PART 1 – PRESSURE, PRESSURE IN A FLUID AND PASCAL'S LAW

1.1 Learning Objectives

After studying this part, you will be able to:

- Understand the meaning of a fluid.
- Distinguish between solids and fluids.
- Define density and relative density.
- Understand pressure and its SI unit.
- Derive the expression for pressure due to a fluid column.
- Understand atmospheric pressure and absolute pressure.
- Apply Pascal's law.
- Understand hydraulic machines and their applications.
- Solve Board, NEET and JEE-level numerical problems.

1.2 What Is a Fluid?

A **fluid** is a substance that can flow and does not have a fixed shape.
Liquids and gases are fluids.
Examples: Water, Oil, Air, Petrol, Mercury
A fluid takes the shape of the container in which it is placed.

1.3 Solids and Fluids

The major difference between solids and fluids is their response to tangential force.
A solid can resist a tangential force and maintain its shape.
A fluid cannot sustain a tangential force in static equilibrium. It continuously deforms or flows when such a force is applied.
Therefore:

Solid: Can sustain shear stress in static equilibrium.

Fluid: Cannot sustain static shear stress.

This is one of the fundamental properties of fluids.

1.4 Density

Density is defined as the mass contained per unit volume of a substance.
If: ,Mass = m, Volume = V then:

$$\rho = m/V$$

Where ρ is the density.

SI Unit: kg/m^3

Dimensional Formula**:** $[\rho] = ML^{-3}$

1.5 Density of Water

The density of water at approximately 4°C is:

$$P_{water} \approx 1000\ kg/m^3$$

It can also be written as:**1 g/cm³**
Therefore:**1000 kg/m³ = 1 g/cm³**

This conversion is frequently useful in numerical problems.

1.6 Relative Density

Relative density is the ratio of the density of a substance to the density of water at the specified reference temperature.

Therefore:

Relative density = Density of substance / Density of water

Or

$$\mathbf{RD = \rho/\rho_{water}}$$

Relative density has **no unit and no dimensions**.

For example, if:$\rho = 800\ kg/m^3$then:

RD = 800/1000

Therefore:**RD = 0.8**

1.7 Pressure

When a force acts normally on a surface, the force per unit area is called **pressure**.

Therefore:

$$\mathbf{p = F/A}$$

Where: F = normal force, A = area over which the force acts

Pressure is a **scalar quantity**.

1.8 SI Unit of Pressure

The SI unit of pressure is:**pascal (Pa)**

Therefore:$\mathbf{1\ Pa = 1\ N/m^2}$

Other commonly used units include:

- atmosphere (atm)
- bar
- torr
- mmHg

Important approximate relations:

$\mathbf{1\ atm \approx 1.013 \times 10^5\ Pa}$

$\mathbf{1\ bar = 10^5\ Pa}$

$\mathbf{1\ torr \approx 133.3\ Pa}$

1.9 Pressure and Area

From:**p = F/A**for a fixed force:

$$p \propto 1/A$$

Therefore:

- Smaller area → greater pressure
- Larger area → smaller pressure

Examples

A sharp knife cuts more easily because its edge has a very small area.

A pointed nail produces large pressure on the surface because its tip has a small area.

A school bag with broad straps produces less pressure on the shoulders than a bag with narrow straps, for the same weight.

1.10 Pressure in a Fluid

A fluid at rest exerts pressure on the walls and surfaces in contact with it.
Fluid pressure acts:

- Normally to a surface.
- In all directions at a given point in a fluid at rest.

The pressure at a point in a static fluid depends on:

- Density of fluid
- Depth below the surface
- Acceleration due to gravity
- Atmospheric pressure at the surface

1.11 Why Does Fluid Pressure Act Normally?

Suppose a fluid exerted a tangential force on a boundary.
The fluid would respond by flowing along the surface.
Therefore, a fluid at rest cannot exert a tangential force on a surface.
Hence, the force exerted by a fluid at rest on a surface is **normal to the surface**.

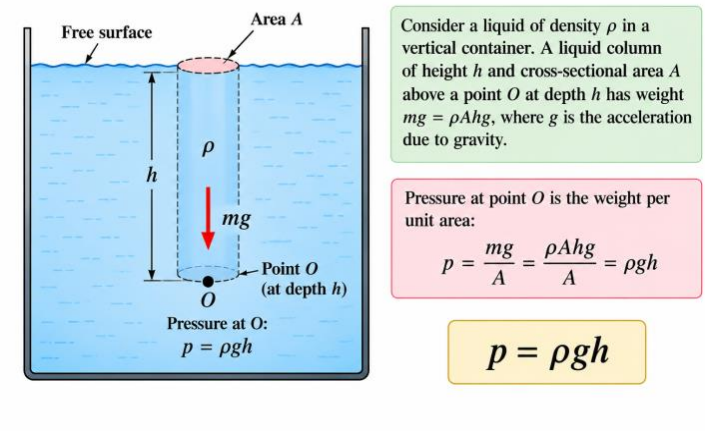
1.12 Pressure Due to a Liquid Column

Consider a liquid of density ρ contained in a vertical vessel.
Let: Depth of liquid = h, Cross-sectional area = A
The mass of the liquid column of height h is: **m = ρV**
Since:**V = Ah**
We get:**m = ρAh**
Therefore, its weight is:**W = mg**= ρAhg
The pressure due to this liquid column is:

p = Force/Area

Therefore:p = ρAhg/A
Hence: **p = ρgh**
This is called the **gauge pressure** due to the liquid column. It gives gauge pressure, not absolute pressure

Figure 9.1: Pressure Due to a Liquid Column



1.13 Total or Absolute Pressure

If the liquid is open to the atmosphere, atmospheric pressure also acts on the liquid surface.

Therefore, the total pressure at depth h is:

$$\mathbf{P = P_0 + \rho gh}$$

Where:

- P_0 = atmospheric pressure
- ρgh = gauge pressure

Thus:

Absolute pressure = Atmospheric pressure + Gauge pressure

1.14 Gauge Pressure

Gauge pressure is the pressure measured relative to atmospheric pressure.
Therefore:

$$\mathbf{P_g = P - P_0}$$

For a liquid column: $\mathbf{P_g = \rho gh}$
Thus:

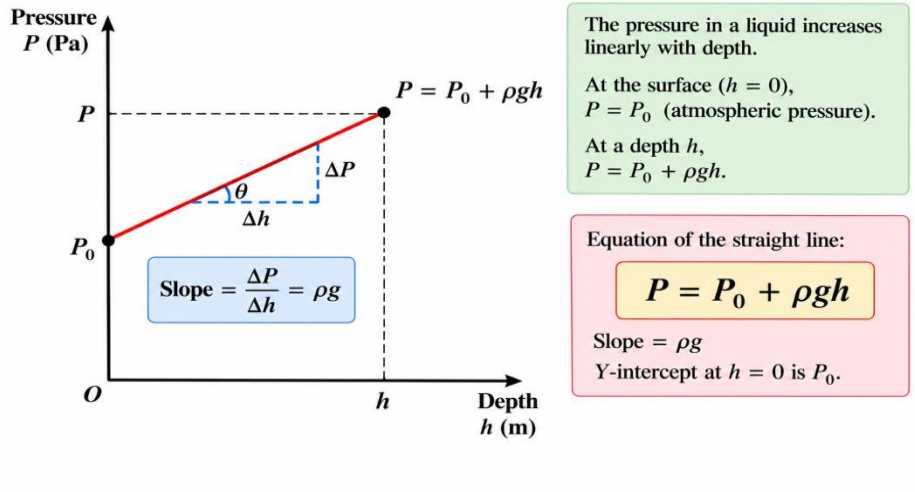
$$P_{absolute} = P_{atmospheric} + P_{gauge}$$

1.15 Important Result: Pressure Depends on Depth

From: $\mathbf{P = P_0 + \rho gh}$ for a particular liquid:
P increases linearly with depth h.
Therefore:

- Pressure at greater depth is greater.
- Pressure at the same depth is the same if the fluid is at rest and connected.

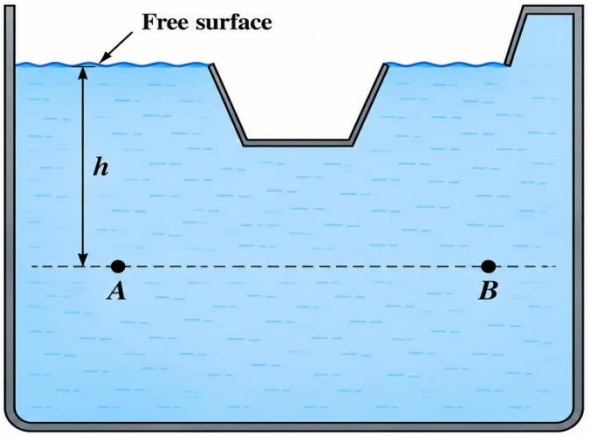
Figure 9.2: Variation of Pressure with Depth



1.16 Pressure at the Same Horizontal Level

For a liquid at rest in a connected container, pressure at the same horizontal level is the same, provided the points are within the same continuous fluid.
This is an important property of hydrostatic equilibrium.

Figure 9.3: Equal Pressure at the Same Depth



In a liquid at rest, the pressure depends only on the depth from the free surface and not on the shape of the container.

Points A and B are at the same depth h, hence the pressure at these points is equal.

Pressure at A:

$$P_A = \rho gh$$

Pressure at B:

$$P_B = \rho gh$$

$$P_A = P_B$$

1.17 Hydrostatic Pressure

The pressure developed in a fluid due to the weight of the fluid above a point is called **hydrostatic pressure**.

For a liquid: $\mathbf{P_g = \rho gh}$

It depends on:

1. Density of liquid
2. Depth
3. Acceleration due to gravity

It does **not** directly depend on the shape of the container.

1.18 Paradox of Hydrostatic Pressure

Consider vessels of different shapes containing the same liquid.

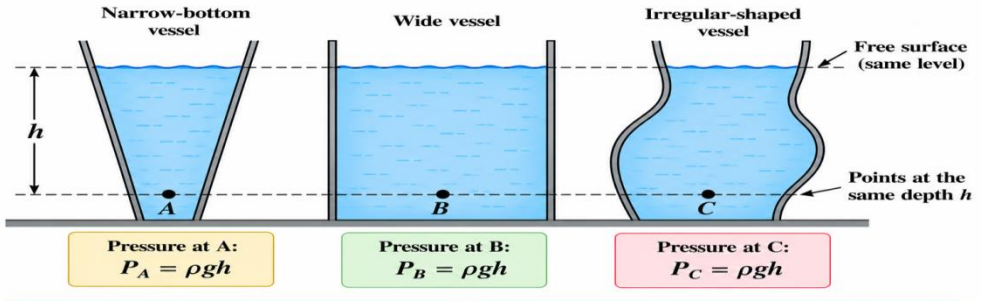
If the liquid level and the depth of a point are the same, the pressure at that point is the same even though the total amount of liquid in the vessels may be different.

This is sometimes called the **hydrostatic paradox**.

The pressure depends on:**ρ, g and h**

not directly on the shape of the container.

Figure 9.4: Hydraulic Paradox



Although the vessels have different shapes and different cross-sectional areas, the pressure at the same depth h is the same in all the vessels:

$$P = \rho gh$$

1.19 Pressure Variation with Density

From: $\mathbf{P_g = \rho gh}$ for the same depth and g:

$$p_g \propto \rho$$

Therefore, a denser liquid produces greater pressure at the same depth.
For example, at the same depth:
Pressure in mercury > pressure in water
because mercury has much greater density.

1.20 Pressure in a Liquid: Important Points

At a particular depth:
Pressure increases with density: $\mathbf{P \propto \rho}$
Pressure increases with depth: $\mathbf{P \propto h}$
Pressure increases with gravitational acceleration: $\mathbf{P \propto g}$
Therefore: $\mathbf{P_g = \rho gh}$

1.21 Atmospheric Pressure

The Earth's atmosphere has mass and therefore exerts pressure on surfaces.
The pressure exerted by the atmosphere is called **atmospheric pressure**.
At sea level, its standard value is approximately:

$$\mathbf{P_{atm} = 1.013 \times 10^5\ Pa}$$

Or approximately:**1 atm**

1.22 Atmospheric Pressure and Height

Atmospheric pressure generally decreases as altitude increases.
This happens because the amount of air above a point decreases with increasing height.
Therefore:

- Pressure is higher near sea level.
- Pressure is lower at high altitude.

This is why atmospheric pressure is not constant everywhere.

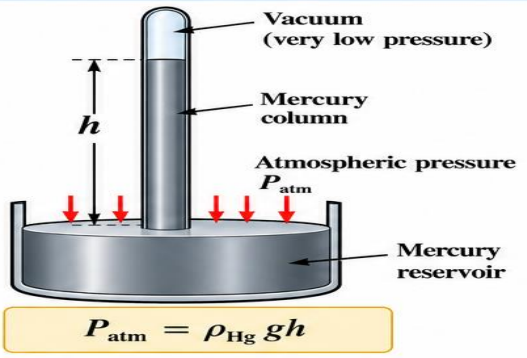
1.23 Barometer

A **barometer** is an instrument used to measure atmospheric pressure.
A mercury barometer consists of:

- A long glass tube
- Mercury
- A mercury reservoir
- Nearly vacuum space above the mercury column

The atmospheric pressure supports the mercury column.

Figure 9.5: Mercury Barometer



Atmospheric pressure is balanced by the weight of mercury column.

1.24 Derivation of Barometric Pressure

At the mercury surface in the reservoir:
Atmospheric pressure acts downward.
At the same horizontal level inside the mercury column:
Pressure due to the mercury column is: **P = ρgh**
For equilibrium: **Patm = $\rho_{mercury}$ gh**
Thus, measuring h allows us to determine atmospheric pressure.
At standard atmospheric pressure: **h ≈ 76 cm of mercury**
Therefore:**1 atm ≈ 76 cm Hg**

Note:"Space above the mercury column is nearly a vacuum and contains only a small amount of mercury vapour."

1.25 Why Is Mercury Used in a Barometer?

Mercury is preferred because:

1. High density

Mercury has high density, so only a short column is needed.

2. Very low vapour pressure

Its vapour pressure is small, so the space above the mercury remains nearly vacuum.

3. Does not wet glass significantly

This helps in obtaining a well-defined column.

4. Easily visible

Mercury is opaque and its level can be observed clearly.

1.26 Pascal's Law

Pascal's law is one of the most important principles of fluid mechanics.

Statement

"For a confined fluid in static equilibrium,pressure applied to an enclosed fluid is transmitted undiminished in all directions to every part of the fluid and to the walls of its container.''

This principle is used in hydraulic machines.

1.27 Hydraulic Press

A hydraulic press consists of:

- A small piston
- A large piston
- An enclosed incompressible fluid
- Connecting fluid chamber

Suppose:

- Force F_1 acts on small piston.
- Area of small piston = A_1.
- Force F_2 acts on large piston.
- Area of large piston = A_2.

Pressure applied to the fluid is: $\mathbf{P = F_1/A_1}$

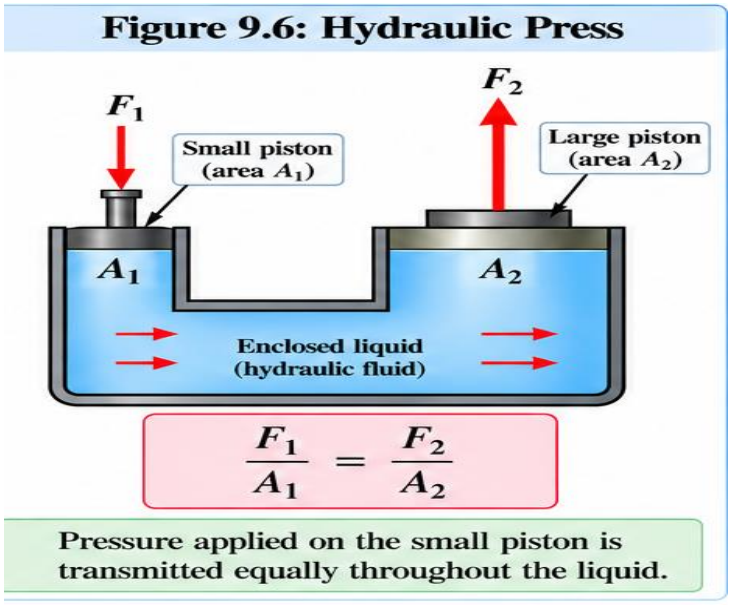
According to Pascal's law, the same pressure is transmitted to the large piston: $\mathbf{P = F_2/A_2}$

Therefore: $F_1/A_1 = F_2/A_2$

Hence: $\mathbf{F_2/F_1 = A_2/A_1}$

Or, $\mathbf{F_2 = F_1(A_2/A_1)}$

Thus, a small applied force can produce a large output force when the output piston has a larger area.



1.28 Mechanical Advantage of Hydraulic Press

Mechanical advantage is:

MA = Output force/Input force

Therefore: $\mathbf{MA = F_2/F_1}$

Using Pascal's law: $\mathbf{MA = A_2/A_1}$

Thus, a larger ratio of piston areas produces a larger mechanical advantage.

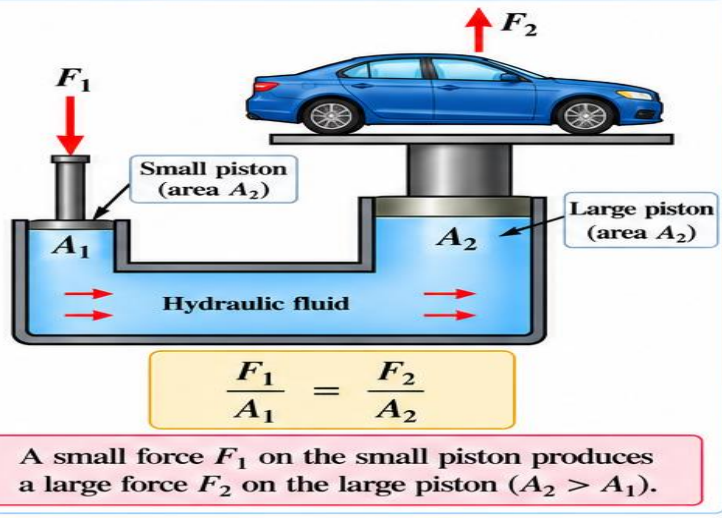
1.29 Hydraulic Lift

A hydraulic lift is based on Pascal's law.
It is used to lift heavy objects such as:

- Cars
- Trucks
- Machinery

A small force applied to a small piston produces a larger force on the larger piston.

Figure 9.7: Hydraulic Lift

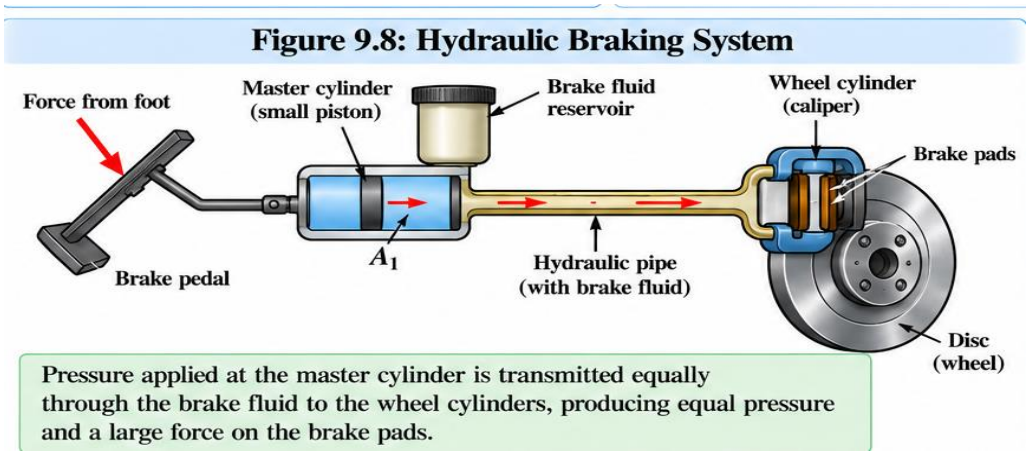


1.30 Hydraulic Brakes

Hydraulic brakes also work on Pascal's law.
When the brake pedal is pressed:

1. A force is applied to the master-cylinder piston.
2. Pressure is produced in the brake fluid.
3. This pressure is transmitted through the brake fluid.
4. The pressure acts on the pistons at the wheels.
5. The brake pads press against the rotating disc or drum.
6. Friction slows or stops the vehicle.

Figure 9.8: Hydraulic Braking System



1.31 Hydraulic Machines and Conservation of Energy

Although a hydraulic press can multiply force, it does not create energy.
If the small piston moves through distance x_1 and the large piston moves through distance x_2:
For an incompressible fluid: $\mathbf{A_1x_1 = A_2x_2}$
Therefore: $\mathbf{x_2 = (A_1/A_2)x_1}$
The larger piston moves through a smaller distance.
Work input: $\mathbf{W_1 = F_1x_1}$
Work output: $\mathbf{W_2 = F_2x_2}$
Using: $F_2/F_1 = A_2/A_1$
And

$$x_2/x_1 = A_1/A_2$$

We obtain ideally: $\mathbf{W_2 = W_1}$

"Thus, an ideal hydraulic machine conserves mechanical work. In a real hydraulic machine, some energy is lost due to friction and other dissipative effects."

1.32 Worked Example 1 – Pressure

Question

A force of 200 N acts normally on an area of 0.02 m². Calculate the pressure.

Solution

Using: **P = F/A** P = 200/0.02
Therefore: $\mathbf{P = 1 \times 10^4\ Pa}$

1.33 Worked Example 2 – Liquid Pressure

Question

Calculate the gauge pressure at a depth of 5 m in water. Take: $\rho = 1000\ kg/m^3$, $g = 9.8\ m/s^2$

Solution

P = ρgh = 1000 × 9.8 × 5
Therefore: $\mathbf{P = 4.9 \times 10^4\ Pa}$

1.34 Worked Example 3 – Absolute Pressure

Question

Find the absolute pressure at a depth of 10 m below the surface of water if atmospheric pressure is 1.013×10^5 Pa.

Solution

Gauge pressure:
$\mathbf{P_g = \rho gh} = 1000 \times 9.8 \times 10 = 9.8 \times 10^4$ Pa
Therefore:

Pabsolute = Patm + Pg

Or,

Pabsolute = $1.013 \times 10^5 + 9.8 \times 10^4$

Hence:

Pabsolute = $\mathbf{1.993 \times 10^5\ Pa}$

1.35 Worked Example 4 – Hydraulic Press

Question

The area of the small piston of a hydraulic press is 5 cm^2 and that of the large piston is 500 cm^2. If a force of 100 N is applied to the small piston, calculate the output force.

Solution

Using: $\mathbf{F_2/F_1 = A_2/A_1}$

Therefore: $F_2 = 100 \times (500/5)$

Hence: $\mathbf{F_2 = 10{,}000\ N}$

Thus, an input force of 100 N can ideally produce an output force of 10,000 N.

1.36 Worked Example 5 – Pressure in Two Liquids

Question

At the same depth, which produces greater pressure: water or mercury?

Solution

Gauge pressure: $\mathbf{P = \rho gh}$

At the same h and g: $\mathbf{P \propto \rho}$

Mercury has a much higher density than water.

Therefore:

Mercury produces greater pressure at the same depth.

1.37 NEET/JEE Quick Insights

Insight 1: Pressure is a scalar quantity.

Insight 2: For the same liquid: $\mathbf{P_g \propto h}$

Insight 3: At the same depth in the same connected fluid: $\mathbf{P_1 = P_2}$

Insight 4: Pressure does not depend directly on the shape of the container.

Insight 5: For a hydraulic machine: **Force ratio = Area ratio**

Insight 6: For an incompressible fluid: $\mathbf{A_1x_1 = A_2x_2}$

Insight 7: A hydraulic press multiplies force but does not multiply work.

1.38 Master Concept – Part 1

The fundamental chain of this part is:

Fluid → Density → Pressure → Hydrostatic Pressure $P_g = \rho gh$ → Atmospheric pressure→ Pascal's Law→ Hydraulic Press / Lift / Brakes

The most important equation to remember is: $\mathbf{P = P_0 + \rho gh}$

And the most important principle is:

Pascal's Law: Pressure applied to an enclosed fluid is transmitted undiminished throughout the fluid.

PART 2 – BUOYANCY, ARCHIMEDES' PRINCIPLE AND FLOTATION

2.1 Learning Objectives

After studying this part, you will be able to:

- Understand buoyant force and its physical origin.
- Derive the expression for buoyant force.
- State and apply Archimedes' principle.
- Understand apparent weight in a fluid.
- Explain why objects float or sink.
- Derive the condition of flotation.
- Calculate the fraction of a floating body submerged in a liquid.
- Understand the principle of a hydrometer.
- Solve Board, NEET and JEE-level numerical problems.

2.2 Buoyant Force

When an object is partially or completely immersed in a fluid, the fluid exerts an upward force on the object.

This upward force is called the **buoyant force** or **upthrust**.

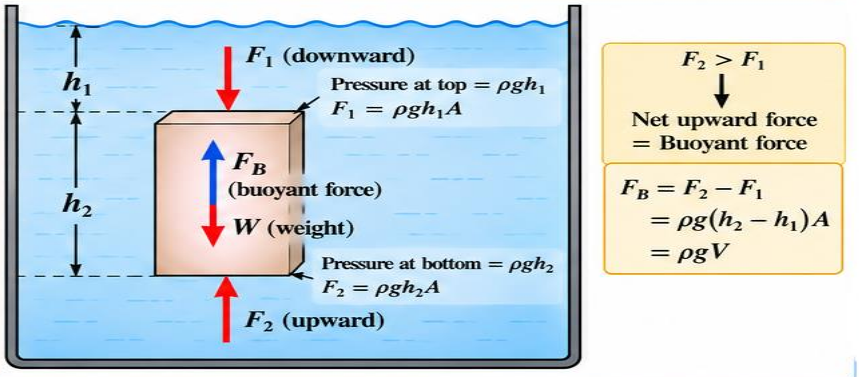
The buoyant force acts vertically upward.

It is a consequence of the fact that fluid pressure increases with depth.

Since the lower surface of an immersed object is generally at greater depth than its upper surface, the pressure on the lower surface is greater.

The resulting net force is upward.

Figure 9.9: Buoyant Force on an Immersed Body



2.3 Origin of Buoyant Force

Consider a cubical body immersed in a liquid.

Let:

- Area of upper and lower surfaces = A
- Depth of upper surface = h_1
- Depth of lower surface = h_2
- Density of liquid = ρ

Pressure at the upper surface: $\mathbf{P_1 = P_0 + \rho g h_1}$
Pressure at the lower surface: $\mathbf{P_2 = P_0 + \rho g h_2}$
Since: $\mathbf{h_2 > h_1}$
We have: $\mathbf{P_2 > P_1}$
Therefore, the upward force on the lower surface is greater than the downward force on the upper surface.
This difference produces the upward buoyant force.

2.4 Derivation of Buoyant Force

Upward force on the lower surface: $\mathbf{F_2 = P_2 A}$
Downward force on the upper surface: $\mathbf{F_1 = P_1 A}$
Net upward force: $\mathbf{F_B = F_2 - F_1}$
Therefore: $\mathbf{F_B = (P_2 - P_1)\, A}$
Now: $P_2 - P_1 = \rho g(h_2 - h_1)$
Therefore: $\mathbf{F_B = \rho g(h_2 - h_1)A}$
But: $\mathbf{(h_2 - h_1)A = V}$
Where, V is the volume of the immersed part of the body.
Hence: $$\mathbf{F_B = \rho V g}$$
Thus, the buoyant force equals the weight of the fluid displaced by the immersed body.

2.5 Archimedes' Principle

Statement

When a body is wholly or partially immersed in a fluid, it experiences an upward buoyant force equal to the weight of the fluid displaced by the body.
Therefore:
F_B = Weight of displaced fluid
Let:

- Density of fluid = ρ_f
- Displaced volume = V_d

Then:

$$F_B = \rho_f V_d g$$

This is the mathematical form of Archimedes' principle.

2.6 Volume of Displaced Fluid

The volume of displaced fluid is equal to the **volume of the portion of the object that is immersed**.
Therefore:

Completely immersed body:

$$V_{displaced} = V_{body}$$

Partially immersed body:

$$V_{displaced} = V_{immersed}$$

This distinction is very important for floating-body problems.

2.7 Buoyant Force on a Completely Immersed Body

If the object is completely immersed:

$$\boldsymbol{V_{displaced} = V_{body}}$$

Therefore,

$$F_B = \rho_f V_{body} g$$

If the body's density is ρ_b:
Weight of body:

$$W = \rho_b V_{body} g$$

Thus:

$$\frac{F_B}{W} = \frac{\rho_f}{\rho_b}$$

This relation is very useful for conceptual and numerical problems.

2.8 Apparent Weight

When a body is immersed in a fluid, the buoyant force acts upward while its weight acts downward.
Therefore, the apparent weight is reduced.
If:

- Actual weight = W
- Buoyant force = F_B

Then:

$$W_{app} = W - F_B$$

For a body of mass m:W = mg
Therefore:

$$W_{app} = mg - F_B$$

2.9 Apparent Loss of Weight

The reduction in weight when an object is immersed in a fluid is called **apparent loss of weight**.
Therefore:

Loss of weight = Buoyant force

Thus:

$$W - W_{app} = F_B$$

According to Archimedes' principle:

Loss of weight = Weight of displaced fluid

This provides an experimental method for determining the volume or density of objects.

2.10 Condition for Floating

Suppose a body floats on the surface of a liquid.
For equilibrium:

Upward force = Downward force

Therefore:

$$F_B = W$$

Using Archimedes' principle:

$$\rho_f V_{immersed} g = \rho_b V_{body} g$$

Cancelling g:

$$\rho_f V_{immersed} = \rho_b V_{body}$$

Hence:

$$\frac{V_{immersed}}{V_{body}} = \frac{\rho_b}{\rho_f}$$

Fraction above the liquid:

$$\frac{V_{above}}{V_{body}} = 1 - \frac{\rho_b}{\rho_f}$$

This is one of the most important formulas for floating bodies.

2.11 Fraction of Volume Submerged

For a floating body:

Fraction submerged = Density of body / Density of liquid

Therefore:

$$\frac{V_{immersed}}{V_{body}} = \frac{\rho_b}{\rho_f}$$

The fraction above the liquid surface is:

$$\frac{V_{above}}{V_{body}} = 1 - \frac{\rho_b}{\rho_f}$$

This is useful for many NEET/JEE numerical problems.

2.12 Conditions for Floating and Sinking

Consider the density of the body ρ_b and density of liquid ρ_f.

- Case 1: $\rho_b < \rho_f$ ------Body floats.

A part of the body remains above the surface.

- Case 2: $\rho_b = \rho_f$-------Body is neutrally buoyant.

The body is neutrally buoyant when fully immersed.
It can remain suspended within the liquid.

- Case 3: $\rho_B > \rho_f$-------- Body sinks.

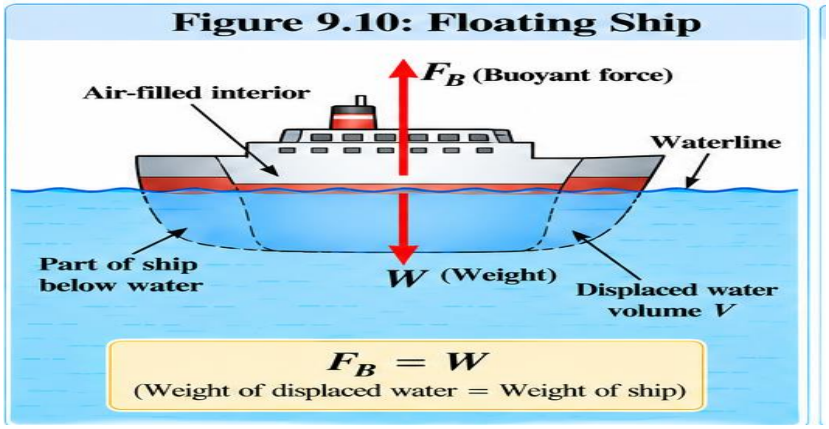
The body sinks if no other upward force supports it.

2.13 Why Does an Iron Ship Float?

Iron has a density greater than that of water.
Therefore, a solid piece of iron generally sinks.
But an iron ship can float because the ship is hollow and contains a large volume of air.
The **average density of the ship and the enclosed air** can be less than the density of water.
The ship displaces enough water so that:

Buoyant force = Weight of ship

Therefore, the ship floats.



2.14 Principle of Flotation

The principle of flotation states:

A floating body displaces a quantity of fluid whose weight is equal to the weight of the body.

Therefore:

Weight of floating body = Weight of displaced fluid

This is a direct consequence of Archimedes' principle and equilibrium.

2.15 Relative Density of a Solid Using Loss of Weight

Suppose a solid object has:

- Weight in air = Wair
- Weight when completely immersed in water = Wwater

Loss of weight: **Wair − Wwater**

This equals the weight of displaced water.

Therefore:

Relative density of solid = Wair/ (Wair − Wwater)

This is an important experimental formula.

"This formula assumes that the solid is completely immersed in water and does not dissolve or react with it."

2.16 Derivation of Relative Density Formula

Let:

- Volume of solid = V
- Density of solid = ρ_s
- Density of water = ρ_w

Weight of solid in air: **Wair = ρ_s Vg**

Loss of weight in water: **Wair − Wwater = ρ_w Vg**

Therefore:

Relative density: **ρ_s / ρ_w = Wair/ (Wair − Wwater)**

Hence:

RD = Wair/ (Wair − Wwater)

2.17 Relative Density of a Liquid

The density of an unknown liquid can also be compared with water.
For a suitable hydrometer or other measurement method, the relative density is:

$$RD = \frac{\rho_{liquid}}{\rho_{water}}$$

Thus:

$$\rho_{liquid} = RD \times \rho_{water}$$

2.18 Hydrometer

A **hydrometer** is an instrument used to measure the relative density or specific gravity of liquids.
It works on the principle of flotation.
A hydrometer floats at different depths in different liquids.

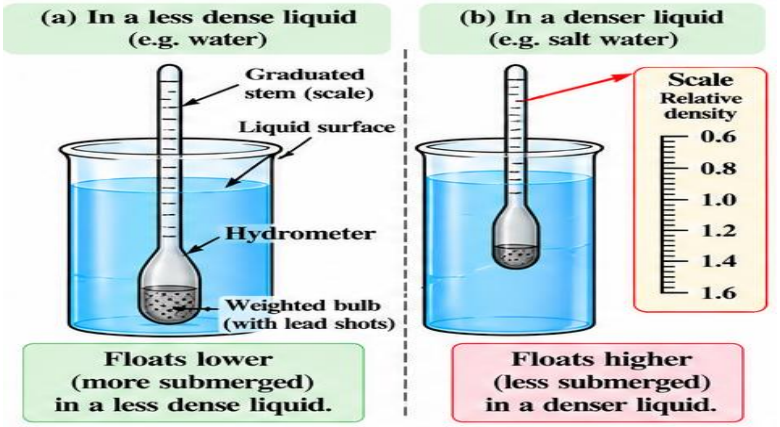
In a denser liquid:

Less volume needs to be submerged to displace a weight equal to the hydrometer's weight.
Therefore, the hydrometer floats higher.

In a less dense liquid:

More volume must be submerged.
Therefore, the hydrometer sinks deeper.

Figure 9.11: Hydrometer



2.19 Applications of Hydrometers

Hydrometers are used to determine the density or relative density of liquids in applications such as:

- Battery electrolyte testing
- Dairy industry
- Chemical industries
- "Measurement of alcohol concentration in suitable liquid mixtures."
- Petroleum products
- Laboratory measurements

2.20 Worked Example 1 – Buoyant Force

Question

A body of volume 0.02 m^3 is completely immersed in water. Calculate the buoyant force.
Take: $\rho_w = 1000\ kg/m^3$, $g = 9.8\ m/s^2$

Solution

Using: $\mathbf{F_B = \rho_f Vg}$
Therefore: $F_B = 1000 \times 0.02 \times 9.8$
Hence: $\mathbf{F_B = 196\ N}$

2.21 Worked Example 2 – Apparent Weight

Question

A body weighs 500 N in air. When completely immersed in water, it experiences an upward buoyant force of 120 N. Find its apparent weight.

Solution

$$\mathbf{Wapp = W - F_B}$$

Therefore: Wapp = 500 − 120
Hence: **Wapp = 380 N**

2.22 Worked Example 3 – Fraction Submerged

Question

A body of density 600 kg/m^3 floats in water. What fraction of its volume is submerged?

Solution

For a floating body:

$$\mathbf{V\ (immersed) / V\ (body) = \rho\ (body) / \rho\ (water)}$$

Therefore: V (immersed) / V (body) = 600/1000
Hence: **V (immersed) / V (body) = 0.6**
Thus, **60% of the volume is submerged**.
The remaining **40%** remains above the water surface.

2.23 Worked Example 4 – Density of a Solid

Question

A solid weighs 20 N in air and 15 N when completely immersed in water. Find its relative density.

Solution

Using: **RD = Wair/ (Wair − Wwater)**
Therefore: RD = 20/(20 − 15)= 20/5
Hence: **RD = 4**
Therefore, the density of the solid is: $\mathbf{\rho_s = 4 \times 1000 = 4000\ kg/m^3}$

2.24 Worked Example 5 – Floating Ice

Question

Ice has density approximately 900 kg/m^3 and water has density 1000 kg/m^3. What fraction of an ice block remains above water?

Solution
Fraction submerged: **Vsub/ V = ρ (ice) /ρ (water)** = 900/1000= 0.9
Therefore, fraction above water: **1 − 0.9 = 0.1**
Hence: **10% of the volume remains above water.**

2.25 Important Concept: Iceberg

Since ice is less dense than water, an iceberg floats with most of its volume below the water surface.
Approximately: **90% of its volume is submerged**
and approximately: **10% remains above the surface**
for the representative densities used above.

2.26 NEET/JEE Quick Insights

Insight 1: For a completely immersed body: $\mathbf{F_B = \rho_f V g}$

Insight 2: For a floating body: $\mathbf{F_B = W}$

Insight 3: For a floating body: $\mathbf{V_{sub}/V = \rho_b/\rho_f}$

Insight 4: If density of body is half the density of liquid:
Half the volume is submerged.

Insight 5: If density of body is 0.8 times density of liquid: **80% volume is submerged.**

Insight 6: Apparent loss of weight: $\mathbf{W - W_{app} = F_B}$

Insight 7: For the same fully immersed volume: $\mathbf{F_B \propto}$ **density of fluid**

2.27 Important Concept – Does Buoyant Force Depend on Depth?

For a completely immersed object in an incompressible liquid of constant density, the buoyant force is: $\mathbf{F_B = \rho_f V g}$
Therefore, if the object's immersed volume remains unchanged, the buoyant force is **independent of the depth**.
However, the individual pressures on the surfaces increase with depth.
Their difference remains such that the net buoyant force remains $\rho_f Vg$.
This is a common conceptual question.

2.28 Master Concept – Part 2

The complete logical chain is:
Fluid Pressure Increases with Depth→ Pressure Difference on an Immersed Body→Upward Buoyant Force→Archimedes' Principle $F_B = \rho_f V_d g$ **→Floating / Suspended / Sinking→Principle of Flotation→Hydrometer and Density Measurement**
The most important result for a floating body is: $\mathbf{F_B = W}$ and therefore:

$$\mathbf{V_{sub}/V\ (body) = \rho\ (body)/\rho\ (fluid)}$$

PART 3 – FLUID FLOW, EQUATION OF CONTINUITY AND BERNOULLI'S THEOREM

3.1 Learning Objectives

After studying this part, you will be able to:

- Understand the difference between steady and unsteady flow.
- Explain streamline and turbulent flow.
- Define streamline and understand its properties.
- Understand volume flow rate.
- Derive the equation of continuity.
- Understand Bernoulli's theorem.
- Derive Bernoulli's equation for an ideal fluid.
- Apply Bernoulli's principle to practical situations.
- Solve Board, NEET and JEE-level numerical problems.

3.2 Fluid Flow

A fluid is said to be in motion when its particles continuously change their positions with time.
Examples:

- Water flowing through a pipe
- Air moving around an aircraft wing
- Water flowing through a river
- Blood flowing through blood vessels

The motion of fluids is called **fluid flow**.
The nature of fluid flow depends on:

- Velocity of fluid
- Density
- Viscosity
- Geometry of the passage

3.3 Steady Flow

A flow is called **steady flow** if the velocity of the fluid at a particular point does not change with time.
At a fixed point:**v = constant with time**
The velocity may be different at different points, but at each fixed point it remains constant with time.

Example

Water flowing smoothly through a pipe at constant conditions can approximately represent steady flow.

3.4 Unsteady Flow

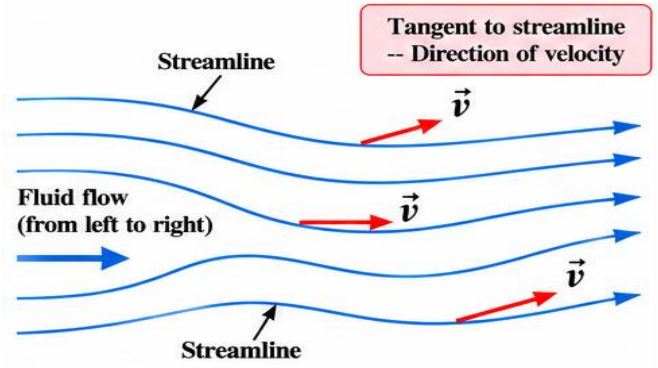
A flow is called **unsteady flow** when the velocity at a particular point changes with time.
Examples:

- Water flow immediately after opening a tap
- Rapidly changing river flow
- Flow during sudden changes in pressure

3.5 Streamline Flow

A **streamline** is an imaginary curve drawn in a flowing fluid such that the tangent at any point gives the direction of the fluid velocity at that point.
In streamline flow, the motion of fluid is smooth and orderly.
A fluid particle follows a well-defined path.

Figure 9.12: Streamline in a Fluid



3.6 Properties of Streamlines

Property 1

The tangent to a streamline at any point gives the direction of fluid velocity at that point.

Property 2

Two streamlines cannot intersect.
If two streamlines intersected, the fluid particle at the intersection would have two different directions of velocity at the same instant, which is impossible.

Property 3

The closer the streamlines are, the greater is the speed of the fluid in that region.

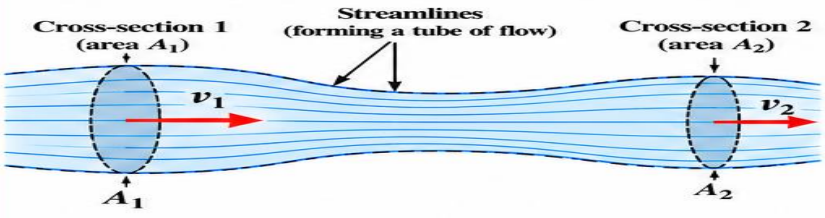
Property 4

For steady flow, the pattern of streamlines does not change with time.

3.7 Tube of Flow

A collection of streamlines forming a tube-like region is called a **tube of flow**.
A fluid particle cannot cross the boundary of a tube of flow because the velocity is tangent to the streamlines.

Figure 9.13: Tube of Flow



The streamlines form a tube of flow. No fluid crosses the streamline boundaries. The same fluid flows through both cross-sections.

3.8 Turbulent Flow

When the flow becomes irregular and chaotic, it is called turbulent flow.
In turbulent flow, fluid particles move irregularly and randomly
In turbulent flow:

- Fluid particles do not follow simple smooth paths.
- There are irregular fluctuations in velocity.
- Mixing occurs between neighbouring layers.
- Eddies are formed in the fluid.
- Velocity fluctuates with time
- Energy dissipation is comparatively large

Examples:

- Smoke rising chaotically from a chimney
- Water flowing rapidly past an obstruction
- Strongly disturbed air flow

3.9 Laminar Flow and Turbulent Flow

Fluid flow can broadly be classified into:

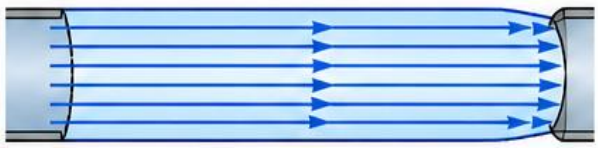
1. Laminar Flow

In laminar flow, fluid particles move in smooth, nearly parallel layers.
Characteristics:

- Smooth flow
- Well-defined streamlines
- Little mixing between layers
- Velocity at a given point remains steady
- Viscous effects are important

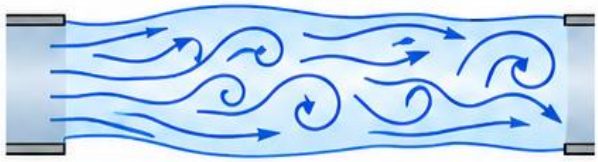
Figure 9.14-1: Streamline and Turbulent Flow

(a) Streamline flow (Smooth flow)



The fluid particles move along smooth, parallel and ordered streamlines.

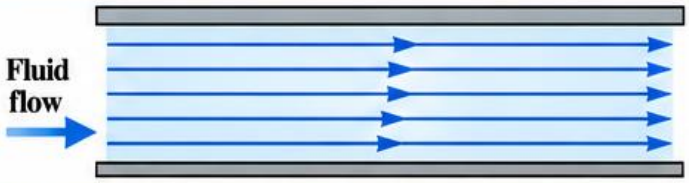
(b) Turbulent flow (Irregular flow)



The fluid particles move in an irregular and chaotic manner with eddies.

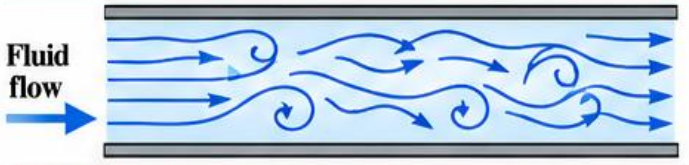
Figure 9.14-2: Laminar and Turbulent Flow

(a) Laminar flow



Laminar flow: Parallel, smooth streamlines. Different layers of fluid slide past each other in an orderly manner.

(b) Turbulent flow



Turbulent flow: Irregular streamlines with eddies and random mixing of fluid particles.

3.10 Volume Flow Rate

The volume of fluid flowing through a cross-section per unit time is called **volume flow rate**.
It is represented by: **$Q = \Delta V/\Delta t$**
If the fluid moves with speed v through an area A:
Volume crossing the area in time Δt is: **$\Delta V = A(v\Delta t)$**
Therefore:

$$Q = \frac{\Delta V}{\Delta t} = Av$$

SI unit:**m^3/s**

3.11 Mass Flow Rate

Mass flow rate is the mass of fluid flowing through a cross-section per unit time.
It is given by: **$\dot{m} = \Delta m/\Delta t$**
Since: **$\Delta m = \rho\Delta V$**
We get: **$\dot{m} = \rho(\Delta V/\Delta t)$**
Therefore: **$\dot{m} = \rho Av$**
For an incompressible fluid:
ρ is constant, so the mass flow rate is proportional to volume flow rate.

3.12 Equation of Continuity

Consider a fluid flowing through a pipe whose cross-sectional area changes.
At section 1:

- Area = A_1
- Velocity = v_1
- Density = ρ_1

At section 2:

- Area = A_2
- Velocity = v_2
- Density = ρ_2

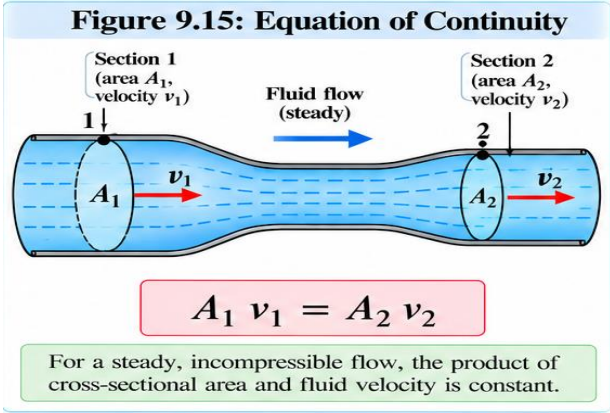
For steady flow, conservation of mass requires:
Mass entering per second = Mass leaving per second
Therefore:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

This is the general **equation of continuity**.

3.13 Continuity Equation for an Incompressible Fluid

For an incompressible fluid:**$\rho_1 = \rho_2 = \rho$**
Therefore:**$A_1v_1 = A_2v_2$**
Hence: **Av = constant**
This is the most commonly used form of the continuity equation.



3.14 Physical Meaning of the Continuity Equation

From: $\mathbf{A_1v_1 = A_2v_2}$

We obtain: $\mathbf{v \propto 1/A}$

Therefore:

Narrower section → Higher velocity

Wider section → Lower velocity

This is an extremely important result.

3.15 Example of Continuity Equation

Suppose a pipe has: $A_1 = 4$ cm² and $A_2 = 2$ cm². If: $v_1 = 3$ m/s then: $A_1v_1 = A_2v_2$

Therefore: $4 \times 3 = 2 \times v_2$

Hence: $\mathbf{v_2 = 6\ m/s}$

Thus, fluid speed doubles when the area is reduced to half.

3.16 Bernoulli's Principle

Bernoulli's theorem is based on the **conservation of mechanical energy** in a flowing ideal fluid.

For steady, incompressible, non-viscous flow along a streamline:

$$\boldsymbol{P + (1/2)\,\rho v^2 + \rho g h = constant}$$

This is called **Bernoulli's equation**.

Here:

- P = pressure energy per unit volume
- (½) ρv² = kinetic energy per unit volume
- ρgh = gravitational potential energy per unit volume

3.17 Statement of Bernoulli's Theorem

For a steady, incompressible and non-viscous fluid flowing along a streamline, the sum of pressure energy per unit volume, kinetic energy per unit volume and gravitational potential energy per unit volume remains constant.

Therefore:

$$\boldsymbol{P + \left(\frac{1}{2}\right)\rho v^2 + \rho g h = constant}$$

3.18 Energy Interpretation of Bernoulli's Equation

Bernoulli's equation is:

$$P + (1/2)\,\rho v^2 + \rho g h = constant$$

The three terms represent:

Pressure energy per unit volume (P)
Kinetic energy per unit volume (1/2) ρv²
Potential energy per unit volume (ρgh)

Therefore:

Pressure energy + Kinetic energy + Potential energy = Constant

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3.19 Bernoulli's Equation Between Two Points

At point 1:

- Pressure = P_1
- Velocity = v_1
- Height = h_1

At point 2:

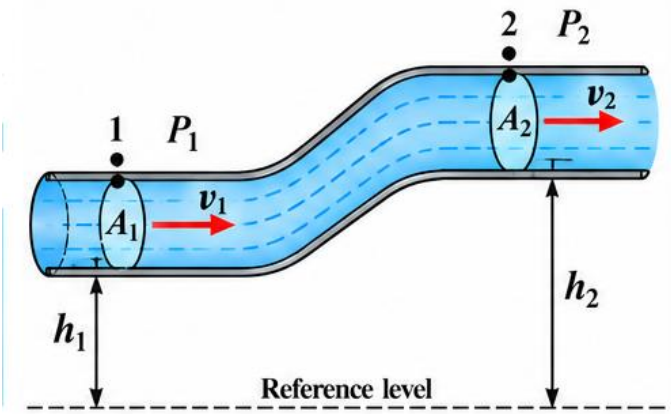
- Pressure = P_2
- Velocity = v_2
- Height = h_2

Then:

$$P_1 + (½)\,\rho v_1^2 + \rho g h_1 = P_2 + (½)\,\rho v_2^2 + \rho g h_2$$

This is the practical form of Bernoulli's equation.

Figure 9.16: Bernoulli's Theorem



$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

3.20 Derivation of Bernoulli's Equation

Consider a fluid flowing steadily through a pipe.
Suppose a volume ΔV of fluid moves from point 1 to point 2.
Let:

- Pressure at 1 = P_1
- Pressure at 2 = P_2
- Velocity at 1 = v_1
- Velocity at 2 = v_2
- Heights = h_1 and h_2

Work Done by Pressure Forces

At point 1, work done on the fluid: $\mathbf{W_1 = P_1 \Delta V}$
At point 2, the fluid does work against pressure P_2: $\mathbf{W_2 = P_2 \Delta V}$
Therefore, net work done on the fluid: $\mathbf{W = (P_1 - P_2)\Delta V}$

Change in Kinetic Energy

Mass of fluid: $\mathbf{m = \rho \Delta V}$

Change in kinetic energy:

$$\boldsymbol{\Delta K = (1/2)\rho \Delta V(v_2^2 - v_1^2)}$$

Change in Potential Energy

Change in gravitational potential energy:

$$\boldsymbol{\Delta U = \rho \Delta V\, g(h_2 - h_1)}$$

Apply Work–Energy Theorem

Net work done: $\mathbf{W = \Delta K + \Delta U}$

Therefore:

$$(P_1 - P_2)\Delta V = (1/2)\rho \Delta V(v_2^2 - v_1^2) + \rho \Delta V g(h_2 - h_1)$$

Dividing by ΔV:

$$P_1 - P_2 = (1/2)\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$

Rearranging:

$$P_1 + (1/2)\rho v_1^2 + \rho g h_1 = P_2 + 1/2\rho v_2^2 + \rho g h_2$$

Hence:

$$P + (1/2)\rho v^2 + \rho g h = constant$$

This is Bernoulli's equation.

3.21 Important Conditions for Bernoulli's Equation

Bernoulli's theorem is valid under idealized conditions:

1. Flow is **steady**.
2. Fluid is **incompressible**.
3. Fluid is **non-viscous**.
4. The equation is applied **along a streamline**.
5. The fluid flow is considered along a streamline, with no energy added or removed by external devices such as pumps or turbines

If significant viscosity or energy loss is present, the simple Bernoulli equation needs modification.

3.22 Bernoulli's Principle: Pressure–Velocity Relation

For a horizontal pipe: $\mathbf{h_1 = h_2}$

Therefore:

$$\boldsymbol{P + \frac{1}{2}\rho v^2 = constant}$$

Thus:

Higher velocity → Lower pressure

Lower velocity → Higher pressure

This is one of the most important applications of Bernoulli's principle.

3.23 Venturi Effect

The reduction in pressure that occurs when a fluid flows through a narrow section of a passage is called the **Venturi effect**.

It follows from:**Continuity equation**

$$\mathbf{Av = constant}$$

And

Bernoulli's equation

$$P + \frac{1}{2}\rho v^2 = constant$$

In the narrow section:

- Area decreases.
- Velocity increases.
- Pressure decreases.

3.24 Venturimeter

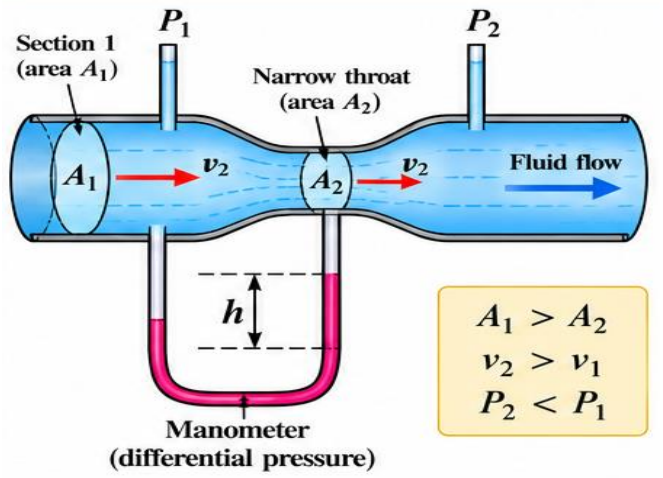
A **Venturimeter** is a device used to measure the flow speed or volume flow rate of a fluid through a pipe.

It consists of:

- Wide section
- Narrow throat
- Pressure measurement points

The pressure difference between the wide section and throat is used to determine the flow rate.

Figure 9.17: Venturimeter



3.25 Torricelli's Law

Consider a large tank containing a liquid.

A small hole is made at depth h below the liquid surface.

If the tank is sufficiently large compared with the hole, the speed of the liquid surface can be neglected.

Applying Bernoulli's equation between the free surface and the hole:

At the free surface:

- P_1 = Patm,
- $v_1 \approx 0$,
- height = h

At the hole:

- P_2 = Patm,
- velocity = v

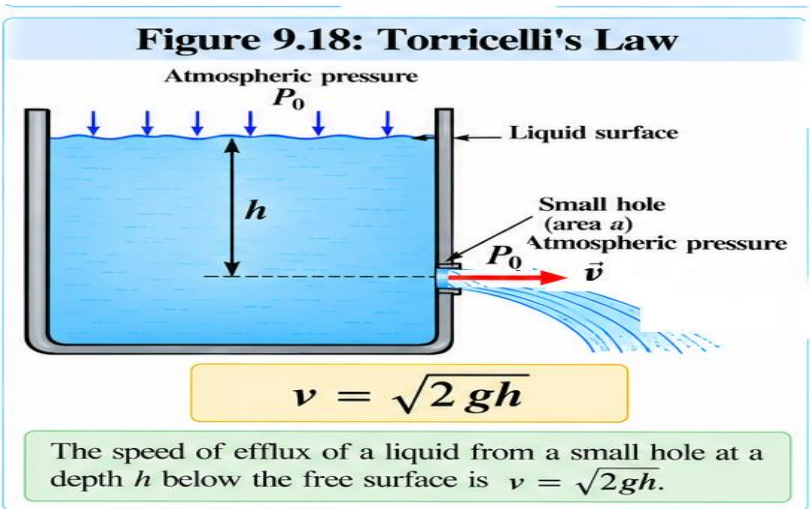
Taking the hole as zero height:
Bernoulli's equation gives:

$$\rho gh = (1/2)\rho v^2$$

Therefore:

$$v = \sqrt{(2gh)}$$

This is called **Torricelli's law**.
The liquid is assumed to be ideal, and viscous losses are neglected



3.26 Applications of Bernoulli's Principle

Bernoulli's principle has many practical applications.

1. Venturimeter: Used to measure flow rate.

2. Atomizer: Fast-moving air creates a region of lower pressure and draws liquid upward.

3. Carburetor: The pressure difference associated with fast air flow helps draw fuel into the air stream.

4. Airplane Wings: Pressure and velocity differences around an airfoil contribute to aerodynamic lift.

5. Chimneys: Fast-moving air above a chimney can reduce pressure and assist the upward movement of smoke.

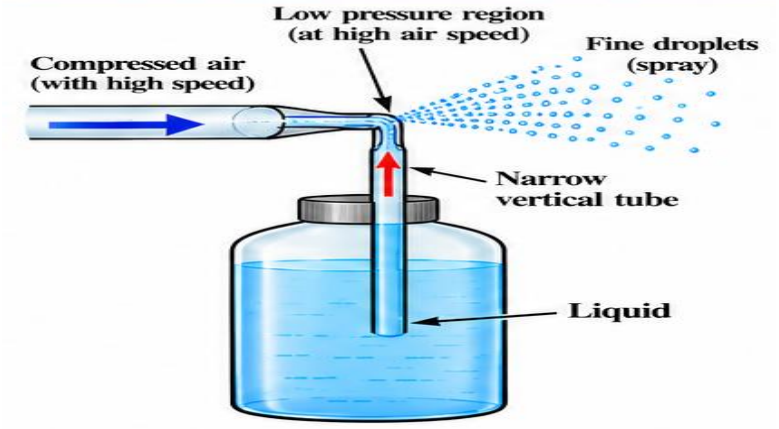
6. Torricelli's Law: The speed of efflux from a small hole is related to the liquid depth.

3.27 Atomizer

An atomizer works using a pressure difference created by rapidly moving air.
When air flows rapidly across the top of a tube:

- Air velocity increases.
- Pressure decreases.
- Atmospheric pressure on the liquid surface pushes the liquid upward.
- The liquid emerges and breaks into fine droplets.

Figure 9.19: Atomizer



3.28 Airplane Wing and Bernoulli's Principle

Air flows around an aircraft wing.
The pressure and velocity distributions around the wing can produce an upward aerodynamic force called **lift**.
In simplified textbook treatment:

- Faster air flow corresponds to lower static pressure.
- Pressure differences across the wing contribute to lift.

However, in actual aerodynamics, lift depends on the complete flow field around the airfoil and is not explained solely by a simplistic "equal transit time" argument.

3.29 Chimney and Bernoulli's Principle

Wind moving rapidly across the top of a chimney can reduce the pressure there.
The relatively higher pressure inside the building then helps push smoke upward.
Thus, air movement can improve the removal of smoke.

3.30 Worked Example 1 – Continuity Equation

Question

Water flows through a pipe of cross-sectional area 4 cm^2 at 2 m/s. If the pipe narrows to 2 cm^2, find the velocity of water in the narrow section.

Solution

For incompressible flow:$\mathbf{A_1v_1 = A_2v_2}$
Therefore:$4 \times 2 = 2 \times v_2$
Hence: $\mathbf{v_2 = 4\ m/s}$

3.31 Worked Example 2 – Bernoulli's Equation

Question

Water flows horizontally through a pipe. At one point its velocity is 2 m/s and pressure is 2×10^5 Pa. At another point its velocity is 4 m/s. Find the pressure there.
Take:$\rho = 1000\ kg/m^3$.

Solution

Since the pipe is horizontal:

$$P_1 + (1/2)\,\rho {v_1}^2 = P_2 + (1/2)\,\rho {v_2}^2$$

Therefore:

$$P_2 = P_1 + (1/2)\,\rho({v_1}^2 - {v_2}^2)$$

Substituting:

$$P_2 = 2 \times 10^5 + (1/2)(1000)(4 - 16) = 2 \times 10^5 - 6000$$

Hence: $P_2 = 1.94 \times 10^5$ Pa

As expected, the pressure decreases when the velocity increases.

3.32 Worked Example 3 – Torricelli's Law

Question

A small hole is 1.25 m below the surface of water in a large tank. Find the speed of efflux.
Take: $g = 9.8$ m/s².

Solution

Using: $\mathbf{v = \sqrt{(2gh)}} = \sqrt{(2 \times 9.8 \times 1.25)} = \sqrt{24.5}$

Hence: $\mathbf{v \approx 4.95\ m/s}$

3.33 Worked Example 4 – Volume Flow Rate

Question

Water flows through a pipe of area 5×10^{-3} m² with velocity 2 m/s. Find the volume flow rate.

Solution

Q = Av

Therefore: $Q = (5 \times 10^{-3})(2)$

Hence: $\mathbf{Q = 1 \times 10^{-2}\ m^3/s}$

3.34 Worked Example 5 – Mass Flow Rate

Question

Water flows through a pipe at a volume flow rate of 0.02 m³/s. Find the mass flow rate.
Take: $\rho = 1000$ kg/m³.

Solution:

Given:

$$Q = 0.02\ m^3/s$$

$$\rho = 1000\ kg/m^3$$

We know that

$$m = \rho Q$$

Substituting:

$$\dot{m} = 1000 \times 0.02$$

Hence:

$$m = 20\ kg/s$$

Answer: Mass flow rate = 20 kg/s

3.35 NEET/JEE Quick Insights

Insight 1: For incompressible fluid:**Av = constant**

Insight 2: If area becomes half:**Velocity becomes twice**

Insight 3: For horizontal flow:

$$P + \frac{1}{2}\rho v^2 = constant$$

Therefore:
Higher velocity → Lower pressure

Insight 4: In a wide section:**Velocity is smaller**and generally:**Pressure is higher**

Insight 5: In a narrow section:**Velocity is greater**and generally:**Pressure is lower**

Insight 6: Torricelli's law: **$v = \sqrt{(2gh)}$** has the same mathematical form as the speed acquired by a body falling freely through height h.

3.36 Master Concept – Part 3

The complete logical chain is:
Fluid Flow→ Streamline Flow→ Continuity Equation ($A_1v_1 = A_2v_2$) → Narrower Area → Higher Velocity→ Bernoulli's Principle ($P + (1/2)\rho v^2 + \rho gh = Constant$) → Higher Velocity → Lower Pressure *(*under the applicable Bernoulli conditions)**→Venturi Effect→Torricelli's Law→Applications: Venturimeter, Atomizer, Airflow, etc.**

PART 4: VISCOSITY, STOKES' LAW AND TERMINAL VELOCITY

4.1 Introduction to Viscosity

Real fluids do not flow without internal resistance.
When different layers of a fluid move with different velocities, an internal frictional force develops between the layers. This property of a fluid is called **viscosity**.

Definition

Viscosity is the property of a fluid by virtue of which it opposes the relative motion between its adjacent layers.
The internal frictional force is called **viscous force**.

Examples

- Honey flows more slowly than water because honey has greater viscosity.
- Oil flows more slowly than water.
- Water flows more easily than glycerine.
- Air also possesses viscosity.

Important Concept

Greater viscosity → greater resistance to flow.
Lower viscosity → easier flow.

4.2 Viscous Force Between Fluid Layers

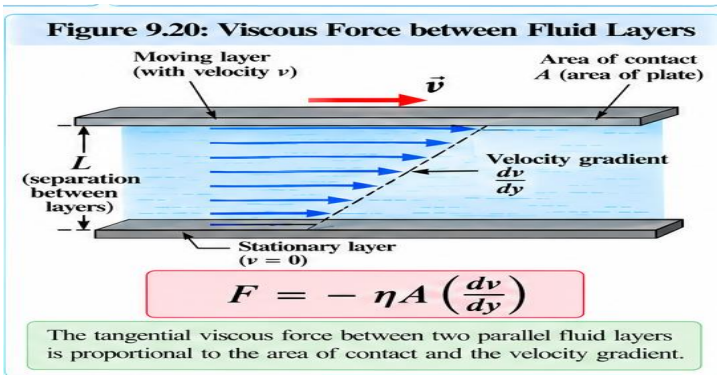
Consider two parallel layers of a fluid.

- The lower layer is stationary.
- The upper layer moves with velocity v.
- The separation between the layers is L.
- Area of contact between the layers is A.

The velocity changes gradually from zero at the stationary layer to v at the moving layer.
The velocity gradient is: **dv/dy,** where y represents the perpendicular distance between the layers.
The viscous force is given by: **F ∝ A**and: **F ∝ dv/dy**
Therefore: Magnitude of viscous force,

$$F = \eta A \left|\frac{dv}{dy}\right|$$

Where, η is the **coefficient of viscosity**.



4.3 Newton's Law of Viscosity

Newton's law of viscosity states that:
The viscous force between two adjacent layers of a fluid is directly proportional to the area of contact and the velocity gradient between the layers.
The magnitude of viscous force is directly proportional to the area of contact and the magnitude of the velocity gradient.
Mathematically:

$$F = \eta A \, |dv/dy|$$

In vector form, viscous force acts in opposite direction of relative motion.
For a uniform velocity gradient:

$$F = \eta A \, (v/L)$$

4.4 Coefficient of Viscosity

The coefficient of viscosity η is a measure of the internal resistance offered by a fluid to the relative motion of its layers.
From:

$$\boldsymbol{F = \eta A(dv/dy)}$$

We get:

$$\boldsymbol{\eta = F/[A(dv/dy)]}$$

SI unit of η = Pa·s
Also: **1 Pa·s = 1 N·s/m²**
Dimensional Formula: [η] = [ML⁻¹T⁻¹]

4.5 Physical Meaning of Coefficient of Viscosity

Suppose two parallel layers of a liquid have:

- area = 1 m²
- separation = 1 m
- velocity difference = 1 m/s

Then the velocity gradient is: **dv/dy = 1 s⁻¹**
If the viscous force between them is 1 N, then: **η = 1 Pa·s**
Thus:
The coefficient of viscosity is numerically equal to the tangential force required to maintain a unit velocity gradient between two unit-area layers separated by unit distance.

4.6 Velocity Profile in a Fluid

When a fluid flows through a narrow pipe, different layers generally move with different velocities.
For laminar flow:

- Velocity is zero at the wall of the pipe. For a fluid satisfying the no-slip condition, the fluid velocity relative to the pipe wall is zero at the wall.
- Velocity increases towards the centre.
- Maximum velocity occurs at the centre.

Thus, for fully developed laminar flow of a Newtonian fluid through a circular pipe, the velocity profile is parabolic."

4.7 Factors Affecting Viscosity

The viscosity of a fluid depends mainly on:

1. Nature of the fluid
2. Temperature
3. Pressure, particularly for gases

Effect of Temperature

For most liquids: **Temperature ↑ → Viscosity ↓**
For gases: **Temperature ↑ → Viscosity ↑**
This difference is important for conceptual questions.

Example

Heating honey makes it flow more easily because its viscosity decreases.

4.8 Reynolds Number

The nature of fluid flow can be predicted using a dimensionless quantity called the **Reynolds number**.
It is defined as:

$$Re = \rho v d/\eta$$

Where:

- ρ = density of fluid
- v = characteristic velocity
- d = characteristic length or diameter
- η = coefficient of viscosity

For flow through a circular pipe, d is usually taken as the pipe diameter.

Important

Reynolds number has **no unit and no dimensions**.

4.9 Physical Significance of Reynolds Number

Reynolds number represents the relative importance of:
Inertial effects / Viscous effects
Therefore:

$$R_e = Inertial\ effects\ /\ Viscous\ effects$$

Hence:

- Small Re → viscous effects dominate → flow tends to be laminar.
- Large Re → inertial effects dominate → flow tends to be turbulent.

For flow through a pipe, approximate values are:

Reynolds Number	Nature of Flow
Re < 2000	Generally laminar
2000 – 4000	Transition region
Re > 4000	Generally turbulent

These values are approximate and depend on the experimental conditions and geometry.

4.10 Critical Velocity

The maximum velocity up to which the flow of a fluid through a pipe remains approximately laminar is called the **critical velocity**.

From:

$$R_e = \frac{\rho v_c d}{\eta}$$

We obtain:

$$v_c = \frac{\eta R_e}{\rho d}$$

Thus, approximately,

$$v_c \propto \eta$$

$$v_c \propto 1/\rho$$

And

$$v_c \propto 1/d$$

These proportionalities are fixed for a given critical Reynolds number.

Important Concept

A fluid with greater viscosity can generally sustain laminar flow up to a higher velocity.

4.11 Stokes' Law

When a small spherical body moves slowly through a viscous fluid, the fluid exerts a resistive force on the sphere.

According to **Stokes' law**,

$$Fv = 6\pi\eta r v$$

Where:

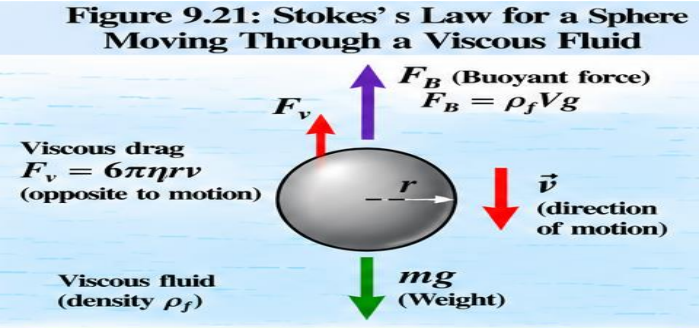
- Fv = viscous drag
- η = coefficient of viscosity
- r = radius of sphere
- v = velocity of sphere

The force acts opposite to the direction of motion.

Conditions for Stokes' Law

Stokes' law is applicable when:

- The body is approximately spherical.
- The sphere is small.
- The velocity is sufficiently low.
- The flow around the sphere is laminar.
- The fluid is sufficiently extensive compared with the sphere.



Forces acting on a sphere moving through a viscous fluid.

4.12 Terminal Velocity

Consider a small spherical body falling through a viscous liquid.
Three forces act on the sphere:

1. Weight

$$W = mg$$

acting downward.

2. Buoyant Force

According to Archimedes' principle,

$$F_B = \rho_f Vg$$

acting upward.

3. Viscous Force

According to Stokes' law,

$$Fv = 6\pi\eta rv$$

acting opposite to the direction of motion.

4.13 Motion of a Sphere Through a Viscous Fluid

Initially, when the sphere starts falling:**v = 0**
Therefore viscous force is zero.
The net downward force is relatively large, so the sphere accelerates.
As the velocity increases:**Fv = 6πηrv**also increases.
Therefore the net downward force decreases.
Eventually,

Weight = Buoyant force + Viscous force

At this stage,

$$mg = F_B + F_v$$

The resultant force becomes zero.
Hence,**a = 0**but the sphere continues to move with constant velocity.
This constant velocity is called **terminal velocity**.

4.14 Derivation of Terminal Velocity

Consider a sphere of: radius = r, density = ρ_s, fluid density = ρ_f, viscosity = η
Volume of sphere: $V = 4/3\ \pi r^3$
Mass of sphere: $m = \rho_s V$

Weight: $\mathbf{W = \rho_s V g}$
Buoyant force: $\mathbf{F_B = \rho_f V g}$
At terminal velocity v_t,

Weight = Buoyant force + Viscous force

Therefore,

$$\rho_s V g = \rho_f V g + 6\pi\eta r v_f$$

Hence,

$$(\rho_s - \rho_f)Vg = 6\pi\eta r v_t$$

Substituting: $V = 4/3\ \pi r^3$
We get

$$(\rho_s - \rho_f)(4/3\ \pi r^3)g = 6\pi\eta r v_t$$

Therefore,

$$v_t = 2r^2(\rho_s - \rho_f)g/(9\eta)$$

Final Result

$$\boxed{v_t = \frac{2r^2 g(\rho_s - \rho_f)}{9\eta}}$$

4.15 Important Dependences of Terminal Velocity

From: $v_t = \frac{2}{9} r^2 g(\rho_s - \rho_f)$
We obtain:

Dependence on radius

$$v_t \propto r^2$$

Therefore, if radius becomes twice: v_t **becomes four times.**

Dependence on viscosity

$$v_t \propto \frac{1}{\eta}$$

Higher viscosity → lower terminal velocity

Dependence on density difference

$$v_t \propto (\rho_s - \rho_f)$$

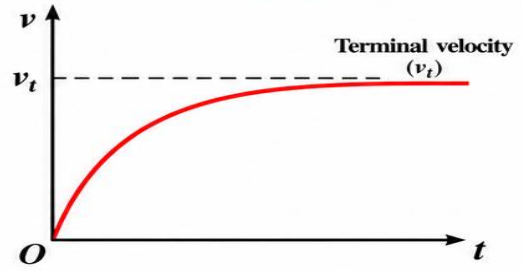
Greater density difference → greater terminal velocity

Dependence on gravitational acceleration

$$v_t \propto g$$

Greater g → greater terminal velocity

Figure 9.22: Velocity-Time Graph for a Falling Sphere



The velocity of a falling sphere increases rapidly at first and then approaches a constant terminal velocity v_t.

4.16 Terminal Velocity and Free Fall

A common misconception is that a sphere falling through a liquid continues to accelerate indefinitely.
This is not true.
As the velocity increases: **Viscous drag ↑**
Eventually: **Net force = 0**
Therefore: **Acceleration = 0** and **Velocity = constant = terminal velocity**

4.17 Applications of Terminal Velocity

Terminal velocity is important in many physical situations.

1. Raindrops
Raindrops accelerate initially, but air resistance increases with velocity. Eventually, they approach a nearly constant terminal speed.

2. Small Particles in Liquids
Fine particles settling through a liquid can reach terminal velocity.
This principle is useful in studying sedimentation.

3. Measurement of Viscosity
The terminal velocity of small spheres can be used to determine the viscosity of a liquid.

4. Separation of Particles
The difference in settling speeds can be used in industrial separation processes.

4.18 Viscosity and Temperature — Exam Concept
Liquids: When temperature increases:**Intermolecular attraction becomes less effective**
Therefore:**Viscosity of liquid decreases.**

Gases: When temperature increases:**Molecular motion increases**and momentum transfer between layers increases.
Therefore:**Viscosity of gases increases.**

Quick Comparison

Property	Temperature Increase
Viscosity of liquids	Decreases
Viscosity of gases	Increases

This is a frequently tested conceptual point.

4.19 Solved Example 1
A fluid has coefficient of viscosity**$\eta = 0.5$ Pa·s.** A layer of area**$A = 2$ m²**moves with velocity **$v = 0.4$ m/s**relative to another layer separated by**$L = 0.02$ m**.Find the viscous force.

Solution
Using: **$F = \eta Av/L$**
Substituting: **$F = (0.5)(2)(0.4)/(0.02)$**
Therefore: **$F = 20$ N**

Answer: $F = 20$ N

4.20 Solved Example 2
A small sphere of radius 1 mm falls through a liquid. If the density difference between the sphere and liquid is 500 kg/m³, viscosity is 1 Pa·s and g = 9.8 m/s², find its terminal velocity.

Given: $r = 1\text{ mm} = 1 \times 10^{-3}$ m, $\rho_s - \rho_f = 500$ kg/m³, $\eta = 1$ Pa·s, $g = 9.8$ m/s²
Using: **$v_t = 2r^2(\rho_s - \rho_f)g/(9\eta)$**
Therefore: **$v_t = 2(1 \times 10^{-3})^2(500)(9.8)/(9 \times 1) \approx 1.09 \times 10^{-3}$ m/s**

Answer: $v_t \approx 1.09$ mm/s

4.21 Solved Example 3
The diameter of a pipe is doubled while the fluid velocity and all other properties remain unchanged. What happens to the Reynolds number?

We know: $\mathbf{Re = \rho vd/\eta}$
Therefore: $\mathbf{Re \propto d}$
If diameter is doubled: $\mathbf{Re' = 2Re}$
Answer: The Reynolds number becomes **twice** its original value.

4.22 Solved Example 4 — NEET/JEE Concept

Two spherical particles have radii r and 2r and fall through the same viscous liquid under identical conditions.Compare their terminal velocities.
Since: $\mathbf{v_t \propto r^2}$
Therefore: $\mathbf{v_t(2r)/\ v_t(r) = (2r/r)^2}$
Hence: $\mathbf{v_t\ (2r) = 4v_t(r)}$
Answer: The larger sphere has **four times** the terminal velocity.

4.23 NEET/JEE Quick Insights

Insight 1: If radius of a sphere becomes 3 times:
$\mathbf{v_t' = 9v_t}$

Insight 2: If viscosity becomes 4 times:
$\mathbf{v_t' = v_t/4}$

Insight 3: If the density difference between the sphere and fluid becomes zero, the terminal velocity due to gravity-driven settling becomes zero in the ideal Stokes model.

Insight 4: For the same fluid and sphere:
$\mathbf{v_t \propto g}$
Therefore, terminal velocity changes if the value of g changes.

Insight 5: At terminal velocity:
$\mathbf{W = F_B + Fv}$
This equation is extremely important for numerical problems.

PART 5: SURFACE TENSION, SURFACE ENERGY AND CAPILLARITY

5.1 Introduction to Surface Tension

The molecules inside a liquid experience attractive forces from neighbouring molecules in all directions.
For a molecule deep inside the liquid, the attractive forces are approximately balanced.
However, a molecule at the free surface of a liquid experiences a net inward attractive force because there are fewer neighbouring molecules above it.
As a result, the liquid surface tends to contract and acquire minimum possible area.
This property of a liquid is called **surface tension**.

Definition

Surface tension is the tangential force acting per unit length perpendicular to an imaginary line drawn on the liquid surface.
It is represented by **T** or **S**.

5.2 Molecular Explanation of Surface Tension

Consider three molecules:

- Molecule A inside the liquid
- Molecule B near the surface
- Molecule C at the surface

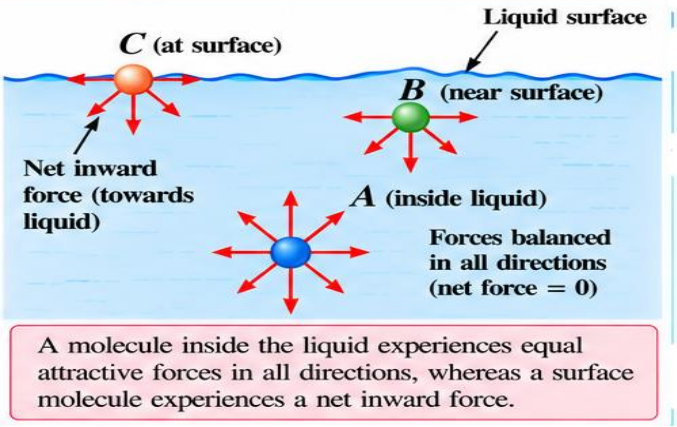
Molecule inside the liquid

It is attracted approximately equally in all directions.
Therefore:**Net force ≈ 0**

Molecule at the surface

It experiences a resultant attractive force towards the interior of the liquid.
Therefore, the surface tends to contract.
This explains why a liquid surface behaves somewhat like a stretched elastic membrane.

Figure 9.23: Molecular Explanation of Surface Tension



A molecule inside the liquid experiences equal attractive forces in all directions, whereas a surface molecule experiences a net inward force.

5.3 Mathematical Definition of Surface Tension

If a force F acts perpendicular to a line of length l on the liquid surface, then

$$T = F/l$$

Therefore,

$$F = Tl$$

SI Unit: N/m

Dimensional Formula: Since**T = Force/Length,** we get

$$[T] = [MT^{-2}]$$

5.4 Surface Tension as a Force

Surface tension can be understood as force per unit length.
For example, if

T = 0.07 N/m then a line of length **l = 0.2 m** experiences force **F = Tl**
Therefore,

$$F = 0.07 \times 0.2 = 0.014 \text{ N}$$

5.5 Surface Energy

The molecules at the surface of a liquid possess greater potential energy than molecules inside the liquid.
Therefore, the liquid surface possesses **surface energy**.

Definition

Surface energy is the energy required to increase the surface area of a liquid by a unit amount, under suitable conditions.
For a simple liquid surface at constant temperature:
Surface energy per unit area = Surface tension
Therefore,

$$T = \Delta W/\Delta A$$

Or,

$$\Delta W = T\Delta A$$

Where:

- ΔW = work done
- ΔA = increase in area of liquid surface

5.6 Why a Liquid Surface Tends to Minimise Area

For a given volume, a liquid tends to have the minimum possible surface area because the surface possesses excess energy.
Since the system tends to minimise its energy, the liquid surface tends to minimise its area.
This is why:

- Small liquid drops tend to become spherical.
- Soap bubbles are approximately spherical.
- Water droplets tend to become nearly spherical in the absence of gravity.

Important Concept

Minimum surface area → minimum surface energy

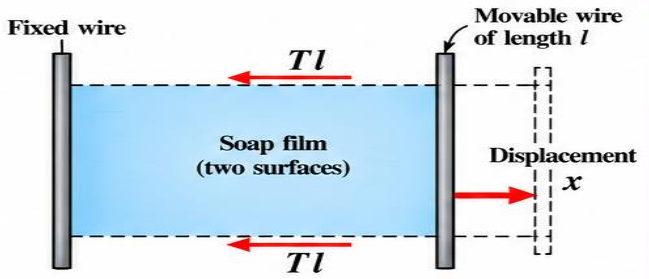
5.7 Surface Tension of a Soap Film

A soap film has **two surfaces**:

- Front surface
- Back surface

If a movable wire of length l is attached to a soap film, the surface tension acts on both surfaces.
Therefore, total force is: **F = 2Tl**
If the wire is displaced by a distance x:
Increase in area of one surface: $\Delta A_1 = lx$
Since there are two surfaces: $\Delta A = 2lx$
Therefore, work done is: $W = T(2lx)$
Thus, $W = 2Tlx$
Since: $F = 2Tl$
We get: $W = Fx$

Figure 9.24: Surface Tension on a Soap Film



Force on movable wire = $2Tl$

The soap film has two surfaces, so the total force due to surface tension on the movable wire is $2Tl$.

5.8 Effect of Temperature on Surface Tension

For most liquids:

Temperature ↑ → Surface tension ↓

As temperature increases, molecular cohesion becomes less effective at the surface.

At the critical temperature, the distinction between liquid and vapour phases disappears and surface tension approaches zero.

Important Exam Point

Surface tension generally decreases with increase in temperature.

5.9 Effect of Impurities on Surface Tension

The effect of impurities depends on their nature and concentration.

Examples:

- Soap generally decreases the surface tension of water.
- Some dissolved substances may increase or decrease surface tension.

Therefore, the statement

"Every impurity decreases surface tension"

is incorrect.

5.10 Excess Pressure Inside a Liquid Drop

Consider a spherical liquid drop of radius R.

Let:

- Surface tension = T
- Pressure inside = Pin
- Pressure outside = Pout

The excess pressure is

$$\Delta P = P_{in} - P_{out}$$

For a liquid drop,

$$\Delta P = 2T/R$$

Therefore,

$$P_{in} = P_{out} + 2T/R$$

5.11 Derivation of Excess Pressure Inside a Liquid Drop

Imagine cutting the spherical drop into two hemispheres.
The excess pressure acts over the circular cross-sectional area.
Pressure force: $\mathbf{F_P = \Delta P \times \pi R^2}$
Surface tension acts along the circumference:

$$\mathbf{F_T = T \times 2\pi R}$$

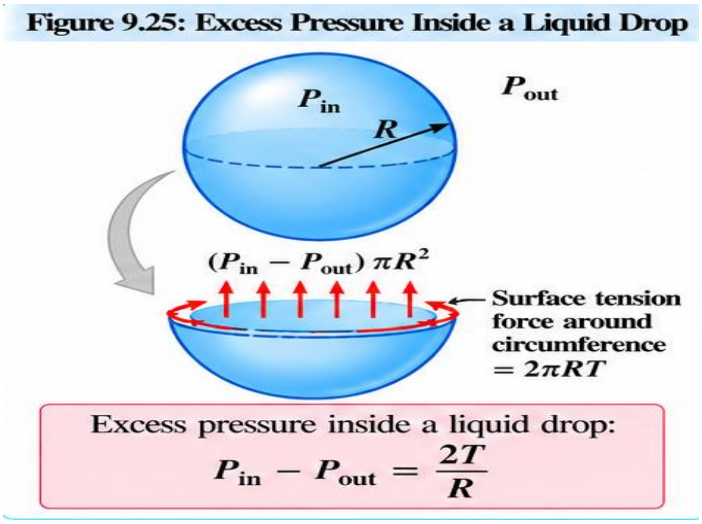
For equilibrium: $\mathbf{\Delta P \times \pi R^2 = 2\pi RT}$
Therefore,

$$\mathbf{\Delta P = 2T/R}$$

Final Result

Excess pressure inside a liquid drop = 2T/R

Figure 9.25: Excess Pressure Inside a Liquid Drop



5.12 Excess Pressure Inside a Soap Bubble

A soap bubble has **two liquid surfaces**.
Therefore, surface tension acts twice.
For a soap bubble of radius R:

$$\mathbf{\Delta P = 4T/R}$$

Thus,

$$\mathbf{Pin - Pout = 4T/R}$$

Or

$$\mathbf{Pin = Pout + 4T/R}$$

Important Comparison

Surface	Excess Pressure
Liquid drop	$\Delta P = 2T/R$
Soap bubble	$\Delta P = 4T/R$

Important NEET/JEE Point

For the same radius and same surface tension:
Excess pressure inside soap bubble = 2 × excess pressure inside liquid drop

5.13 Derivation for Soap Bubble

For a hemispherical soap bubble:

Pressure force: $\mathbf{F_P = \Delta P \times \pi R^2}$

Since the bubble has two surfaces, surface tension force is: $\mathbf{F_T = 2T \times 2\pi R}$

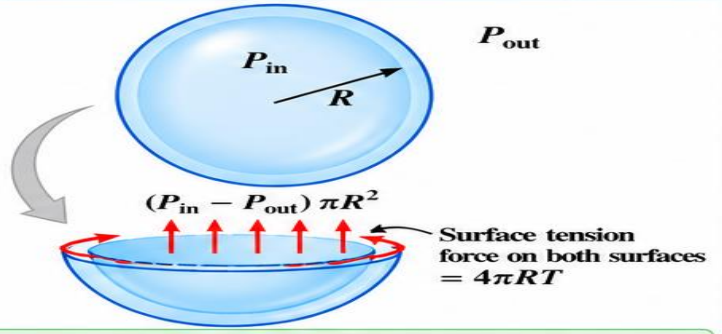
Therefore,

$$\mathbf{\Delta P \times \pi R^2 = 4\pi RT}$$

Hence,

$$\mathbf{\Delta P = 4T/R}$$

Figure 9.26: Excess Pressure Inside a Soap Bubble



Excess pressure inside a soap bubble:

$$P_{in} - P_{out} = \frac{4T}{R}$$

5.14 Dependence of Excess Pressure on Radius

For a liquid drop: $\mathbf{\Delta P = 2T/R}$

Therefore: $\mathbf{\Delta P \propto 1/R}$

For a soap bubble: $\mathbf{\Delta P = 4T/R}$

Therefore: $\mathbf{\Delta P \propto 1/R}$

Important Conclusion

Smaller drop/bubble → greater excess pressure

Larger drop/bubble → smaller excess pressure

5.15 Coalescence of Liquid Drops

Suppose several small liquid drops combine to form a larger drop.

The total volume remains constant.

If small drops combine:

Total volume before = Volume after

But the total surface area generally decreases.

Therefore:

Surface area decreases → Surface energy decreases

The decrease in surface energy is converted into other forms of energy, often including kinetic energy and heat.

5.16 Coalescence of Soap Bubbles

When two soap bubbles are connected, air flows from the smaller bubble towards the larger bubble, provided the bubbles have the same surface tension and the usual idealized conditions apply.

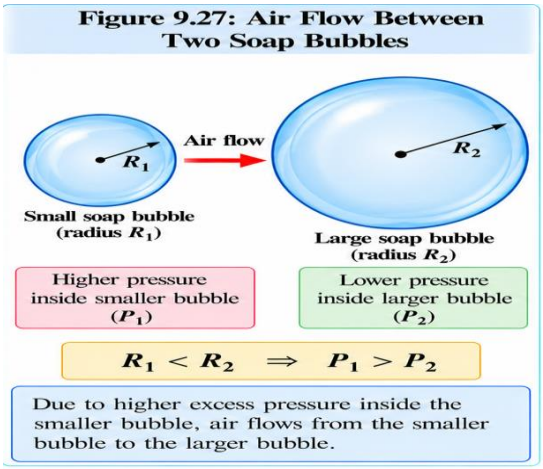
Why?

For a soap bubble: $\Delta P = 4T/R$

Therefore, the smaller bubble has greater internal pressure.

Hence, air flows:

Smaller bubble → Larger bubble



Important Concept

Smaller bubble has greater excess pressure.

5.17 Angle of Contact

When a liquid is placed in contact with a solid surface, the liquid surface near the point of contact forms an angle with the solid surface.

This angle is called the **angle of contact**.

It is represented by **θ**.

The angle of contact is measured through the liquid.

For Water and Clean Glass:

θ < 90°, Water wets glass:

For Mercury and Glass:

θ> 90°, Mercury does not wet glass.

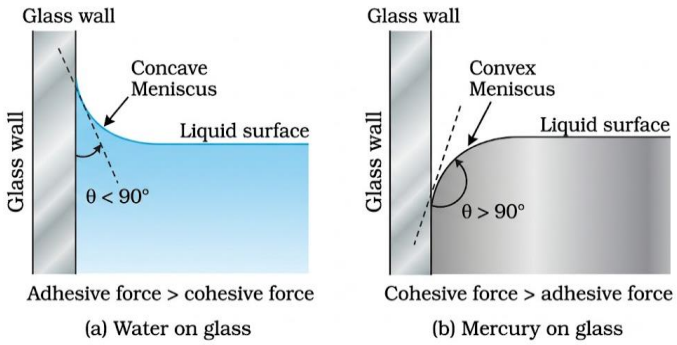


Figure 9.28: Angle of Contact and Meniscus for Water and Mercury

5.18 Cohesive and Adhesive Forces

Cohesive Force

The attractive force between molecules of the same substance is called **cohesive force**.

Example:

Attraction between water molecules

Adhesive Force

The attractive force between molecules of different substances is called **adhesive force**.
Example:
Attraction between water and glass

Relationship with Wetting

If adhesive force > cohesive force

The liquid tends to spread over the solid.
The relative strengths of adhesive and cohesive interactions influence the wetting behaviour and the angle of contact.
Example:
Water on clean glass
If cohesive force > adhesive force
The liquid tends to contract and form a convex surface.
Example:**Mercury on glass**

5.19 Meniscus

The curved surface of a liquid near the walls of a container is called the **meniscus**.
Water in Glass: Meniscus is **concave**.
Mercury in Glass: Meniscus is **convex**.
The shape depends on the relative strengths of adhesive and cohesive forces.

5.20 Capillarity

The phenomenon of rise or fall of a liquid in a narrow tube is called **capillarity** or **capillary action**.
A narrow tube is called a **capillary tube**.

Examples

- Rise of water in plants
- Absorption of ink by blotting paper
- Rise of kerosene in a wick
- Movement of water through soil
- Absorption of liquid by a sponge

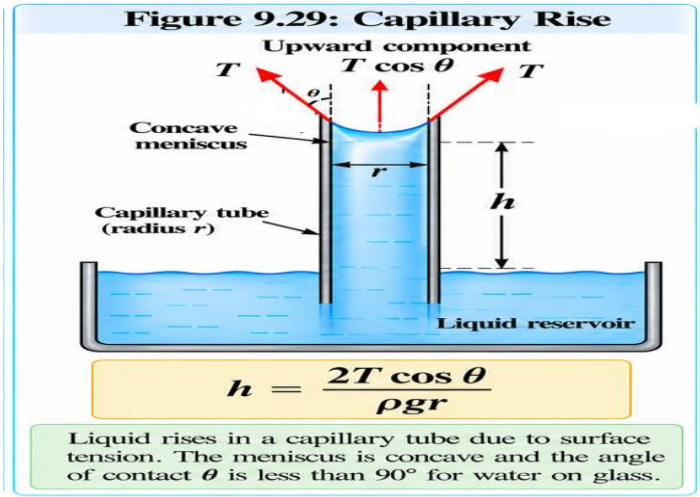
5.21 Capillary Rise

Consider a capillary tube of radius r inserted vertically into a liquid.
If the liquid wets the tube, the liquid rises to a height h.
The upward component of surface tension balances the weight of the liquid column.
Surface tension force: $\mathbf{F_T = 2\pi r T \cos\theta}$
Weight of liquid column: $\mathbf{W = \pi r^2 h \rho g}$
At equilibrium: $\mathbf{2\pi r T \cos\theta = \pi r^2 h \rho g}$
Therefore,

$$\mathbf{h = 2T \cos\theta/(\rho g r)}$$

Final Formula

$$h = \frac{2T\cos\theta}{\rho g r}$$



5.22 Capillary Depression

If the liquid does not wet the tube, the liquid level inside the capillary falls below the external liquid level.

This is called **capillary depression**.

For a non-wetting liquid: **θ > 90°**

Therefore, **cosθ < 0**

Hence, **h < 0**

The negative sign represents depression.

Example: Mercury shows capillary depression in a glass capillary.

5.23 Factors Affecting Capillary Rise

From: **h = 2T cosθ/(ρgr)**

We obtain:

1. Surface tension: h ∝ T

Higher surface tension → greater rise, under otherwise identical conditions

2. Radius: h ∝ 1/r

Smaller capillary radius → greater rise

3. Density: h ∝ 1/ρ

Higher density → smaller rise

4. Gravitational acceleration: h ∝ 1/g

Greater g → smaller capillary rise

5. Angle of contact: h ∝ cosθ

The sign and magnitude depend on θ.

5.24 Capillary Rise in Two Tubes

Suppose two capillary tubes have radii r_1 and r_2.
For the same liquid and same conditions: $\mathbf{h_1 r_1 = h_2 r_2}$
Therefore,

$$\mathbf{h_1/h_2 = r_2/r_1}$$

Important

If radius becomes half:**Capillary rise becomes twice.**

5.25 Applications of Capillarity

1. Plants: Water rises through narrow vessels in plants due partly to capillary effects along with other transport mechanisms.
2. Lamp Wick: Kerosene rises through the narrow spaces in the wick.
3. Fountain Pen: Ink moves through narrow passages due to capillary effects.
4. Blotting Paper: Ink spreads through the fine pores of blotting paper.
5. Soil: Water can move through narrow spaces between soil particles.
6. Towel and Cotton Cloth
Water spreads through the fine capillary spaces in fibres.

5.26 Surface Tension and Small Liquid Drops

For a liquid drop: $\mathbf{\Delta P = 2T/R}$
Therefore, if the radius decreases, the excess pressure increases.
This is particularly important in questions involving:

- Formation of small drops
- Sprays
- Atomization
- Coalescence of drops

5.27 Work Done in Increasing Surface Area

For a liquid surface: $\mathbf{W = T\Delta A}$
For a soap film having two surfaces:

$$\boldsymbol{W = 2T\,\Delta A_{single}}$$

If a rectangular soap film has dimensions l and x:
Area of one surface:$\mathbf{A = lx}$
Total area of two surfaces: $\mathbf{A\ (total) = 2lx}$
Therefore,

$$\mathbf{W = 2Tlx}$$

5.28 Solved Example 1 — Surface Tension

The surface tension of a liquid is: $\mathbf{T = 0.06\ N/m}$
Find the force acting on a line of length l = 0.5 m on the liquid surface.
Solution:
Using: $\mathbf{F = Tl}$
Therefore,

$$\mathbf{F = 0.06 \times 0.5 = 0.03\ N}$$

Answer: $F = 3 \times 10^{-2}$ N

5.29 Solved Example 2 — Liquid Drop

A liquid drop has radius: **R = 2 mm**and surface tension: **T = 0.05 N/m**.
Find the excess pressure inside the drop.
Using: $\Delta P = 2T/R$
Therefore: $\Delta P = 2(0.05)/(2 \times 10^{-3}) = 50$ Pa
Answer: $\Delta P = 50$ Pa

5.30 Solved Example 3 — Soap Bubble

A soap bubble has radius**R = 2 mm**and surface tension**T = 0.05 N/m**. Find the excess pressure inside it.
Using: $\Delta P = 4T/R$
Therefore: $\Delta P = 4(0.05)/ (2 \times 10^{-3}) = 100$ Pa
Answer: $\Delta P = 100$ Pa
Observation: For the same T and R:**Soap bubble excess pressure = 2 × liquid drop excess pressure**

5.31 Solved Example 4 — Capillary Rise

Water rises in a capillary tube of radius**r = 0.5 mm**
If:**T = 0.072 N/m, ρ = 1000 kg/m³, θ = 0°**and**g = 9.8 m/s²,** find the capillary rise.
Using: $h = 2T\cos\theta/(\rho g r)$
Since, $\cos 0° = 1$
Therefore, $h = 2(0.072)/ (1000 \times 9.8 \times 0.5 \times 10^{-3}) \approx 0.0294$ m $h \approx 2.94$ cm
Answer: $h \approx 2.94$ cm

5.32 Solved Example 5 — NEET/JEE Concept

Two capillary tubes have radii r and 3r and are inserted into the same liquid.
Compare the heights of liquid rise.
Since: $h \propto 1/r$
Therefore, $h_1/h_2 = r_2/r_1$
Thus, $h_1/h_2 = 3$
Answer: The liquid rises **three times higher** in the tube of radius r.

5.33 Solved Example 6 — Bubble Comparison

Two soap bubbles have radii R and 2R.
Compare their excess pressures.
For a soap bubble: $\Delta P = 4T/R$
Therefore: $\Delta P_1/\Delta P_2 = (1/R)/(1/2R)$
Hence: $\Delta P_1/\Delta P_2 = 2$
Answer: The smaller bubble has **twice the excess pressure** of the larger bubble.

5.34 Liquid Drop vs Soap Bubble

Property	Liquid Drop	Soap Bubble
Number of surfaces	1	2
Excess pressure	2T/R	4T/R
Surface tension contribution	T	2T
For same T and R	ΔP	$2\Delta P_drop$

5.35 Cohesive vs Adhesive Force

Cohesive Force	Adhesive Force
Between same molecules	Between different molecules
Water-water attraction	Water-glass attraction
Helps maintain liquid cohesion	Helps liquid stick to another surface

5.36 NEET/JEE Quick Insights

Insight 1: If the radius of a liquid drop is reduced to half:**ΔP becomes twice.**

Insight 2: If the radius of a soap bubble is reduced to one-third:**ΔP becomes three times.**

Insight 3: If the radius of a capillary tube is doubled:**h becomes half.**

Insight 4: If surface tension becomes twice:
Capillary rise becomes twice, provided all other quantities remain unchanged.

Insight 5: For a soap bubble: **$\Delta P = 4T/R$**
The factor 4 arises because the bubble has **two surfaces**.

Insight 6: For a liquid drop: **$\Delta P = 2T/R$**
The factor 2 arises from **one surface**.

5.37 Quick Revision — Part 5

Concept	Key Result
Surface tension	$T = F/l$
Surface energy	$\Delta W = T\Delta A$
Temperature effect	T generally decreases as temperature increases
Liquid drop	$\Delta P = 2T/R$
Soap bubble	$\Delta P = 4T/R$
Angle of contact	θ
Capillary rise	$h = 2T\cos\theta/(\rho gr)$
Smaller radius	Greater capillary rise
Water-glass	Concave meniscus
Mercury-glass	Convex meniscus
Cohesive force	Same molecules
Adhesive force	Different substances