin lens.

The Cartesian sign convention must be applied consistently to the radii of curvature and distances at both refracting surfaces.

When light travels from rarer medium of refractive index (μ_1) to denser medium of refractive index (μ_2), then,

For the first surface:

$$\frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} - - - (1)$$

When light travels from denser medium to rarer medium, then for the second surface:

$$\frac{\mu_1}{v} - \frac{\mu_2}{v'} = \frac{\mu_1 - \mu_2}{R_2} - - - - (2)$$

Adding equations (1) and (2), we have

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - - - - (3)$$

Or,

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - - - - (4)$$

For object to be at infinity, its image is formed at focus. Therefore, as $u \to \infty; v \to f$

Then equation (4) becomes as

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - - - - (5)$$

Or,

$$\frac{\mu_1}{f} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - - - - (6)$$

We can also write equation (5) as

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - - - -(7)$$

For air: $\mu_1 = 1$ and let $\mu_2 = \mu$, then

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - - - (8)$$

This is one of the **most important derivations of Chapter 9**.

3.18 Power of a Lens

The power of a lens is defined as the reciprocal of its focal length measured in metres.

$$\boxed{P = \frac{1}{f}}$$

Where, f is in metres.
SI unit: diopter (D)
Thus: 1 D = 1 m^{-1}

3.19 Sign of Power

Convex lens

For a converging convex lens:

$$f > 0$$

Therefore:

$$P > 0$$

Concave lens

For a diverging concave lens:

$$f < 0$$

Therefore:

$$P < 0$$

3.20 Power in Terms of Focal Length in cm

If focal length is given in centimetres:

$$P = \frac{100}{f\ (in\ cm)}$$

For example, for: f =20 cm,

$$P = 100/20 = +5\ D$$

for a convex lens.

3.21 Combination of Thin Lenses in Contact

If two thin lenses of powers P_1 and P_2 are placed in contact, their equivalent power is:

$$P = P_1 + P_2$$

Since:

$$P = \frac{1}{f}$$

Where f is the equivalent focal length,

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

For n thin lenses in contact:

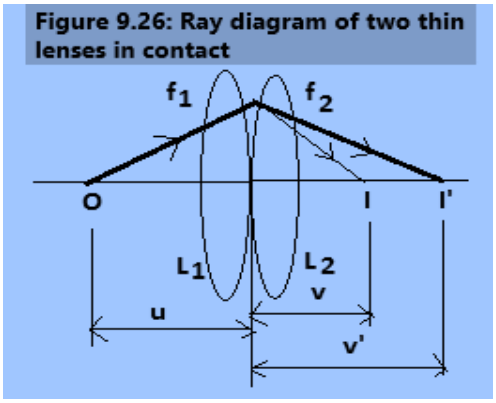
$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ ----$$

or

$$P=P_1+P_2+P_3+\ -----$$

Derivation-9:

Derivation of the combined focal length of two thin lenses in contact:



Using Lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

For Lens L_1:

$$\frac{1}{v'}-\frac{1}{u}=\frac{1}{f_1}----(1)$$

The image formed by lens L_1 will act as virtual image for lens L_2.

For Lens L_2:

$$\frac{1}{v}-\frac{1}{v'}=\frac{1}{f_2}----(2)$$

Adding equations (1) and (2), we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}----(3)$$

Assuming two lenses as one lenses and for object placed at u , its image formed at v, the its focal length will be F as,

$$\frac{1}{F}=\frac{1}{v}-\frac{1}{u}----(4)$$

Comparing equations (3) and (4), we obtain

$$\boxed{\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}}$$

3.22 Numerical Example — Lens Formula

Question

A convex lens has focal length 20 cm. An object is placed 30 cm in front of it. Find the image position.

Given

F = +20 cm u = −30 cm

Using:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

We get:

$$\frac{1}{v}=\frac{1}{20}-\frac{1}{30}=\frac{1}{60}$$

Hence: v = + 60 cm

The image is formed **60 cm on the other side of the lens**.

3.23 Numerical Example — Power

Question

Find the power of a concave lens having focal length 50 cm.

For a concave lens: f = −50 cm

Therefore:

$$P=\frac{100}{f}=\frac{100}{-50}=-2\ D$$

Answer: P = −2 D

Questions on Lens Maker Formula

1. A convex lens has focal length 20 cm air. When it is dipped in water, what will be its focal length in water? Given refractive index of glass material is 3/2 and and that of water is 4/3.

Solution:

Given,

$f_a = 20\ cm, \mu_g = 3/2, \mu_w = 4/3$

To find f_w-

Using

$$\frac{1}{f_a}=(\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)---(1)$$

$$\frac{1}{f_w}=\left(\frac{\mu_g}{\mu_w}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)---(2)$$

Dividing equation (1) by (2), we have

$$\frac{f_w}{f_a}=\frac{\mu_g-1}{\frac{\mu_g}{\mu_w}-1}=\frac{\frac{3}{2}-1}{\frac{\frac{3}{2}}{\frac{4}{3}}-1}=\frac{\frac{1}{2}}{\frac{9}{8}-1}=\frac{1\times 8}{2\times 1}=4$$

Or,

$$f_w = 4f_g = 4\times 20 = 80\ cm$$

Answer: 80 cm
Hence, if a convex lens is immersed in a liquid (water) of lower refractive index, its focal length increases and here increases 4 times.

---

2. A convex lens of refractive index (μ_g) is dipped in a liquid of refractive indices (μ_l). What will be the nature, focal length and power of the lens in each case?
(a) $\mu_g > \mu_l$
(b) $\mu_g = \mu_l$
(c) $\mu_g < \mu_l$
Answer:

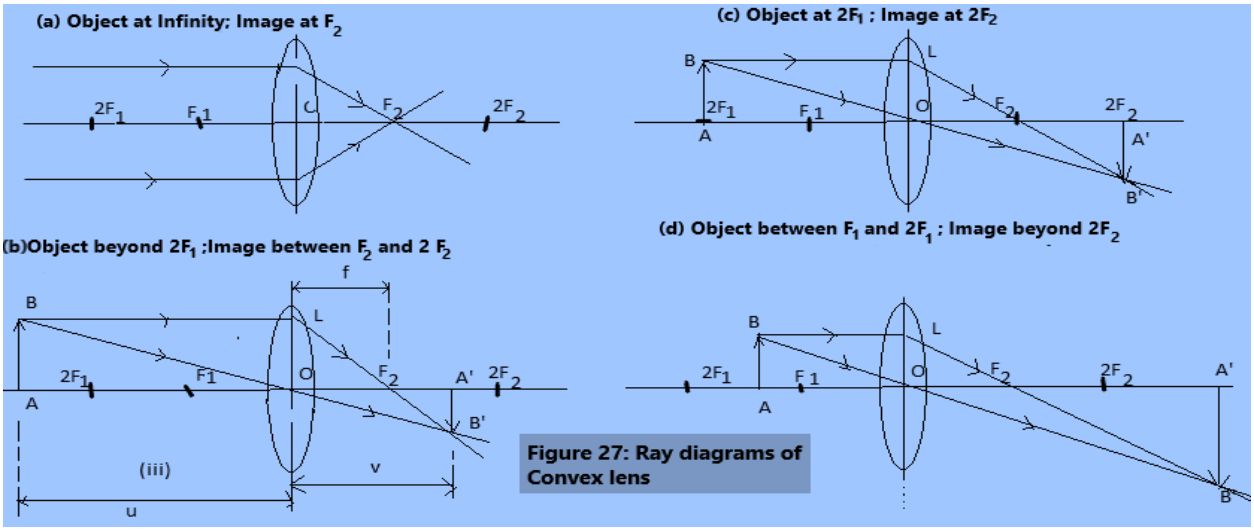
(a) Nature does not change; focal length increases and power decreases.
(b) Lens disappears, becomes plane glass sheet, focal length infinity; power becomes zero
(c) Convex lens becomes concave lens; focal length decreases; power increases.

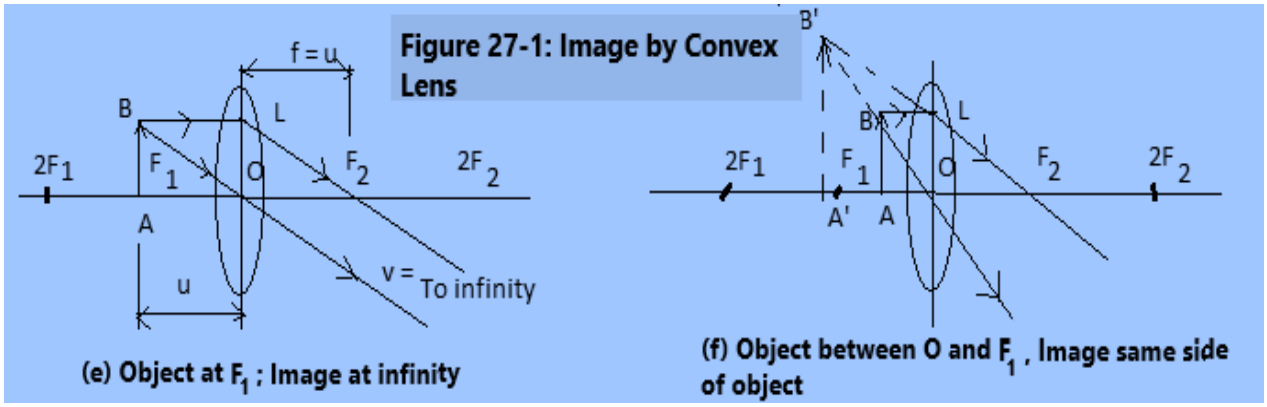
3.24 High-Yield Comparison

Quantity	Convex Lens	Concave Lens
Nature	Converging	Diverging
Focal length	Positive	Negative
Power	Positive	Negative
Real image	Possible	Not for a real object in ordinary conditions
Virtual image	Possible	Always virtual for a real object
Principal focus	Real	Virtual

PART–4: Image Formation by Lenses + Ray Diagrams + Important Applications

Object Positions and Its Image Formation by Convex Lens





4.1 Image Formation by a Convex Lens

A convex lens is a **converging lens**.

The nature and position of the image depend upon the position of the object with respect to: F_1, $2F_1$, O

4.2 Object at Infinity

When an object is effectively at infinity, the incident rays are approximately parallel to the principal axis.

After refraction through a convex lens, the rays converge at the principal focus F_2.

Image

- Position: F_2
- Nature: Real and inverted
- Size: Highly diminished

4.3 Object Beyond $2F_1$

When the object is placed beyond $2F_1$:

Image

- Position: Between F_2 and $2F_2$
- Nature: Real and inverted
- Size: Diminished

4.4 Object at $2F_1$

When the object is placed at $2F_1$:

Image

- Position: At $2F_2$
- Nature: Real and inverted
- Size: Same as object

Therefore: $|m|=1$

4.5 Object Between F_1 and $2F_1$

When the object is placed between F_1 and $2F_1$:

Image

- Position: Beyond $2F_2$
- Nature: Real and inverted
- Size: Enlarged

Therefore: $|m|>1$

4.6 Object at F_1

When the object is placed exactly at the principal focus:
The refracted rays become parallel.
Therefore, the image is formed at infinity.

Image

- Position: Infinity
- Nature: Real, inverted
- Size: Highly enlarged

4.7 Object Between F_1 and Optical Centre O

When the object is placed between F_1 and O:

Image

- Position: Same side of lens as object
- Nature: Virtual and erect
- Size: Enlarged

This is the basic working principle of a **simple magnifying glass.**

4.8 Complete Convex-Lens Image Table

Object position	Image position	Nature	Size
At infinity	At F_2	Real, inverted	Highly diminished
Beyond $2F_1$	Between F_2 and $2F_2$	Real, inverted	Diminished
At $2F_1$	At $2F_2$	Real, inverted	Same
Between F_1 and $2F_1$	Beyond $2F_2$	Real, inverted	Enlarged
At F_1 (for ideal thin lens)	At infinity	Real, inverted	Highly enlarged
Between F_1 and O	Same side as object	Virtual, erect	Enlarged

---

NEET Memory Rule

For a convex lens:

Object moves from infinity towards O, image moves from F_2 towards infinity, and the image size increases.

4.9 Principal Rays for Convex Lens

For drawing ray diagrams, remember only three standard rays.

Ray 1

Parallel to principal axis → passes through F_2.

Ray 2

Through F_1 → emerges parallel to principal axis.

Ray 3

Through optical centre → approximately undeviated.

Two rays are sufficient to locate the image.

A concave lens is a **diverging lens**.

For a real object, it always forms:

- virtual image
- erect image
- diminished image
- image between O and F, on the same side as the object.

Object Positions and Image Formation by Concave lens

4.10 Object at Infinity — Concave Lens

Parallel rays incident on a concave lens diverge after refraction.

Their backward extensions meet at the principal focus on the object side.

Image

- Position: F_1
- Nature: Virtual, erect
- Size: Highly diminished

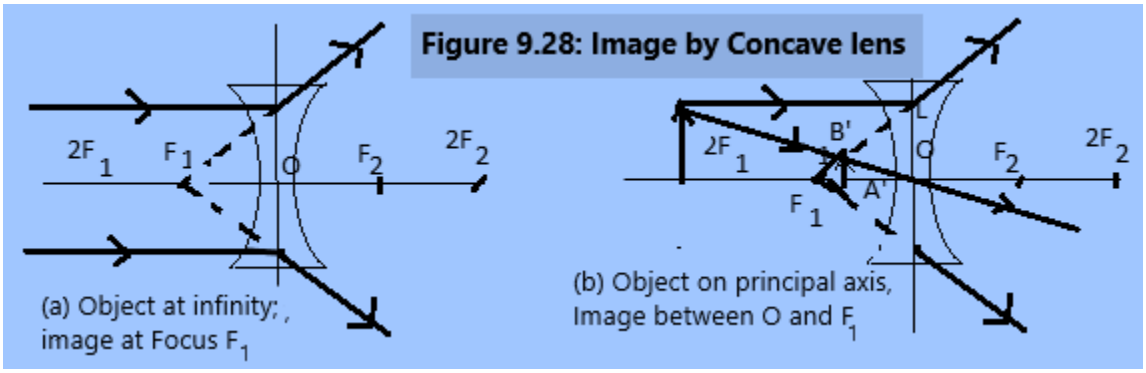
4.11 Object at Finite Distance — Concave Lens

For any real object placed at a finite distance:

Image

- Position: Between O and F_1
- Nature: Virtual and erect
- Size: Diminished

Image Formation by Concave Lens for different positions of object



4.12 Complete Concave-Lens Result

For a real object:

Virtual + Erect + Diminished

This is a very important NEET fact.

4.13 Comparison: Convex vs Concave Lens

Property	Convex lens	Concave lens
Shape	Thick at centre	Thin at centre
Nature	Converging	Diverging
Focal length in air	Positive	Negative
Power	Positive	Negative
Real image	Possible	Not for a real object
Virtual image	Possible	Always for a real object
Magnifying action	Possible	No
Common use	Magnifier, camera, optical instruments	Spectacles for myopia

---

4.14 Lens Formula — Numerical Strategy

Always begin with:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

Then write the signs.

For a real object

Usually: u < 0

Convex lens, f > 0
Concave lens, f < 0
Then calculate v.
Finally determine:

- real/virtual,
- erect/inverted,
- magnified/diminished.

Derivation -10:

Derivation of lens formula for a convex lens

Let an extended object AB be placed between F_1 and 2 F_1 on the principal axis in front of a spherical thin convex lens. Its image A'B" is formed between 2 F_2 and infinity. The ray diagram shows the position of object (u), the position of image (v), its focal length (f). The height of object is h_o and the height of image is h_I.

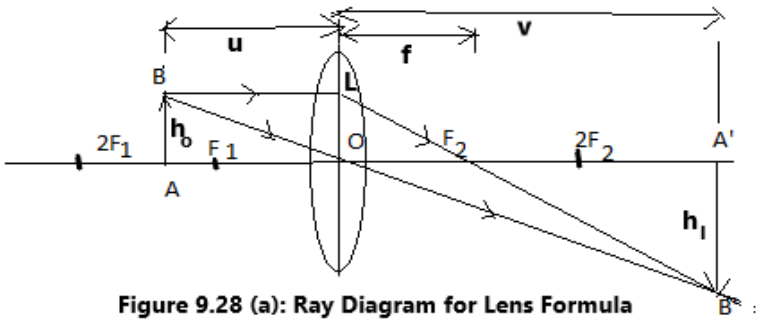


Figure 9.28 (a): Ray Diagram for Lens Formula Derivation of Convex Lens

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From the ray diagram, using sign conventions,

Since, ΔABO and ΔA'B'O are similar triangles. Therefore,

$$\frac{A'B'}{AB} = \frac{-h_I}{h_O} = \frac{OA'}{OA} = \frac{v}{-u}$$

Or,

$$\frac{h_I}{h_O} = \frac{v}{u} - - - -(1)$$

Moreover, AB ∥ OL and also, Δ OLF_2 and Δ$A'B'F_2$ are similar triangles. Therefore,

$$\frac{A'B'}{OL = AB} = \frac{-h_I}{h_O} = \frac{A'F_2}{OF_2} = \frac{v-f}{f}$$

Or

$$\frac{h_I}{h_O} = \frac{f-v}{f} - - - -(2)$$

From equations (1) and (2), we get

$$\frac{v}{u} = \frac{f-v}{f}$$

Or

$$vf = uf - uv$$

Or

$$uf - vf = uv$$

Or

$$f\,(u - v) = uv$$

Or

$$\frac{u - v}{uv} = \frac{1}{f}$$

Or

$$\frac{u}{uv} - \frac{v}{uv} = \frac{1}{f} - - - (3)$$

Hence, from equation (3), we obtain

$$\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$$

Derivation -11:

Derivation of lens formula for a Concave lens

Let an extended object AB be placed on the principal axis at distance u from its optical centre O in front of a spherical concave lens. Its image is formed between optical centre O and F_1. The ray diagram is shown. Its focal length (f) is shown. The height of object is h_o and the height of image is h_I.

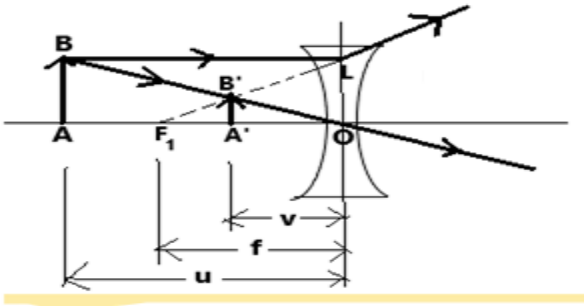


Figure 9.28 (b): Ray Diagram for Lens Formula Derivation of Concave Lens

Using the sign conventions and as shown in the Figure above,
$\Delta A'B'O$ and ΔABO are similar triangles. Therefore,

$$\frac{A'B'}{AB} = \frac{h_I}{h_O} = \frac{A'O}{AO} = \frac{-v}{-u}$$

Or

$$\frac{h_I}{h_O} = \frac{v}{u} - - - - - (1)$$

Similarly, ΔOLF_1 and $\Delta A'B'F_1$ are similar triangles. Therefore,

$$\frac{A'B'}{OL = AB} = \frac{h_I}{h_O} = \frac{A'F_1}{OF_1} = \frac{-f - (-v)}{-f} = \frac{f - v}{f}$$

Or

$$\frac{h_I}{h_O} = \frac{f - v}{f} - - - (2)$$

From equations (1) and (2), we get

$$\frac{v}{u} = \frac{f - v}{f}$$

Or

$$vf = uf - uv$$

Or

$$\frac{1}{f} = \frac{u - v}{uv}$$

Hence, we obtain

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

4.15 Numerical Example — Convex Lens

Question

An object is placed 40 cm in front of a convex lens of focal length 20 cm. Find the image distance and magnification.

Given

u = −40 cm f = +20 cm

Using:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Therefore:

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}$$

Hence:

v = +40 cm

Now,

$$m = \frac{v}{u} = \frac{40}{-40} = -1$$

Therefore: m = −1

Interpretation

The image is:

- real
- inverted
- same size

This agrees with the $2F_1$ case.

4.16 Numerical Example — Concave Lens

Question

A concave lens has focal length 20 cm. An object is placed 30 cm in front of it. Find the image distance.

Given
u = −30 cm, f = −20 cm
Using:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

Therefore:

$$\frac{1}{v}=-\frac{1}{30}-\frac{1}{20}=-\frac{5}{60}$$

Thus: v = −12 cm
The negative value shows that the image is formed on the **same side of the lens as the object**.

4.17 Magnification — Important Interpretation

For a lens:

$$m=\frac{v}{u}$$

If m < 0, then image is inverted.
If m > 0, then image is erect.
If |m| >1, then image is magnified.
If |m| < 1, then image is diminished.
If |m| =1, then image has the same size.

4.18 Combination of Lenses in Contact

For two lenses:

$$P=P_1+P_2$$

Therefore:

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

Important conceptual point
The powers must be added **algebraically**.
For example:
Convex lens: P_1 = +4D
Concave lens: P_2 = −1D
Then:

$$P=P_1+P_2=(4-1)=3D$$

Thus the combination behaves as a converging system.

4.19 Numerical — Combination of Lenses

Question
Two thin lenses of powers +5D and −2D are placed in contact. Find their equivalent power and focal length.
Solution
$P = P_1 + P_2$ or, P = 5−2 = 3
Hence, P = +3D
Since

$$P = \frac{1}{F}$$

Or

$$F = \frac{1}{3} = 0.333\ m = 33.3\ cm$$

Therefore:
F= 0.333 m≈33.3 cm

4.20 Important Concept — Zero Power Combination

If:

$$P_1 + P_2 = 0$$

Then:

$$P = 0$$

Therefore: $F \rightarrow \infty$
The combination has **zero net optical power**.
For example:

$$+5D + (-5D) = 0$$

4.21 Lens Combination — Three Lenses

For three lenses in contact:

$$P = P_1 + P_2 + P_3$$

and

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

This extends directly to any number of thin lenses in contact.

PART–5: Prism, Dispersion and Scattering of Light

5.1 Prism

A **prism** is a transparent refracting medium bounded by two plane refracting surfaces inclined to each other.

The angle between the two refracting surfaces is called the **angle of prism**, denoted by A.

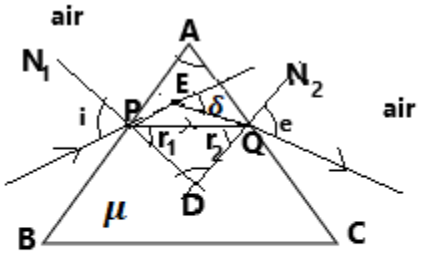


Figure 9.29 (a): Refraction of Light Through a Prism

5.2 Angle of Deviation

When light passes through a prism, its direction changes.

The angle between the original direction of the incident ray and the direction of the emergent ray is called the **angle of deviation**.

It is denoted by δ.

For a prism:

$$\boxed{\delta = i + e - A}$$

Where:

- i = angle of incidence
- e = angle of emergence
- A = angle of prism.

5.3 Relation Between Prism Angle and Refraction Angles

From the geometry of the prism:

$$\boxed{A = r_1 + r_2}$$

Where:

- r_1 = angle of refraction at first surface
- r_2 = angle of refraction at second surface.

Thus the two most important prism relations are:

$$A = r_1 + r_2$$

And

$$\delta = i + e - A$$

5.4 Refraction through a Prism

At the first surface, Snell's law gives:

$$(1)\ \sin i = \mu \sin r_1$$

for a prism in air.
At the second surface:

$$\mu \sin r_2 = (1) \sin e$$

These equations are useful in numerical problems.

5.5 Minimum Deviation

As the angle of incidence is gradually changed, the angle of deviation first decreases, reaches a minimum value and then increases.
The minimum value is called the **angle of minimum deviation**, denoted by:

$$\delta_{min}$$

At minimum deviation, the path of the ray through the prism becomes symmetrical.
Therefore:

$$i = e - - - (1)$$

And

$$r_1 = r_2 = r\ (say) - - - -(2)$$

Since:

$$A = r_1 + r_2 = 2r$$

Or

$$\boxed{r = \frac{A}{2}}$$

5.6 Prism Formula at Minimum Deviation

At minimum deviation:

$$\delta_{min} = i + e - A$$

Since:

$$i = e$$

We have:

$$\delta_{min} = 2i - A$$

Therefore:

$$\boxed{i = \frac{A + \delta_{min}}{2}}$$

Also:

$$r = 2A$$

Using Snell's law:

$$\mu_a \sin i = \mu_g \sin r_1$$

Since, $\mu_1 = 1$ (for air) and let $\mu_g = \mu$; also, $r_1 = r_2 = r$ (say)
Therefore, we obtain:

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\frac{A}{2}}$$

$$\boxed{\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}}$$

This is the **prism formula at minimum deviation.**

5.7 Derivation — Prism Formula

Refractive Index of Prism at Minimum Deviation

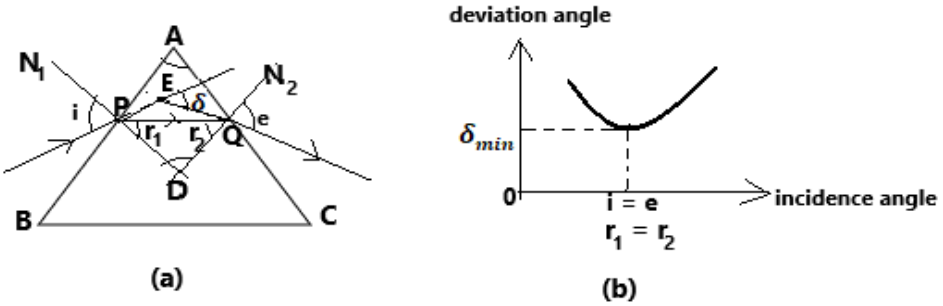


Figure 9.29 (b): Derivation of Prism Formula

Derivation-12:

Derivation of the expression of minimum deviation and obtaing the refractive index of a prism in case of minimum deviation:

From the Figure 9.29 (b), above,

A: angle of prism or apex angle or refracting angle; BC: is the base of the prism; i = incidence angle; r = angle of refraction; e = angle of emergence; N_1 and N_2 are normal at refracting faces AB and AC. The angle of refractions r_1 and $r_2$between the normal N_1, N_2 and refracted ray PQ.

$$\delta = \delta_1 + \delta_2 - - - (1)$$

Where, $\delta_1 = i - r_1$; and $\delta_2 = e - r_2$

Therefore, from equation (1), we get

$$\delta = i - r_1 + e - r_2$$

Or,

$$\delta = (i + e) - (r_1 + r_2) - - - - (2)$$

In the quadrilateral APDQ,

$$A + \angle D = 180^0$$

Also, in ΔPQD,

$$r_1 + r_2 + \angle D = 180^0$$

Thus, we get

$$A = r_1 + r_2 - - - - (2)$$

Therefore, from equation (2), deviation angle,

$$\boxed{\delta = i + e - A} - - - - (3)$$

Minimum deviation angle occurs when refracted ray PQ becomes parallel to base of the prism.

In this case,

$$i = e \; and \; r_1 = r_2 \; and \; \delta = \delta_{min}$$

Therefore, from equation (3),

$$\boxed{\delta_{min} = 2i - A = 2e - A}$$

To derive the expression of Refractive index in Minimum Deviation:

Let the prism is kept in air ($\mu_a = 1$). Suppose the absolute refractive index of glass of prism is μ_g.

Moreover, in the case of minimum deviation,

$$r_1 = r_2 = r \; (say)$$

And,

$$i = e$$

Also, in the prism,

$$A = r_1 + r_2 = 2r$$

Hence,

$$r = \frac{A}{2} - - - - (1)$$

From the expression of minimum deviation,

$$\delta_{min} = 2i - A$$

Or

$$i = \frac{A + \delta_{min}}{2} - - - - (2)$$

Now, using Snell's law at the face AB of the prism in Figure-9.29 above,

$$\mu_a \sin i = \mu_g \sin r_1$$

Or

$$\mu_g = \frac{\sin i}{\sin r} - - - - - (3)$$

As, $r_1 = r$ and $\mu_a = 1$

Using equation (1), (2) and (3), we obtain

$$\boxed{\mu_g = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}}$$

Note-1:

If the prism is kept in water, then refractive index of prism with respect to water is

$$\frac{\mu_g}{\mu_w} = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Where, $\mu_w =$ absolute refractive index of water

Note-2:

If prism is immersed in water, then minimum deviation angle decreases.
This is because,

$$\delta_{min}{}' \propto \left(\frac{\mu_g}{\mu_w} - 1\right) A$$

And

$$\delta_{min} \propto (\mu_g - 1)A$$

Since, $\mu_w = \frac{4}{3} and\ \mu_g = 3/2$
Therefore,

$$\frac{\delta_{min}{}'}{\delta_{min}} = \frac{\frac{9}{8} - 1}{\frac{3}{2} - 1} = 1/4$$

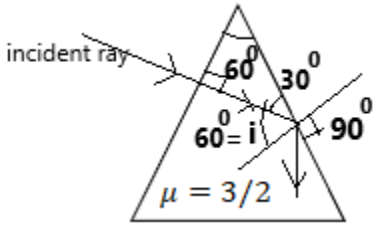
Hence, it decreases.

Graph between deviation angle and incidence angle

A graph is plotted as shown the figure 9.29 (b) as shown above.
From the graph, deviation angle first decreases and at i = e, it becomes minimum and then it increases as incidences angle increases.

Note-3:

If incident ray is perpendicular on the face AB of the equilateral triangular prism and prism is kept in air, then for $\mu_g = 3/2$
The ray of light makes TIR. The emergent ray will not pass from the face AC of the prism.



Explanation:
For TIR (total internal reflection),

$$i > i_c$$

Therefore,

$$\sin i > \sin i_c$$

Where, i_c = critical angle
At the face AB, $i = r = 0^0$
At the face AC,

$$\sin i = \sin 60^0 = 0.866$$

Since,

$$\sin i_c = \frac{1}{\mu} = \frac{2}{3} = 0.67$$

Thus, we see that

$$\sin i > \sin i_c$$

Hence, the ray will not emerge out from the prism. It makes TIR.

Note-4:

If the same equilateral triangular prism is immersed in water of refractive index 4/3, then the ray will be emerging out.

Explanation:

For TIR (total internal reflection),

$$i > i_c$$

Therefore,

$$\sin i > \sin i_c$$

$$\sin i = \sin 60^0 = 0.866$$

And

$$\sin i_c = \frac{\mu_R}{\mu_D} = \frac{4/3}{3/2} = \frac{8}{9} = 0.89$$

Where, $\mu_R =$ refractive index of rarer medium (here water) and $\mu_D =$ refractive index of denser medium (here glass)

Hence, we notice that

$$\sin i < \sin i_c$$

Therefore, the ray will emerge out.

Note-5:

For what angle of incidence, a ray of light fall on an equilateral triangle makes TIR?

Answer: For a prism of angle A and refractive index μ placed in air, the condition for total internal reflection to just occur at the second face is:

$$\boxed{i = sin^{-1}[\mu \sin(A - i_c)]}$$

Where: i_C is the critical angle; $\sin i_C = 1/\mu$; $i =$ angle of incidence at first surface of the prism; r_2 is angle of incidence at second surface of the prism.

For an equilateral triangle, $A = 60^0$. The limiting angle of incidence depends on the refractive index of the prism. Total internal reflection occurs when the angle of incidence at the second face exceeds the critical angle.

Deriving the expression of relation between angle of incidence, critical angle, refractive index of prism and angle of prism when placed in air in case TIR just occurs.

If TIR just occurs at the face AC of an equilateral triangular prism ABC of refractive index μ, then, at face AB,

$$(1)\ \sin i = \mu \sin r_1 - - - (1)$$

At face AC,

If ray of light grazes on it (just occurs TIR),

$$r_2 = i_c\ and\ e = 90^0$$

Then

$$\mu \sin i_c = \sin 90^0 = 1$$

Or

$$\sin i_c = \frac{1}{\mu}$$

Now,

$$A = r_1 + r_2$$

Or

$$r_1 = A - i_c$$

Hence, from equation (1), we get

$$\sin i = \mu \sin(A - i_c)$$

Hence,

$$\boxed{i = sin^{-1}[\mu \sin(A - i_c)]}$$

NEET/JEE Point

The graph is approximately symmetric around the minimum-deviation condition for a given prism and wavelength.

5.8 Thin Prism

For a **thin prism**, the prism angle A is small.
For small angles, using the approximation:

$$\sin\theta \approx \theta$$

the deviation is approximately:

$$\delta = (\mu - 1)A$$

Where:

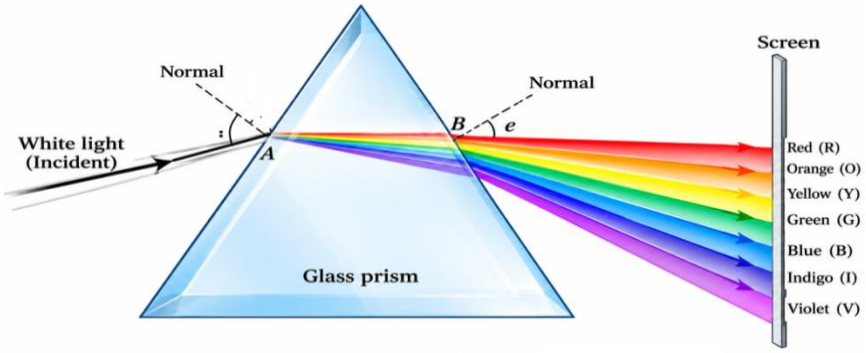
- μ = refractive index of prism material
- A = prism angle in radians.

This is an important approximation for numerical questions.

5.9 Dispersion of Light

Dispersion of light is the phenomenon of splitting of white light into its constituent colours when it passes through a prism.
White light contains a range of wavelengths.
Because the refractive index of the prism material depends on wavelength, different colours deviate by different amounts.

Figure 9.30 : Dispersion of Light through a Prism



5.10 Order of Colours

For visible light:

Increasing wavelength

$$V < I < B < G < Y < O < R$$

Increasing frequency

$$R < O < Y < G < B < I < V$$

Increasing deviation through a normal glass prism

$$R < O < Y < G < B < I < V$$

Therefore:

Violet

- shortest wavelength
- highest frequency
- maximum deviation
- highest refractive index among visible colours

Red

- longest wavelength
- lowest frequency
- minimum deviation
- lowest refractive index among visible colours

5.11 Why Does Dispersion Occur?

The refractive index depends upon wavelength:

$$\mu\lambda = Constant$$

For ordinary transparent materials:

$$\boxed{shorter\ waterlength \Rightarrow larger\ refractive\ index}$$

shorter wavelength ⇒ larger refractive index

Therefore violet light deviates more than red light.

5.12 Angular Dispersion

The angular separation between the extreme colours, usually violet and red, is called **angular dispersion**.

For a thin prism:

$$\delta_v = (\mu_V - 1)A$$

And

$$\delta_r = (\mu_r - 1)$$

Therefore:

$$\delta_v - \delta_r = (n_v - n_r)A$$

This is the angular dispersion between violet and red.

5.13 Dispersive Power

The **dispersive power** of a prism material is defined as the ratio of angular dispersion to the mean deviation.

It is commonly written as:

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

Where:

- μ_v = refractive index for violet
- μ_r = refractive index for red
- μ_y = refractive index for yellow/mean wavelength.

Important

Dispersive power is **dimensionless**.

5.14 Spectrum

The band of colours obtained after dispersion of white light is called the **spectrum**.
The visible spectrum consists approximately of:
Violet, Indigo, Blue, Green, Yellow, Orange, Red

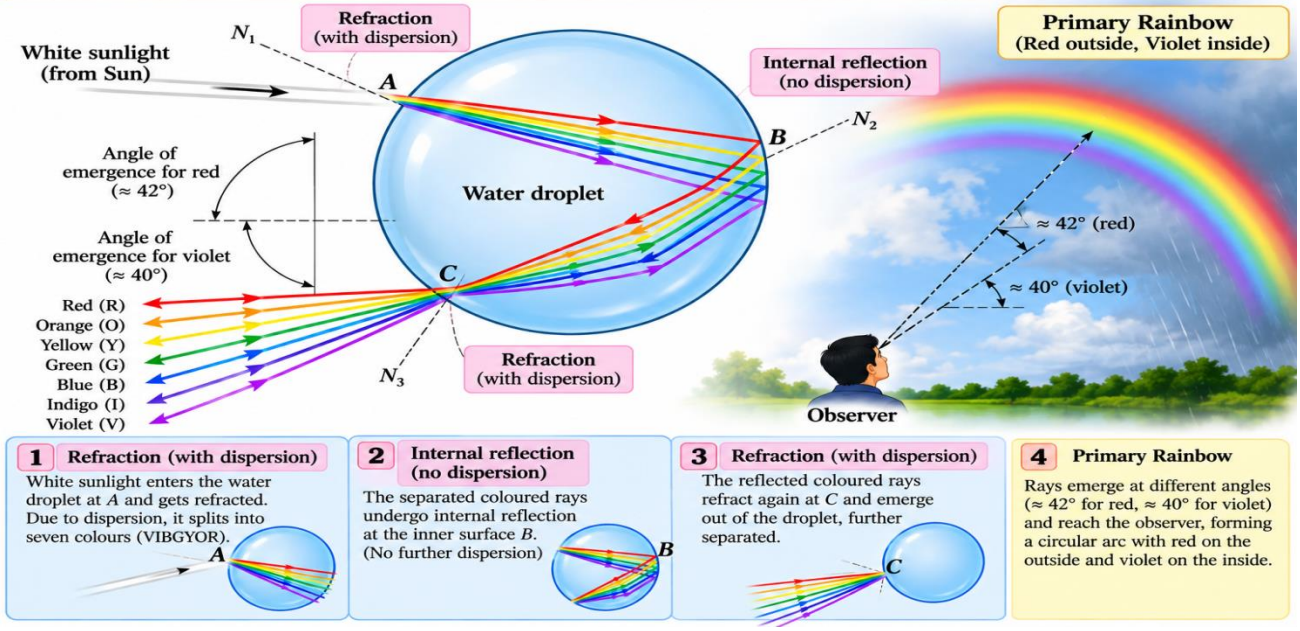
5.15 Primary Rainbow — Concept

A rainbow is a natural optical phenomenon involving:

- refraction
- dispersion
- internal reflection
- refraction again during emergence.

For a primary rainbow, sunlight undergoes **one internal reflection** inside raindrops.

Figure 9.30-1 : Formation of Primary Rainbow



5.16 Scattering of Light

Scattering of light is the phenomenon in which light is redirected in different directions by particles present in a medium.

The scattering depends strongly on wavelength.
For very small particles, the intensity of scattered light approximately follows:

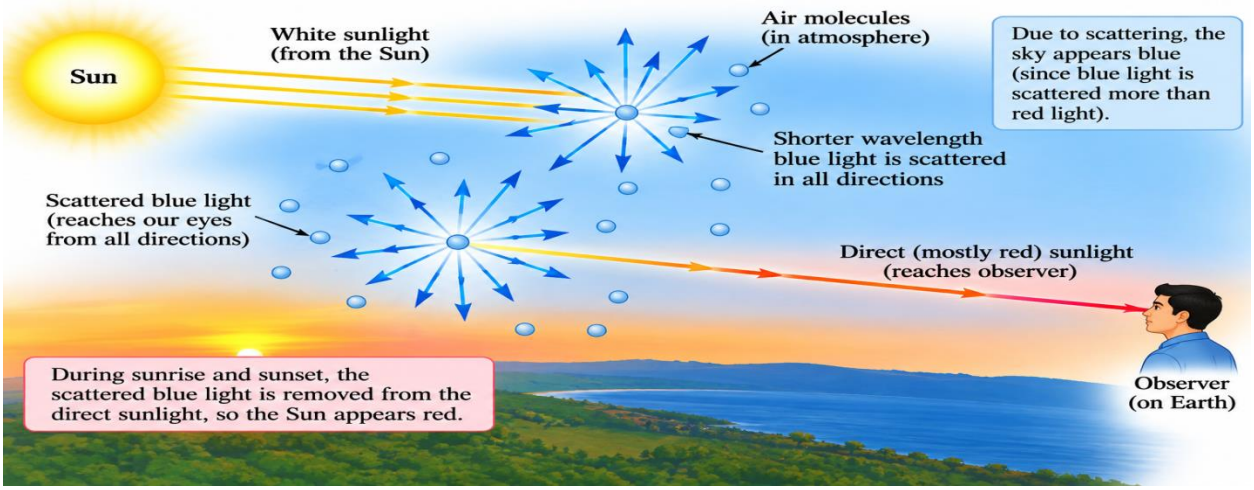
$$I_s \propto \frac{1}{\lambda^4}$$

This is the **Rayleigh scattering dependence**.

Important consequence

Shorter wavelengths are scattered much more strongly than longer wavelengths.

Figure 9.31 : Scattering of Light



5.17 Why Is the Sky Blue?

Sunlight contains different colours.
The shorter-wavelength components, especially blue light, are scattered more strongly by the molecules of the atmosphere than the longer-wavelength red component.
Therefore, the sky appears predominantly blue when viewed away from the Sun.

Important

It is not simply because blue has the "highest energy"; the key point is its **stronger scattering due to shorter wavelength**.

5.18 Why Does the Sun Look Reddish at Sunrise and Sunset?

At sunrise and sunset, sunlight travels through a comparatively longer path in the atmosphere.
The shorter wavelengths are scattered more strongly out of the direct beam.
The light reaching the observer directly from the Sun therefore contains a relatively greater proportion of longer wavelengths, making the Sun appear reddish/orange.

5.19 Why Are Clouds White?

Cloud droplets are relatively large compared with atmospheric molecules.
They scatter different visible wavelengths more nearly uniformly.
As a result, the combined scattered light appears approximately white.

NEET Concept
Do not apply the simple $1/\lambda^4$ Rayleigh-scattering relation blindly to all types of particles.

5.20 Important Numerical — Prism

Question
A prism has angle: A= 60^0 and its minimum deviation is: δ_{min} = 30^0
Find its refractive index.

Solution:
Using:

$$\mu = \frac{\sin\frac{A+\delta_{min}}{2}}{\sin\frac{A}{2}}$$

Or

$$\mu = \frac{\sin\frac{60^0+30^0}{2}}{\sin\frac{60^0}{2}} = \frac{\sin 45^0}{\sin 30^0} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2} = 1.414$$

Therefore: Answer: $\mu \approx 1.41$

5.21 Important Numerical — Thin Prism

Question
A thin prism has apex angle =5^0, and refractive index: 1.5. Find its approximate deviation angle.
Using:

$$\delta = (\mu - 1)$$

Or

$$\delta = (1.5 - 1) \times 5 = 2.5^0$$

Therefore:
$\delta \approx 2.5^0$

5.22 NEET/JEE Quick Concepts

Q1. Which colour deviates most in a glass prism?
Answer: Violet
Q2. Which colour deviates least?
Answer: Red
Q3. Which visible colour has the shortest wavelength?
Answer: Violet
Q4. Which visible colour has the longest wavelength?
Answer: Red
Q5. What is the condition of minimum deviation?
At minimum deviation:
Answer: i = e; $r_1 = r_2$
Q6. What causes the blue colour of the sky?
Answer: Preferential scattering of shorter wavelengths

PART–6: Optical Instruments: Simple Microscope, Compound Microscope & Astronomical Telescope

6.1 Optical Instruments

Optical instruments are devices that use lenses, mirrors or other optical elements to form images and help us observe objects that are too small, too far away or otherwise difficult to see clearly.
Important optical instruments in Class XII Physics are:

1. Simple microscope
2. Compound microscope
3. Astronomical telescope

6.2 Simple Microscope

A **simple microscope** is essentially a convex lens of short focal length used to obtain a magnified virtual image of a small object.
It is commonly called a **magnifying glass**.

Principle

When an object is placed between the optical centre O and the principal focus F of a convex lens, the lens forms a virtual, erect and magnified image.

6.3 Magnifying Power of a Simple Microscope

Magnifying power is defined as the ratio of the angle subtended by the image at the eye when the instrument is used to the angle subtended by the object at the unaided eye.

$$M = \frac{\beta}{\alpha}$$

Where:

- β = angle subtended by the image at the eye
- α = angle subtended by the object at the unaided eye.

6.4 Magnifying Power — Final Image at Near Point

Let:

- f = focal length of the lens
- D = least distance of distinct vision.

For a normal eye:

$$D \approx 25\text{cm}$$

When the final image is formed at the near point:

$$M = 1 + \frac{D}{f_e}$$

This gives the maximum magnification of a simple microscope under the usual near-point condition.

Derivation-13:

Deriving the expression of Magnifying Power of Simple Microscope when final image formed at D (least distance of distinct vision D)

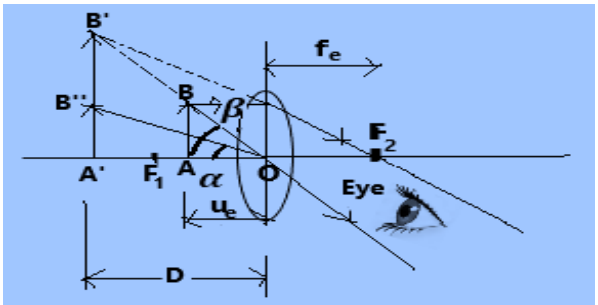


Figure 9.32: Ray diagram of Simple Microscope

As shown in the Figure 9.32,
AB is the object length, its virtual image A'B' is formed towards the object when object is placed between optical centre and focus F_1.
Let OA = -u; OA; = -D
Magnifying Power is

$$M = \frac{\beta}{\alpha} - - - (1)$$

Where: $\beta =$ angle subtended by image at least distance of distinct vision; $\alpha =$ angle subtended by object at least distance of distinct vision
For small angles,

$$\beta = \tan\beta \ \ and \ \alpha = \tan\alpha$$

Therefore, from equation (1),

$$M = \frac{\tan\beta}{\tan\alpha} = \frac{\frac{AB}{OA}}{\frac{A'B''}{OA'}} = \frac{\frac{AB}{-u}}{\frac{AB}{-D}} = \frac{D}{u_e} - - - -(2)$$

(i)When image is formed at least distance of distinct vision: magnifying power(M_D)
Using Lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} - - - - - (3)$$

Substituting $v = -D, u = -u_e; f = f_e$ in equation (2),

$$\frac{1}{-D} - \frac{1}{(-u_e)} = \frac{1}{f_e}$$

Or,

$$\frac{1}{u_e} = \frac{1}{D} + \frac{1}{f_e} - - - -(4)$$

Using equation (2) and (4), we get,

$$M_D = D\left(\frac{1}{D} + \frac{1}{f_e}\right)$$

Hence, we obtain,

$$\boxed{M_D = 1 + \frac{D}{f_e}}$$

(ii)When image is formed at infinity (relaxed eye): magnifying power(M_∞)

In the equation (2) above, we get

$$v = \infty; u = u_e, f = f_e$$

We get

$$\frac{1}{\infty} - \frac{1}{(-u_e)} = \frac{1}{f_e}$$

Or,

$$\frac{1}{u_e} = \frac{1}{f_e}$$

Therefore, from equation (1) above, we get magnifying power when final image formed at D,

$$\boxed{M_\infty = \frac{D}{f_e}}$$

Note-6:

Magnifying Power or Angular Magnification:

In the tensed eye (final image at D):

$$M_D = 1 + \frac{D}{f_e}$$

In the relaxed eye (final image at infinity):

$$M_\propto = \frac{D}{f_e}$$

Thus we see that

$$M_D > M_\infty$$

6.5 Magnifying Power — Final Image at Infinity

When the final image is formed at infinity, the object is placed approximately at the principal focus.

Then:

$$M = \frac{D}{f_e}$$

This condition gives relaxed viewing.

6.6 Comparison — Simple Microscope

Final image	Magnifying power
At near point D	$M = 1 + D/f_e$
At infinity	$M = D/f_e$

Important NEET Point
For the same lens:

$$\boxed{1+\frac{D}{f_e}>\frac{D}{f_e}}$$

Therefore, the near-point adjustment gives greater magnifying power.

6.7 Compound Microscope

A **compound microscope** uses two convex lenses:

1. Objective
2. Eyepiece

The objective has:

- short focal length
- small aperture

The eyepiece acts as a magnifying lens.

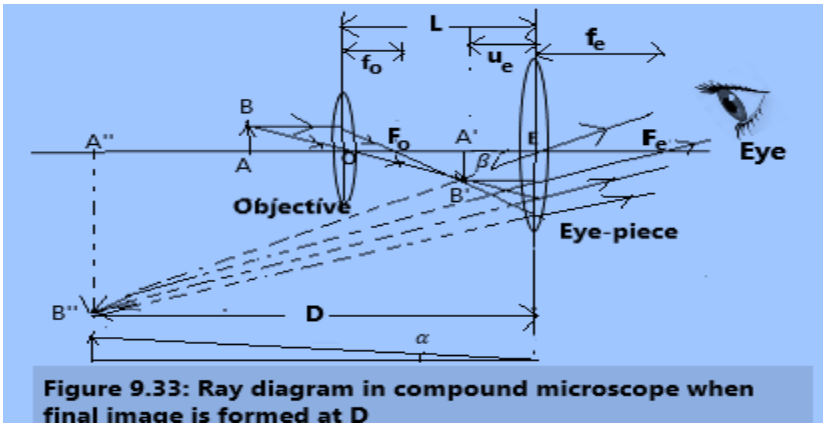


Figure 9.33: Ray diagram in compound microscope when final image is formed at D

6.8 Working of a Compound Microscope

The object is placed just beyond the principal focus of the objective.
The objective produces a real, inverted and magnified intermediate image.
This intermediate image acts as the object for the eyepiece.
The eyepiece further magnifies it and produces the final image.
Thus the compound microscope provides much greater magnification than a simple microscope.

6.9 Magnifying Power of Compound Microscope

The total magnification is the product of:

$$M = M_0 \times M_e$$

Where:

- Mo = magnification produced by objective
- Me = magnification produced by eyepiece.

6.10 Final Image at Infinity

For normal adjustment, the final image is formed at infinity.
The magnifying power is approximately:

$$M = -\frac{L}{f_0} \times \frac{D}{f_e}$$

Where:

- L = tube length
- f_o = focal length of objective
- f_e = focal length of eyepiece
- D = least distance of distinct vision.

Important observation

To obtain high magnification: fo should be small and fe should be small.

6.11 Final Image at Near Point

When the final image is formed at the near point:

$$M = -\frac{L}{f_o} \times \left(1 + \frac{D}{f_e}\right)$$

This gives greater magnification than normal adjustment for the same approximate conditions.

6.12 Important Microscope Derivation

Derivation 14:

Magnifying Power of Compound Microscope

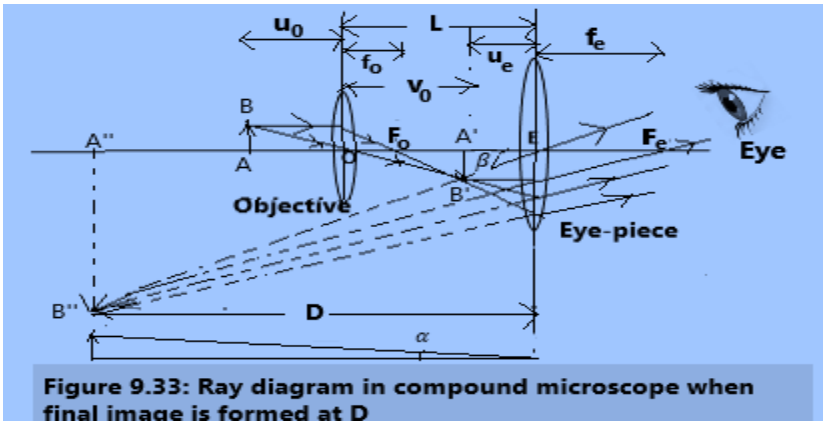


Figure 9.33: Ray diagram in compound microscope when final image is formed at D

In the figure 9.33, L = Tube length; $EA' = u_e$; OA = u_0

In compound microscope, the objective forms a real, inverted and magnified intermediate image. The eyepiece acts as a magnifier and produces a virtual final image. The final image is inverted relative to the original object.

There is an intermediate image formed by objective (A'B') which acts a virtual object for Eye-piece. The image formed by eye piece is virtual and erect.

(i)The magnifying Power when final image is formed at least distance of distinct vision (D):

Magnifying Power

$$M = \frac{\beta}{\alpha} - - - -(1)$$

Where: β = angle subtended by image at least distance

Distinct vision; α = angle subtended by object at least distance of distinct vision

For small angles,

$$\beta = \tan\beta \ \ and \ \alpha = \tan\alpha$$

Therefore, from equation (1),

$$M = \frac{\tan\beta}{\tan\alpha} = \frac{\frac{A'B'}{EA'}}{\frac{AB}{EA}} = \frac{\frac{A'B'}{-u_e}}{\frac{AB}{-D}} = \frac{-h_I}{h_O} \times \frac{(-D)}{(-u_e)} - - - -(2)$$

Since, for an Objective Lens,

$$Linear\ Magnification = \frac{h_I}{h_O} = \frac{v_0}{u_0}$$

Then, equation (2) becomes as

$$M = -\frac{v_0}{u_0} \times \frac{D}{u_e} - - - - - (3)$$

Using Lens formula for Eye-piece,

$$\frac{1}{\boldsymbol{v}} - \frac{1}{u} = \frac{1}{f} - - - -(4)$$

Substituting $v = -D; u = -u_e; f = f_e$ in equation (3), we get

$$\frac{1}{-D} - \frac{1}{(-u_e)} = \frac{1}{f_e}$$

Or,

$$\frac{1}{u_e} = \frac{1}{D} + \frac{1}{f_e} - - - -(5)$$

Substituting $1/u_e$ in equation (3), we get

$$M_D = -\frac{v_0}{u_0}\left[D\left(\frac{1}{D} + \frac{1}{f_e}\right)\right]$$

Hence, we obtain magnifying power when final image is formed at least distance of distinct vision, D is

$$\boxed{M_D = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)}$$

(ii)Tube Length L when final image is formed at D:

$$\boxed{L_D = v_0 + |u_e|}$$

Derivation 15:

(i)The Magnifying Power when final image is formed at infinity (normal adjustment)

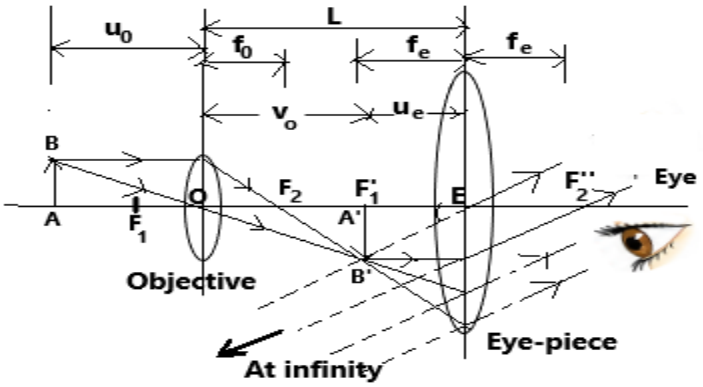


Figure 9.34: Ray diagram of compound microscope when final image formed at infinity

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Using equation (3) above as obtained

$$M = -\frac{v_0}{u_0} \times \frac{D}{u_e} - - - - - (1)$$

Using Lens formula for Eye-piece,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} - - - - (2)$$

Substituting $v = \infty; u = -u_e; f = f_e$ in equation (2), we get

$$\frac{1}{\infty} - \frac{1}{(-u_e)} = \frac{1}{f_e}$$

Or

$$\frac{1}{u_e} = \frac{1}{f_e}$$

Substituting $1/u_e$ in equation (1), we get

$$M_\infty = -\frac{v_0}{u_e} \times \frac{D}{f_e}$$

Hence, we obtain magnifying power when final image is formed at infinity is

$$\boxed{M_\infty = -\frac{v_0}{u_0} \times \frac{D}{f_e}}$$

(ii)Tube Length when final image is formed at infinity

$$\boxed{L_\infty = v_0 + f_e}$$

Note-7:

In compound microscope, the objective forms a real, inverted and magnified intermediate image. The eyepiece acts as a magnifier and produces a virtual final image. The final image is inverted relative to the original object.

Note-8:
The object distance from optical centre O of Objective lens when final image formed at D is slightly beyond its focal length and is less that the object distance when final image is formed at infinity.

6.13 Astronomical Telescope (Refracting Type)

An **astronomical telescope** is an optical instrument used to observe distant astronomical objects such as stars and planets.
A refracting astronomical telescope consists of two convex lenses:

1. Objective
2. Eyepiece

6.14 Objective of Telescope

The objective has:

- large focal length fo
- large aperture

Its function is to collect a large amount of light and form a real image of the distant object.

6.15 Eyepiece of Telescope

The eyepiece has:

- small focal length fe
- comparatively smaller aperture.

It acts as a magnifying glass for the image formed by the objective.

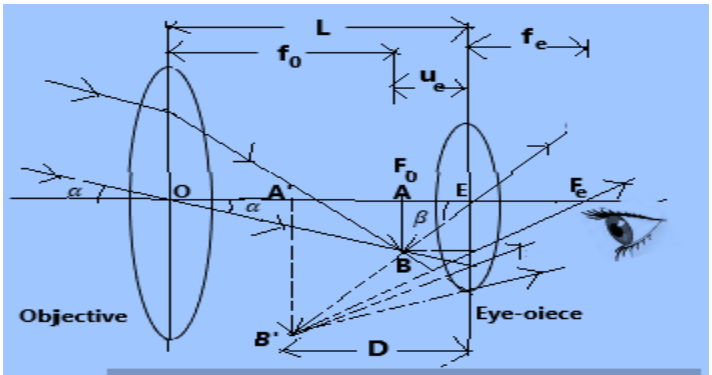


Figure 9.35: Ray diagram of Astronomical telescope refracting type ; final image at D

Derivation 16: Magnifying Power of a Refracting telescope using two convex lenses when final image formed at least distance of distinct vision:
From the figure above,
Magnifying power,

$$M = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{\frac{AB}{AE}}{\frac{AB}{OA}} = \frac{OA}{AE} = \frac{f_0}{-u_e}$$

Or,

$$M = -\frac{f_e}{u_e} - - - - - (1)$$

Using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} - - - (2)$$

For Eye-piece, $v = -D; u = -u_e; f = f_e$, we have

$$\frac{1}{-D} - \frac{1}{(-u_e)} = \frac{1}{f_e}$$

Or

$$\frac{1}{u_e} = \frac{1}{D} + \frac{1}{f_e} - - - - - (3)$$

Substituting $1/u_e$ from equation (3) in the equation (1) above, we get

$$M = -f_0\left(\frac{1}{D} + \frac{1}{f_e}\right) - - - -(4)$$

Hence, we obtain

$$\boxed{M_D = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)}$$

Tube Length at least distance of distinct vision (L_D) in Astronomical refracting telescope

$$\boxed{L_D = f_0 + |u_e|}$$

6.16 Normal Adjustment of Telescope

In normal adjustment: Final image is formed at infinity

The objective forms the image approximately at its focal plane.

The eyepiece is positioned such that this intermediate image lies at the focal plane of the eyepiece.

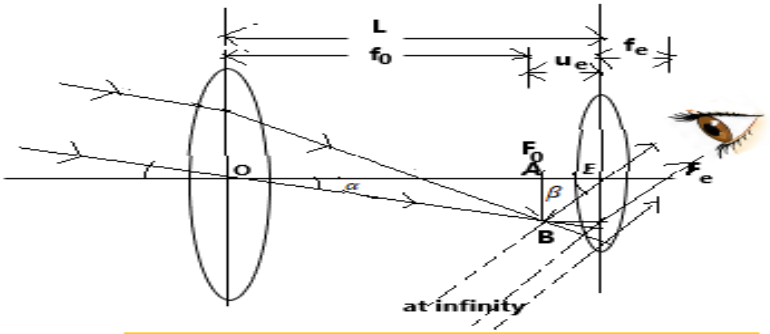


Figure 9.36: Ray diagram of Astronomical telescope ; final image at infinity

Derivation- 17: To obtain magnifying power of Astronomical Telescope when final image is formed at infinity (Normal adjustment)

From the figure above,
Magnifying Power

$$M = -\frac{f_0}{u_e} - - - - - - (1)$$

Using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

For eye-piece, (using sign convention)
$v = \infty; u = -u_e ; f = f_e$, we get

$$\frac{1}{\infty} - \frac{1}{(-u_e)} = \frac{1}{f_e}$$

Or

$$\frac{1}{u_e} = \frac{1}{f_e}$$

Putting the value of $1/u_e$ in equation (1), we have

$$M_\infty = -f_0 \times \frac{1}{f_e}$$

Hence, we obtain

$$\boxed{M_\infty = -\frac{f_0}{f_e}}$$

Tube Length for the final image at (L_∞) in Astronomical refracting telescope

$$\boxed{L_\infty = f_0 + f_e}$$

6.17 Magnifying Power of Astronomical Telescope

For normal adjustment:

$$M = -\frac{f_0}{f_e}$$

The negative sign indicates that the final image is inverted.
The magnitude is:

$$|M| = \frac{f_0}{f_e}$$

Therefore, a telescope should have:

$$f_0 \gg f_e$$

for high angular magnification.

6.18 Length of Telescope in Normal Adjustment

When the telescope is in normal adjustment:
Tube Length

$$\boxed{L = f_0 + f_e}$$

Where:

- L = length of telescope
- f_0 = focal length of objective
- f_e = focal length of eyepiece.

6.19 Telescope — Final Image at Near Point

When the final image is formed at the least distance of distinct vision D, the magnifying power is approximately:

$$M = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

The negative sign again indicates inversion.

6.20 Telescope vs Microscope

Feature	Compound Microscope	Astronomical Telescope
Main purpose	Observe tiny nearby objects	Observe distant objects
Objective focal length	Small	Large
Objective aperture	Small	Large
Eyepiece	Short focal length	Short focal length
Objective forms	Magnified intermediate image	Real image of distant object
Normal final image	Infinity	Infinity

Cassegrain Type (Reflecting Telescope)

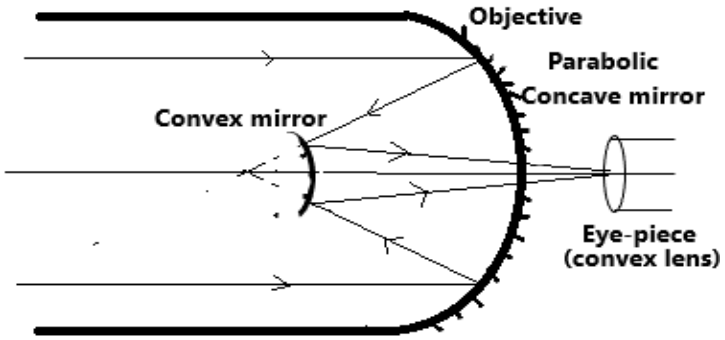


Figure 9.37: Cassegrain Type (Reflecting Telescope)- Ray Diagram

In normal adjustment,

$$M = -\frac{f_0}{f_e}$$

Where,f_0:focal length of Concave Mirror = R/2; f_e = focal length of eye-piece (convex lens)

Advantage of Reflecting Telescope over Refracting telescope:

(a) Spherical aberration is reduced
Spherical aberration can be eliminated for on-axis parallel rays by using a suitably designed parabolic primary mirror

(b) Chromatic aberration is reduced
A reflecting telescope does not suffer from chromatic aberration because reflection is independent of wavelength.

6.21 Numerical Example — Simple Microscope

Question

A simple microscope has focal length 5cm. Find its magnifying power when the final image is formed at the near point. Take: D =25cm

Solution:

Using:

$$M = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 6$$

6.22 Numerical Example — Compound Microscope

An objective has focal length: f_0 = 1cm, and eyepiece focal length: f_e = 5cm.
The tube length is: L = 20cm. Find magnifying power for normal adjustment.

Solution:

Using:

$$|M| = \frac{L}{f_0}\left(\frac{D}{f_e}\right)$$

Or,

$$|M| = \left(\frac{20}{1}\right)\left(\frac{25}{5}\right) = 100$$

Answer: $|M| = 100$

6.23 Numerical Example — Telescope

A telescope has objective focal length: f_0 =100 cm, and eyepiece focal length: fe = 5cm. Find its magnifying power in normal adjustment.

Solution:

$$M = -\frac{f_0}{f_e} = -\frac{100}{5} = -20$$

Therefore:

$$M = -20$$

Magnitude of magnifying power:

$$|M| = 20$$

The negative sign indicates an inverted final image.

6.24 Important Conceptual Questions

Q1. Why does a telescope objective have a large aperture?

Answer: To collect more light from distant and faint astronomical objects.

Q2. Why is the focal length of the telescope objective large?

Because:

$|M| = f_0/f_e$

A large focal length increases angular magnification.

Q3. Why does a microscope objective have a small focal length?

Because its magnification increases when fo decreases.

Q4. Why is the final image of an astronomical telescope inverted?

The objective forms an inverted real image, and the eyepiece magnifies this image without reversing it.

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1. Telescope — normal adjustment
2. Telescope — final image at near point
