PART 1 – INTRODUCTION TO ATOMIC STRUCTURE AND RUTHERFORD'S ATOMIC MODEL

1.1 Introduction

Matter is made up of atoms. The word **atom** originally means "indivisible," but modern experiments have shown that an atom has a complex internal structure.
An atom mainly consists of:

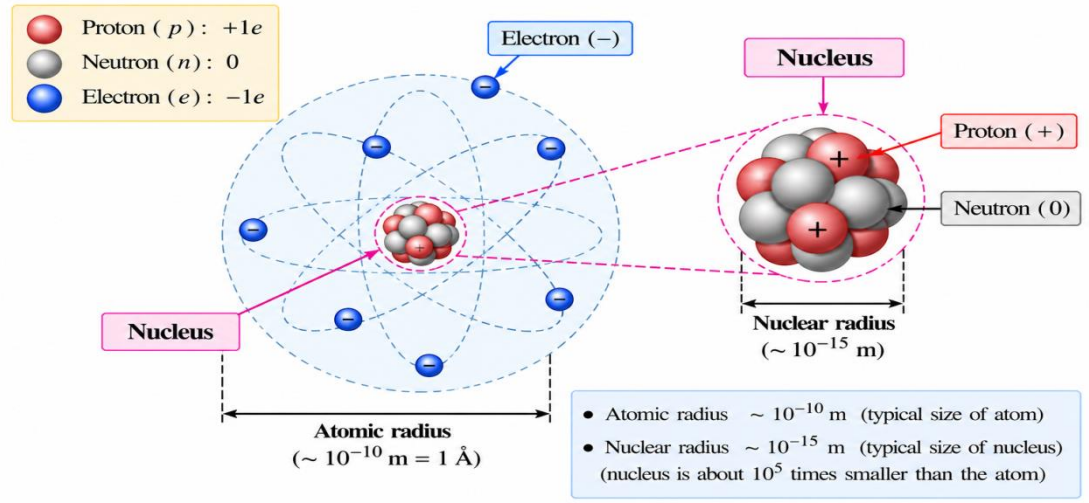
- A small, dense, positively charged **nucleus**
- Negatively charged **electrons** surrounding the nucleus

The nucleus contains:

- Protons
- Neutrons

The size of an atom is of the order of: 10^{-10} m, while the typical nuclear size is of the order of: 10^{-15} m. Thus, most of the volume of an atom is empty space.

Figure 12.1 : Basic Structure of an atom



1.2 Early Models of the Atom

Before Rutherford's experiment, several models were proposed to explain atomic structure.
The important models for this chapter are:

1. Thomson's atomic model
2. Rutherford's nuclear model
3. Bohr's atomic model

These models developed as experimental evidence became available.

1.3 Thomson's Atomic Model

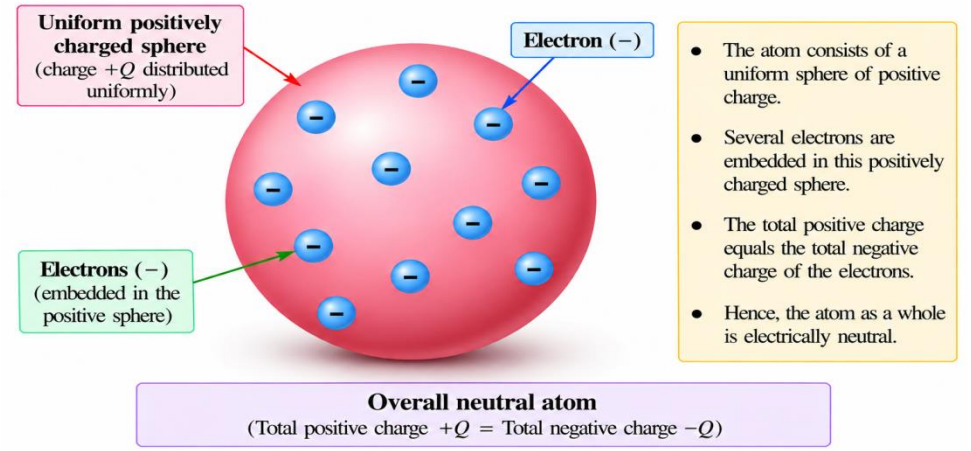
J. J. Thomson proposed a model of the atom after the discovery of the electron.
According to Thomson:

- An atom is approximately spherical.
- Positive charge is distributed uniformly throughout the atom.

- Electrons are embedded within this positively charged sphere.
- The total positive charge equals the total negative charge.
- Therefore, the atom as a whole is electrically neutral.

This model is often called the: **"Plum Pudding Model"** or **"Watermelon Model"** in simplified descriptions.

Figure 12.2 : Thomson's model of atom



1.4 Limitations of Thomson's Model

Thomson's model could explain the overall electrical neutrality of the atom, but it could not explain the results of later scattering experiments.

The major difficulty was:

How is the positive charge actually distributed inside the atom?

Rutherford's alpha-particle scattering experiment provided important evidence about this question.

1.5 Rutherford's Alpha-Particle Scattering Experiment

Ernest Rutherford and his collaborators performed the famous **alpha-particle scattering experiment**.

The experiment is also called the: **Gold Foil Experiment** or **Geiger–Marsden Experiment**

Basic Idea:

A beam of fast-moving alpha particles was directed towards a very thin gold foil.

The scattered alpha particles were detected using a fluorescent screen.

1.6 Why Gold Foil Was Used

A very thin gold foil was used because gold can be beaten into extremely thin sheets.

The foil was only a few hundred nanometres thick in the classic experiment.

A thin foil was important because an alpha particle should have a reasonable chance of passing through the foil without undergoing multiple successive scattering events.

1.7 Alpha Particles

An alpha particle is essentially the nucleus of a helium atom.
It contains:

- 2 protons
- 2 neutrons
- No electrons

Therefore its charge is: q = +2e and its mass is approximately: $m_\alpha \approx 4u$
Alpha particles are relatively massive and positively charged.

1.8 Experimental Arrangement

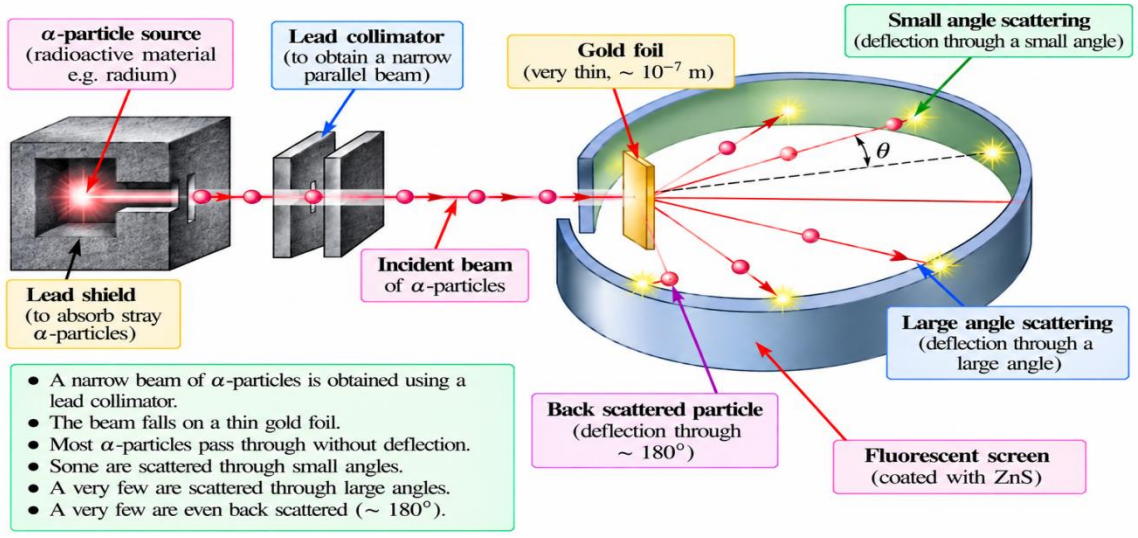
The essential arrangement consisted of:

1. Radioactive source of alpha particles
2. Lead block with a narrow slit
3. Thin gold foil
4. Fluorescent screen
5. Microscope/detector arrangement

The alpha particles were directed towards the gold foil.
When an alpha particle struck the foil, it could:

- Continue almost straight
- Be deflected through a small angle
- Occasionally be deflected through a large angle
- In a very small number of cases, be scattered backwards

Figure 12.3 : Rutherford's alpha- particle scattering experiment



1.9 Observations of Rutherford's Experiment

Rutherford's experiment produced three major observations.

Observation 1: Most Alpha Particles Passed Straight Through

A very large majority of alpha particles passed through the gold foil without significant deflection.

Conclusion
Most of the atom is **empty space**.

Observation 2: Some Alpha Particles Were Deflected Through Small Angles
A small fraction of alpha particles were deflected through small angles.

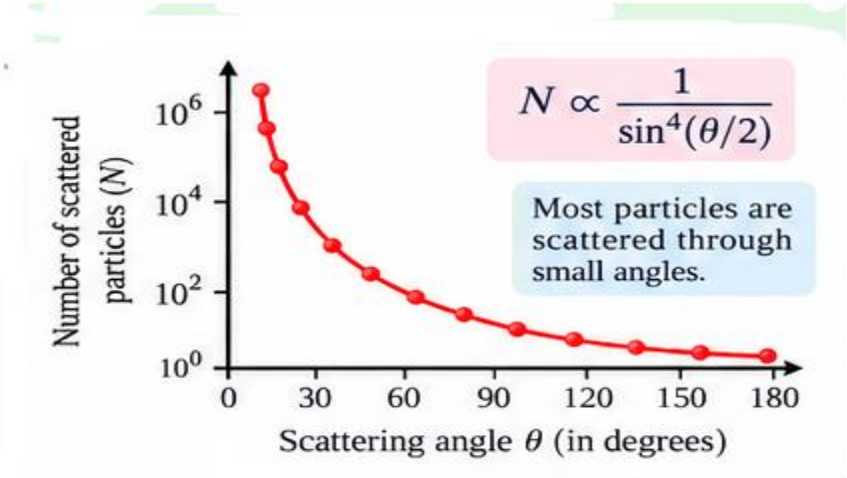
Conclusion
There must be a concentrated positive charge inside the atom that can repel the positively charged alpha particles.

Observation 3: Very Few Alpha Particles Were Deflected Through Large Angles
A very small fraction of alpha particles underwent large-angle scattering. A very tiny fraction were scattered backward. This was the most surprising observation.

Conclusion
Almost all the positive charge and nearly all the mass of the atom are concentrated in a very small central region called the nucleus.

Graph between scattering angle and number of scattered particles:

Figure 12.3-1: Scattering Curve



1.10 Rutherford's Nuclear Model of Atom
Based on the scattering experiment, Rutherford proposed the nuclear model.
According to this model:

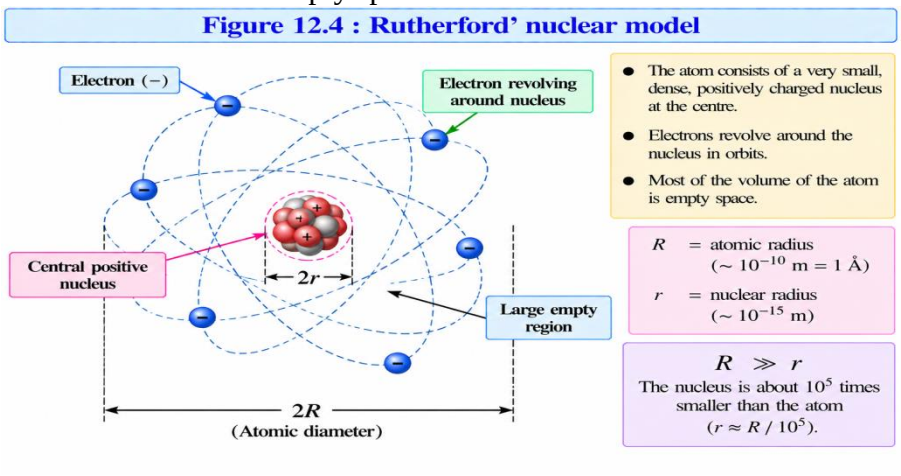
1. Central Nucleus
The atom contains a very small, dense, positively charged nucleus at its centre.

2. Positive Charge
Almost all the positive charge of the atom is concentrated in the nucleus.

3. Mass
Almost the entire mass of the atom is concentrated in the nucleus.

4. Electrons
Electrons revolve around the nucleus.

5. Empty Space
Most of the volume of the atom is empty space.



1.11 Size of Atom and Nucleus
Typical atomic radius: Ratom ≈ 10^{-10} m
Typical nuclear radius: Rnucleus ≈ 10^{-15} m
Therefore: Ratom / Rnucleus ≈ 10^5
So the atomic radius is about **100,000 times larger** than the nuclear radius.

Important Concept
The nucleus occupies an extremely small fraction of the total volume of the atom.
This explains why most alpha particles passed through the foil without significant deflection.

1.12 Why Alpha Particles Are Deflected
Both the alpha particle and nucleus are positively charged. Therefore, there is electrostatic repulsion between them.
According to Coulomb's law:

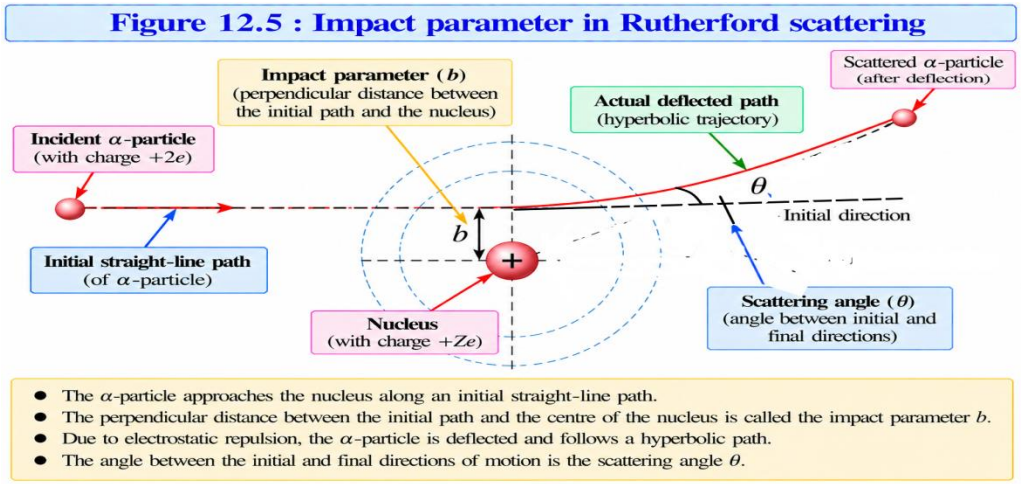
$$F = (1/4\pi\varepsilon_0) \times (q_1q_2/r^2)$$

For an alpha particle: $q_1 = +2e$
For a nucleus of atomic number Z: $q_2 = +Ze$
Therefore:

$$F = (1/4\pi\varepsilon_0) \times (2Ze^2/r^2)$$

Thus: $F \propto 1/r^2$
As the alpha particle comes closer to the nucleus, the repulsive force becomes very large.

1.13 Impact Parameter
The **impact parameter**, usually denoted by b, is the perpendicular distance between the initial direction of the alpha particle and the centre of the nucleus, if the particle were to continue undeflected.



Important Relationship

Smaller impact parameter: b ↓ → alpha particle passes closer to nucleus, → electrostatic repulsion increases; → scattering angle increases.
Therefore:
Smaller b → larger scattering angle

1.14 Closest Approach

Consider a head-on collision between an alpha particle and a nucleus. The alpha particle initially has kinetic energy. As it approaches the positively charged nucleus, its kinetic energy is converted into electrostatic potential energy.
At the point of closest approach:
Kinetic energy becomes momentarily zero for the ideal head-on case.
Therefore:
Initial kinetic energy = electrostatic potential energy at closest approach
For an alpha particle of charge +2e approaching a nucleus of charge +Ze:

$$K = (1/4\pi\varepsilon_0) \times (2Ze^2/r_{min})$$

Therefore:

$$r_{min} = (1/4\pi\varepsilon_0) \times (2Ze^2/K)$$

This expression applies to a head-on approach of an alpha particle towards a stationary nucleus, assuming electrostatic interaction and neglecting nuclear forces.
This is an important competitive-exam relation.

1.15 Derivation of Distance of Closest Approach

Suppose an alpha particle has initial kinetic energy K.
At a very large distance: Potential energy ≈ 0
Therefore: Total energy = K
At closest approach: Kinetic energy = 0
Potential energy: $U = (1/4\pi\varepsilon_0) \times (2Ze^2/r_{min})$
By conservation of energy: $K = (1/4\pi\varepsilon_0) \times (2Ze^2/r_{min})$
Therefore:

$$\boxed{r_{min} = \frac{1}{4\pi\epsilon_0}\left(\frac{2Ze^2}{K}\right)}$$

1.16 Special Case: Alpha Particle Incident on Gold

For gold: Z = 79
Therefore: $q_{nucleus}$ = +79e
Alpha particle charge: q_α = +2e
Electrostatic potential energy:

$$U = (1/4\pi\varepsilon_0) \times (158e^2/r)$$

For a head-on collision:

$$K = (1/4\pi\varepsilon_0) \times (158e^2/r_{min})$$

Hence:

$$r_{min} = (1/4\pi\varepsilon_0) \times (158e^2/K)$$

This type of numerical problem is useful for **NEET/JEE preparation**.

1.17 Rutherford Scattering Formula

For Rutherford scattering, the impact parameter b and scattering angle θ are related by:

$$\boxed{b = \frac{1}{4\pi\epsilon_0}\left(\frac{Zze^2}{2K}cot\frac{\theta}{2}\right)}$$

Where:
Z = atomic number of target nucleus
z = charge number of incident particle
For an alpha particle: z = 2
Therefore: $b = (1/4\pi\varepsilon_0) \times (2Ze^2/2K)\ \cot(\theta/2)$
Hence:

$$\boxed{b = \frac{1}{4\pi\epsilon_0}\left[\frac{Ze^2 cot(\theta/2)}{K}\right]}$$

Important
This formula is mainly useful for **JEE-level competitive problems**.

1.18 Qualitative Meaning of Rutherford Scattering Formula

From: $b \propto cot(\theta/2)$
We can understand:
If b increases: θ decreases.
If b decreases: θ increases.
Therefore:
Close encounter → large deflection
Distant encounter → small deflection

1.19 Rutherford Scattering Cross-Section — Competitive Extension

The Rutherford scattering law gives the angular dependence of scattered alpha particles.
The differential scattering cross-section is proportional to: $d\sigma/d\Omega \propto 1/\sin^4(\theta/2)$
More completely: $d\sigma/d\Omega = [(1/4\pi\varepsilon_0)(Zze^2/4K)]^2 \times 1/\sin^4(\theta/2)$
This is a **JEE/advanced-level extension** and is not required for basic Board-level understanding.

Important Trend

Large scattering angles are much less probable than small scattering angles.

1.20 Successes of Rutherford's Model

Rutherford's model successfully explained:

1. Existence of a small nucleus.
2. Concentration of positive charge in nucleus.
3. Concentration of most atomic mass in nucleus.
4. Large amount of empty space in an atom.
5. Alpha-particle scattering observations.

1.21 Limitations of Rutherford's Model

Although Rutherford's model was a major improvement, it had a serious problem.
According to classical electromagnetic theory, a charged particle moving in an orbit with centripetal acceleration should continuously radiate electromagnetic energy.
An electron revolving around the nucleus is an accelerating charge.
Therefore:
Electron → loses energy continuously
↓
Orbital radius decreases
↓
Electron spirals inward
↓
Electron should eventually fall into the nucleus.
This would make the atom unstable.
But ordinary atoms are stable.
Therefore, Rutherford's model could not explain: **The stability of atoms**

1.22 Second Major Problem: Atomic Spectra

Rutherford's model also could not explain the discrete line spectra observed from atoms.
Experimentally, hydrogen does not emit all possible wavelengths continuously.
Instead, it produces specific spectral lines.

1.23 Continuous Spectrum vs Line Spectrum

Continuous Spectrum

All wavelengths within a certain range are present continuously.
Example: White-light spectrum

Line Spectrum
Only certain discrete wavelengths are present.
Hydrogen produces characteristic spectral lines.

Important Concept
The line spectrum indicates that atomic energy states are not arbitrary; they are associated with specific allowed energies.
This idea becomes central in **Bohr's model**, which we will study in Part 2.

1.24 Solved Numerical

Example 1 – Closest Approach

Question
An alpha particle of kinetic energy 5 MeV approaches a gold nucleus (Z = 79) head-on. Calculate the distance of closest approach.

Given: K = 5 MeV; Z = 79
For alpha particle: z = 2
Using: $r_{min}=1/(4\pi\epsilon_0)\ ((2Ze^2)/K)$
Using the convenient nuclear-physics relation: $e^2/(4\pi\varepsilon_0) \approx 1.44$ MeV·fm
Therefore: $r_{min} = (1.44 \times 2 \times 79)/5$ fm, Or $r_{min} \approx 45.5$ fm
Since: 1 fm = 10^{-15} m
Therefore: $r_{min} \approx 4.55 \times 10^{-14}$ m
Answer: $r_{min} \approx 4.55 \times 10^{-14}$ m

Example 2 – Atomic and Nuclear Size

Question
The atomic radius is approximately 10^{-10} m and nuclear radius is approximately 10^{-15} m. Find the ratio of their radii.

Solution
We know that Ratom/Rnucleus = $10^{-10}/10^{-15} = 10^5$

Answer: Ratom: Rnucleus = 10^5: 1
The atomic radius is approximately **100,000 times** the nuclear radius.

Part 1 Memory Line
"Rutherford discovered the nuclear structure of the atom; Bohr later explained atomic stability and line spectra."

PART 2 – BOHR'S ATOMIC MODEL AND HYDROGEN ATOM

2.1 Introduction to Bohr's Model

Rutherford's nuclear model established that:

- The atom has a small, positively charged nucleus.
- Electrons revolve around the nucleus.
- Most of the atom is empty space.

However, Rutherford's model could not explain:

1. Why atoms are stable.
2. Why electrons do not continuously lose energy.
3. Why atoms produce discrete line spectra.

To overcome these difficulties, **Niels Bohr** proposed his atomic model in 1913.
Bohr combined:

- Rutherford's nuclear model
- Planck's quantum theory
- Experimental observations of atomic spectra

2.2 Bohr's Postulates

Bohr's model is based on three important postulates.
The Bohr model is applicable to hydrogen and hydrogen-like species containing only one electron.

Postulate 1: Stationary Orbits

Electrons revolve around the nucleus only in certain permitted circular orbits called:
Stationary or stable orbits
While an electron remains in a stationary orbit:
It does not radiate energy.
Therefore, the electron does not continuously lose energy while moving in an allowed orbit.

Important

These orbits are also called:
Allowed orbits or **Stationary states**

Postulate 2: Quantisation of Angular Momentum

Only those orbits are permitted for which the angular momentum of the electron is quantised.
The condition is: $mvr = nh/2\pi$
Where: n = 1, 2, 3, ...
The integer n is called the: **Principal quantum number**

Therefore:

For first orbit: n = 1
For second orbit: n = 2
For third orbit: n = 3 and so on.

Important Formula

$$\boxed{mvr = \frac{nh}{2\pi}}$$

This is one of the most important formulas of the chapter.

2.3 Postulate 3: Emission or Absorption of Radiation

An electron can emit or absorb radiation only when it moves from one stationary orbit to another.

If the electron moves from higher energy E_2 to lower energy E_1: $E_2 > E_1$ then a photon is emitted.

Photon energy: $hf = E_2 - E_1$

Therefore:

$$\boxed{hf = E_{high} - E_{low}}$$

If the electron absorbs energy and moves from lower energy E_1 to higher energy E_2: $hf = E_2 - E_1$

Thus:

Absorption → electron moves to higher energy level

Emission → electron moves to lower energy level

Figure 12.5-1: Bohr's Frequency Condition

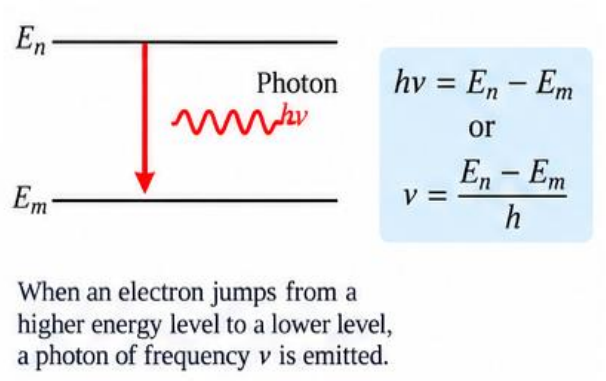


Figure 12.5-2: Photon Excitation and Emission

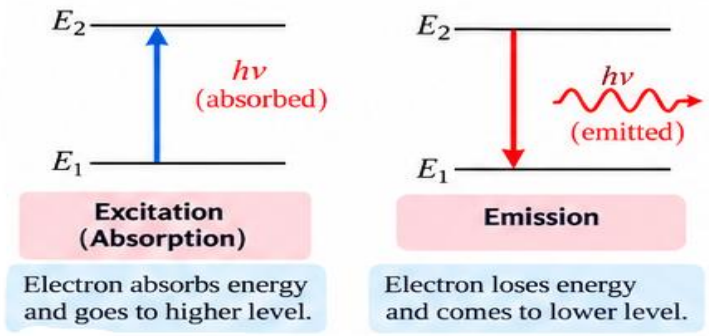
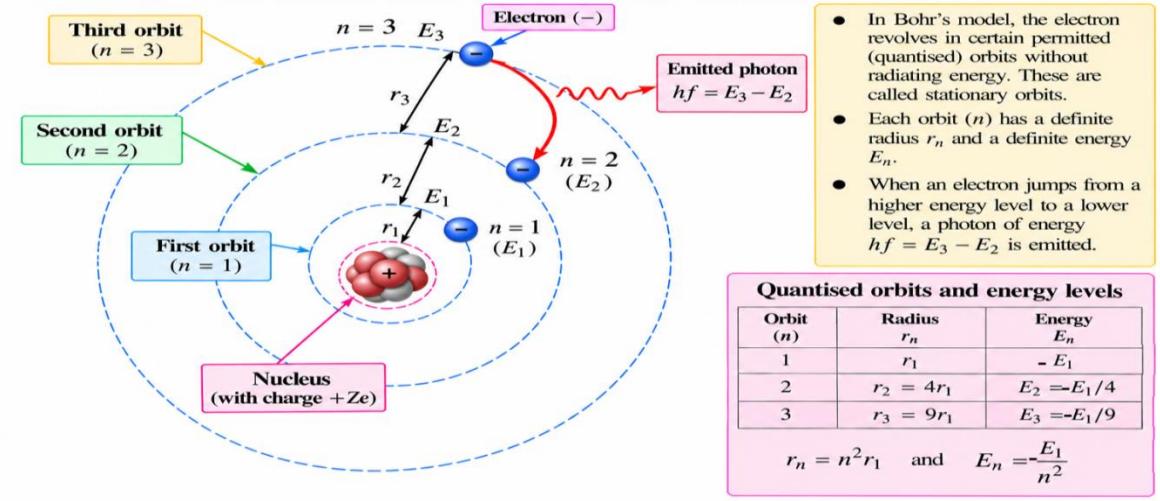


Figure 12.6 : Bohr's model showing quantised stationary orbits



- In Bohr's model, the electron revolves in certain permitted (quantised) orbits without radiating energy. These are called stationary orbits.
- Each orbit (n) has a definite radius r_n and a definite energy E_n.
- When an electron jumps from a higher energy level to a lower level, a photon of energy $hf = E_3 - E_2$ is emitted.

Quantised orbits and energy levels

Orbit (n)	Radius r_n	Energy E_n
1	r_1	$-E_1$
2	$r_2 = 4r_1$	$E_2 = -E_1/4$
3	$r_3 = 9r_1$	$E_3 = -E_1/9$

$r_n = n^2 r_1$ and $E_n = -\frac{E_1}{n^2}$

2.4 Force Acting on the Electron

Consider an electron of mass m and charge −e revolving around a nucleus of charge +Ze.
The electrostatic attraction provides the centripetal force.
Electrostatic force: $F_e = (1/4\pi\varepsilon_0) \times (Ze^2/r^2)$
Centripetal force: $F_c = mv^2/r$
For circular motion: $F_e = F_c$
Therefore: $(1/4\pi\varepsilon_0)\,(Ze^2/r^2) = mv^2/r$
Cancel r: we get: $(1/4\pi\varepsilon_0)(Ze^2/r) = mv^2$
Hence:

$$\boxed{mv^2 = \frac{Ze^2}{4\pi\epsilon_0 r}}$$

This is the basic dynamical equation of the Bohr model.

2.5 Derivation of Radius of the nth Orbit

We have: $mvr = nh/2\pi$
Therefore: $v = nh/(2\pi mr)$
From the force equation: $mv^2 = Ze^2/(4\pi\varepsilon_0 r)$
Substitute the value of v: $[nh/(2\pi mr)]^2 = Ze^2/(4\pi\varepsilon_0 r)$
After simplification: $r_n = n^2h^2\varepsilon_0/(\pi mZe^2)$
Therefore:

$$\boxed{\boldsymbol{r_n = \frac{n^2 a_0}{Z}}}$$

Where: $a_0 = h^2\varepsilon_0/(\pi me^2)$ is called the: **Bohr radius**

2.6 Bohr Radius

For hydrogen: Z = 1
For the first orbit: n = 1
Therefore: $r_1 = a_0$
The Bohr radius is approximately: $\mathbf{a_0 = 5.29 \times 10^{-11}\ m}$ or: $a_0 = 0.529$ Å

General Result: For a hydrogen-like atom: $\mathbf{r_n = a_0\, n^2/Z}$

2.7 Important Radius Relationships

For the same hydrogen-like atom: $r_n \propto n^2$
Therefore: $r_1 : r_2 : r_3 : r_4 = 1 : 4 : 9 : 16$

Example:
For hydrogen: $r_2 = 4r_1$; $r_3 = 9r_1$; $r_4 = 16r_1$

Effect of Nuclear Charge

For the same orbit number n: $r_n \propto \frac{1}{Z}$
Therefore: Higher Z → smaller orbit radius

2.8 Velocity of Electron in the nth Orbit

From: $mv^2 = Ze^2/(4\pi\varepsilon_0 r)$ and: $r_n = n^2h^2\varepsilon_0/(\pi mZe^2)$
Substitution gives: $v_n = Ze^2/(2\varepsilon_0 nh)$
Therefore:

$$\boxed{v_n = \frac{Ze^2}{2\epsilon_0 nh}}$$

Using the fine-structure constant α:

$$\boxed{v_n = \frac{Z\alpha c}{n}}$$

For hydrogen: Z = 1
Therefore: $v_n = \alpha c/n$
Since: $\alpha \approx 1/137$ and: $c = 3 \times 10^8$ m/s
For first orbit: $v_1 \approx 2.18 \times 10^6$ m/s

2.9 Velocity Relationship

For a hydrogen-like atom: $v_n \propto Z/n$
Therefore:

- Z increases → electron velocity increases.
- n increases → electron velocity decreases.

For hydrogen: $v_1 : v_2 : v_3 = 1 : 1/2 : 1/3$

2.10 Energy of Electron in nth Orbit

The total energy of the electron is: $E = K + U$
Where: K = kinetic energy, U = electrostatic potential energy

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Kinetic Energy

From: $mv^2 = Ze^2/(4\pi\varepsilon_0 r)$
Therefore: $K = \frac{1}{2}mv^2$
So:

$$\boxed{K = \frac{Ze^2}{8\pi\epsilon_0 r}}$$

Potential Energy

For an electron and nucleus:

$$\boxed{U = -\frac{Ze^2}{4\pi\epsilon_0 r}}$$

The negative sign indicates that the electron–nucleus system is bound.

Total Energy

Total energy: $E = K + U$
Therefore: $E = Ze^2/(8\pi\varepsilon_0 r) - Ze^2/(4\pi\varepsilon_0 r)$

Hence:

$$\boxed{E = -\frac{Ze^2}{8\pi\epsilon_0 r}}$$

Hence, we get

$$\boxed{K_n = -E_n}$$

$$\boxed{U_n = 2E_n}$$

$$\boxed{U_n = -2K_n}$$

Substituting the radius of nth orbit: we get

$$\boxed{E_n = -\frac{mZ^2e^4}{8{\epsilon_0}^2n^2h^2}}$$

This is the general energy formula for a hydrogen-like atom.

2.11 Energy of Hydrogen Atom

For hydrogen: Z = 1

$$\boxed{E_n = -\frac{me^4}{8{\epsilon_0}^2n^2h^2}}$$

Or,

$$\boxed{E_n(in\ eV) = -\frac{13.6}{n^2}}$$

Here,
By putting the values of

- $m = m_e = 9.1 \times 10^{-31}\ kg$
- $e = 1.6 \times 10^{-19} C$
- $\epsilon_0 = 8.85 \times 10^{-12}\ \ C^2N^{-1}m^2$
- $h = 6.6 \times 10^{-34}\ Js$

We get,

$$\frac{me^4}{1.6 \times 10^{-19} \times 8{\epsilon_0}^2h^2} = -13.6\ eV$$

This is equal to:

$$Rhc = 13.6\ eV$$

Where: $Rhc = Rydberg\ energy$
This is one of the most important formulas in Chapter 12.

Energy of First Orbit: For: n = 1, $E_1 = -13.6$ eV

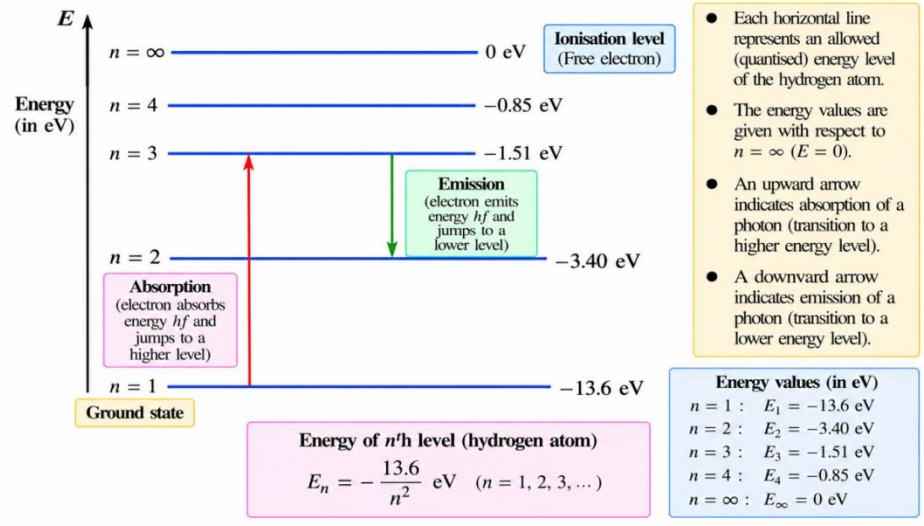
Energy of Second Orbit: For: n = 2, $E_2 = -13.6/4$ Or $E_2 = -3.4$ eV

Energy of Third Orbit: For: n = 3, $E_3 = -13.6/9$ Or $E_3 \approx -1.51$ eV

2.12 Energy-Level Diagram

↑

Figure 12.7 : Energy- level diagram of hydrogen atom



2.13 Why Is the Energy Negative?

The zero of energy is taken when the electron is infinitely far away from the nucleus.
At: $r \to \infty$, $E = 0$
Inside the atom: $E < 0$
Therefore, the electron is in a bound state.

Important

Negative energy means: **The electron is bound to the nucleus.**
It does not mean that the electron possesses "negative kinetic energy."

2.14 Ground State

The lowest energy state is: n = 1
It is called the: **Ground state**
For hydrogen: $E_1 = -13.6$ eV
The electron is most strongly bound in this state.

2.15 Excited States

When the electron absorbs energy, it can move to a higher allowed energy level.
For example: $n = 1 \to n = 2$ or: $n = 1 \to n = 3$
Such higher-energy states are called: **Excited states**

2.16 Excitation Energy

Energy required to take an electron from level n_1 to level n_2, where: $n_2 > n_1$ is:

$$\Delta E = En_2 - En_1$$

For hydrogen: $\Delta E = 13.6\ [1/n_1^2 - 1/n_2^2]$ eV

This is an important numerical formula.

2.17 Example: Excitation from n = 1 to n = 2

For hydrogen: $E_1 = -13.6$ eV; $E_2 = -3.4$ eV
Therefore: $\Delta E = E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV
Answer: 10.2 eV energy is required.

2.18 Ionisation Energy

Ionisation means removing the electron completely from the atom.
For hydrogen in ground state: $E_1 = -13.6$ eV
At infinity: $E\infty = 0$
Therefore: $\Delta E = 0 - (-13.6)$
Ionisation energy = 13.6 eV
Thus: **13.6 eV is required to ionise ground-state hydrogen.**

2.19 Ionisation Energy from nth Orbit

For hydrogen: $En = -13.6/n^2$ eV
Energy required to take the electron from nth orbit to infinity:
Eion = 13.6/n² eV
Therefore: For n = 1: E ion = 13.6 eV
For n = 2: E ion = 3.4 eV
For n = 3: Eion ≈ 1.51 eV
Important: Ionisation energy decreases as n increases.

2.20 Photon Emission during Transition

Suppose an electron moves from: $n_2 \rightarrow n_1$; here: $n_2 > n_1$
Then: E photon = $En_2 - En_1$
For hydrogen: $\mathbf{hf = 13.6\ [1/n_1^2 - 1/n_2^2]\ eV}$
Since: $E = hc/\lambda$
We get: $\mathbf{hc/\lambda = 13.6\ [1/n_1^2 - 1/n_2^2]\ eV}$

2.21 Rydberg Formula

The wavelength of spectral radiation emitted by hydrogen is given by:

$$\boxed{\frac{1}{\lambda} = R_H\left[\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right]}$$

Where: $n_2 > n_1$ and: R = Rydberg constant
Approximately: $R_H = 1.097 \times 10^7\ m^{-1}$
This formula is one of the most important formulas for hydrogen spectrum.

2.22 Spectral Series

Different values of n_1 produce different spectral series.

Series	Final level
Lyman	$n_1 = 1$
Balmer	$n_1 = 2$
Paschen	$n_1 = 3$
Brackett	$n_1 = 4$
Pfund	$n_1 = 5$

The initial level n_2 is always greater than n_1.

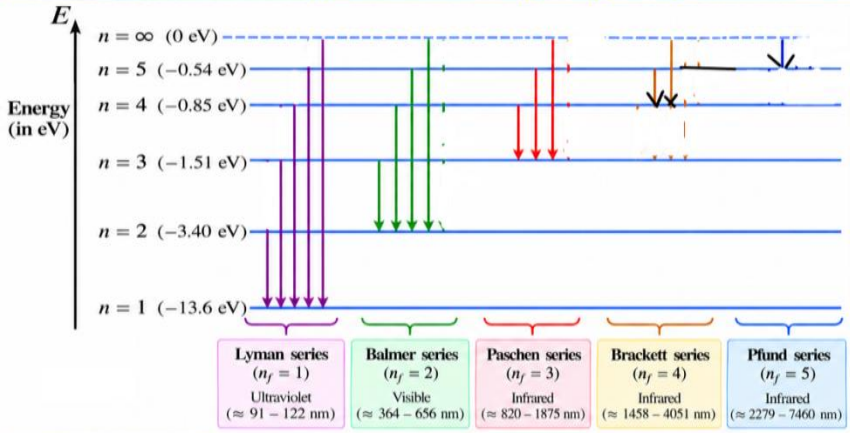
2.23 Lyman Series

For Lyman series: $n_1 = 1$
Therefore: $1/\lambda = R\,[1 - 1/n_2^2]$
Where: $n_2 = 2, 3, 4...$
The Lyman series lies mainly in the **ultraviolet region**.

2.24 Balmer Series

For Balmer series: $n_1 = 2$
Therefore: $1/\lambda = R\,[1/4 - 1/n_2^2]$
Where: $n_2 = 3, 4, 5...$
The Balmer series lies in the **visible region** for its prominent lines.

Figure 12.8 : Hydrogen spectral series



- Each horizontal line represents a quantised energy level (n) of hydrogen atom.
- Transitions from higher level (n_i) to a fixed lower level (n_f) produce a spectral series.
- The wavelength of emitted photon is given by Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$R = 1.097 \times 10^7\ \text{m}^{-1}$$

- Different series correspond to different lower levels (n_f).

Series name	Lower level (n_f)	Transitions ($n_i \rightarrow n_f$)	Spectral region	Wavelength range (approx.)
Lyman series	1	$n = 2, 3, 4, \cdots \rightarrow 1$	Ultraviolet (UV)	91 – 122 nm
Balmer series	2	$n = 3, 4, 5, \ldots \rightarrow 2$	Visible	364 – 656 nm
Paschen series	3	$n = 4, 5, 6, \ldots \rightarrow 3$	Infrared (IR)	820 – 1875 nm
Brackett series	4	$n = 5, 6, 7, \ldots \rightarrow 4$	Infrared (IR)	1458 – 4051 nm
Pfund series	5	$n = 6, 7, 8, \ldots \rightarrow 5$	Infrared (IR)	2279 – 7460 nm

---

2.25 Spectral Series Memory Trick

Remember the final orbit: **Lyman → 1; Balmer → 2; Paschen → 3; Brackett → 4; Pfund → 5**

A useful sequence: **1 – 2 – 3 – 4 – 5**

2.26 Solved Numerical

Example 1 – Energy of an Electron

Question

Find the energy of an electron in the third orbit of hydrogen.

Solution

Using: $E_n = -13.6/n^2$ eV

For: n = 3, $E_3 = -13.6/9 \approx -1.51$ eV

Answer: $E_3 \approx -1.51$ eV

Solved Example 2 – Photon Energy

An electron in hydrogen falls from n = 3 to n = 2. Find the energy of the emitted photon.

Solution

$E_3 = -1.51$ eV and $E_2 = -3.40$ eV

Therefore: $E_{photon} = E_3 - E_2 = -1.51 - (-3.40) = 1.89$ eV

Answer: E photon ≈ 1.89 eV

Solved Example 3 – Radius of Second Orbit

Find the radius of the second orbit of hydrogen.

Solution

For hydrogen: $r_n = a_0 n^2$

Given: $a_0 = 5.29 \times 10^{-11}$ m, n = 2

Therefore: $r_2 = 5.29 \times 10^{-11} \times 4 = 2.116 \times 10^{-10}$ m

Answer: $r_2 \approx 2.12 \times 10^{-10}$ m

Part 2 Memory Line

"Bohr quantised the electron's angular momentum and energy, thereby explaining the stability of the hydrogen atom and its line spectrum."

PART 3 – HYDROGEN SPECTRUM AND SPECTRAL SERIES

3.1 Introduction to Atomic Spectra

When atoms interact with energy, they can emit or absorb electromagnetic radiation.
The radiation is not emitted at all possible wavelengths. Instead, atoms generally produce radiation at **specific wavelengths**. The resulting pattern is called an: **Atomic spectrum**
The spectrum provides important information about the energy levels of an atom.

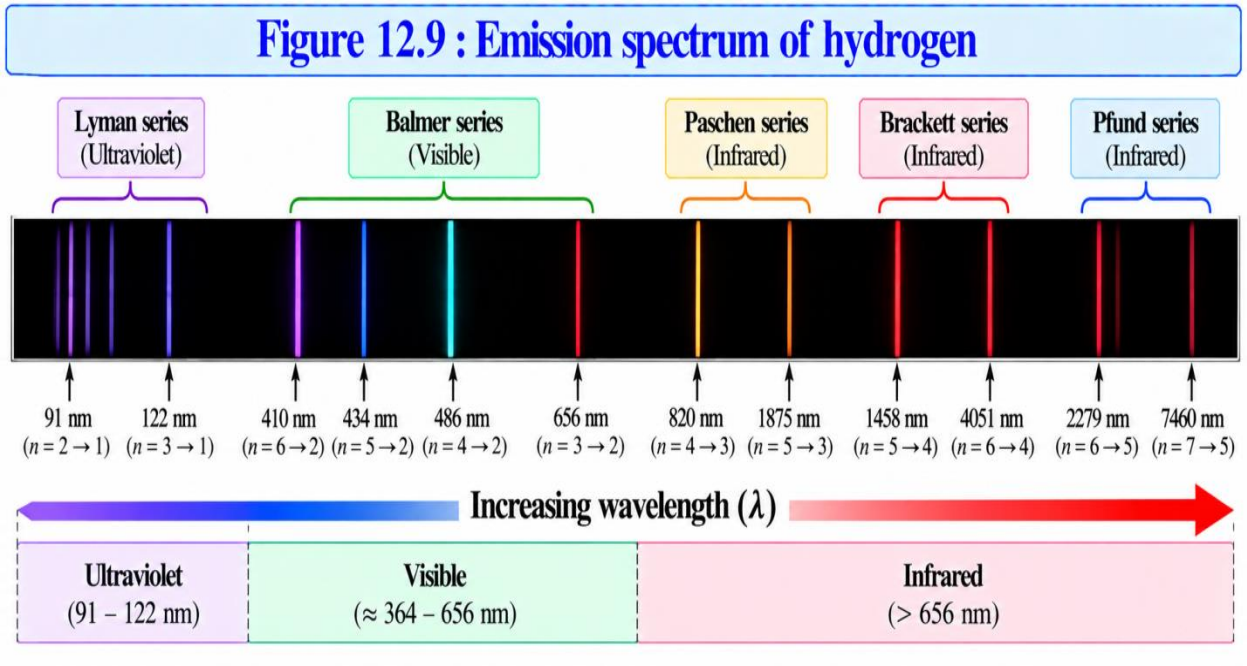
3.2 Types of Spectra

Atomic spectra can broadly be classified as:

1. **Emission spectrum**
2. **Absorption spectrum**

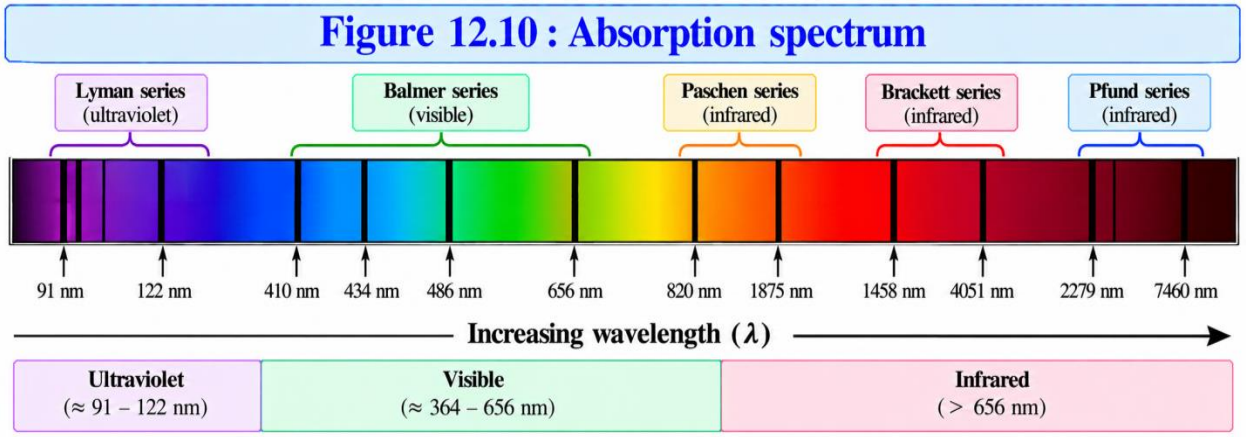
3.2.1 Emission Spectrum

When an excited atom emits radiation, bright spectral lines are observed against a dark background. This is called an: **Emission spectrum**



3.2.2 Absorption Spectrum

When continuous radiation passes through a cooler gas containing atoms, certain wavelengths may be absorbed. The resulting spectrum contains dark lines at specific wavelengths.
This is called an: **Absorption spectrum**



3.3 Continuous Spectrum vs Line Spectrum

Continuous Spectrum

A continuous spectrum contains all wavelengths within a given range without gaps.
Example: Spectrum of white light from a suitable continuous source.

Line Spectrum

A line spectrum consists of isolated lines corresponding to specific wavelengths.
Hydrogen produces a characteristic line spectrum.

Important Concept

The existence of discrete spectral lines indicates that atomic energy is quantised.

3.4 Hydrogen Atomic Spectrum

Hydrogen is the simplest atom and has only one electron. Its emission spectrum consists of several spectral series. The wavelengths of these spectral lines are described by the **Rydberg formula**.

For hydrogen: $1/\lambda = R\,[1/n_1^2 - 1/n_2^2]$
Where: $n_2 > n_1$ and: $R = 1.097 \times 10^7\ m^{-1}$

3.5 Derivation of Rydberg Formula from Bohr Model

This is an important derivation.
For hydrogen: $E_n = -13.6/n^2$ eV
Suppose an electron makes a transition: $n_2 \rightarrow n_1$
Where: $n_2 > n_1$
The energy of the emitted photon is: $E_{photon} = E_{n_2} - E_{n_1}$
Substituting: $E_{photon} = -13.6/n_2^2 - (-13.6/n_1^2)$
Therefore: $E_{photon} = 13.6\,[1/n_1^2 - 1/n_2^2]$ eV
But: $E_{photon} = hf$ and: $f = c/\lambda$
Therefore: $E_{photon} = hc/\lambda$
Hence: $hc/\lambda = 13.6\,[1/n_1^2 - 1/n_2^2]$ eV
After converting the energy constant into SI units:

$$\frac{1}{\lambda} = R_H\left[\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right]$$

This is the **Rydberg formula for hydrogen**.

Where: $R_H = 1.097 \times 10^7\ m^{-1}$

3.6 General Rydberg Formula

For a hydrogen-like species: $\mathbf{1/\lambda = R_Z\ [1/n_1^2 - 1/n_2^2]}$

Where: R_Z = R Z^2

Therefore: $\mathbf{1/\lambda = RZ^2\ [1/n_1^2 - 1/n_2^2]}$ for: $n_2 > n_1$

Important

This formula applies to **hydrogen-like species**, such as: H; He^+, Li^{2+} ; Be^{3+} provided the system contains only one electron.

3.7 Meaning of Hydrogen-Like Species

A hydrogen-like species has: **Only one electron**

Examples: H → 1 electron; He^+ → 1 electron; Li^{2+} → 1 electron; Be^{3+} → 1 electron

Important

The nuclear charge may be different, but there is only one electron.

3.8 Spectral Series

A spectral series is obtained when the electron transitions from various higher levels to the same lower energy level.

For example:

If all transitions terminate at: $n_1 = 1$; we obtain the: **Lyman series**

If all transitions terminate at: $n_1 = 2$; we obtain the: **Balmer series,** and so on.

3.9 Lyman Series

For Lyman series: $n_1 = 1$ and: $n_2 = 2, 3, 4...$

Therefore: $\mathbf{1/\lambda = R\ [1 - 1/n_2^2]}$

The Lyman series lies in the: **Ultraviolet region**

3.10 Balmer Series

For Balmer series: $n_1 = 2$ and: $n_2 = 3, 4, 5...$

Therefore: $\mathbf{1/\lambda = R\ [1/4 - 1/n_2^2]}$

The prominent Balmer lines occur in the: **Visible region**

Important Balmer Transitions

Hα: $n = 3 \rightarrow n = 2$

Hβ: $n = 4 \rightarrow n = 2$

Hγ: $n = 5 \rightarrow n = 2$

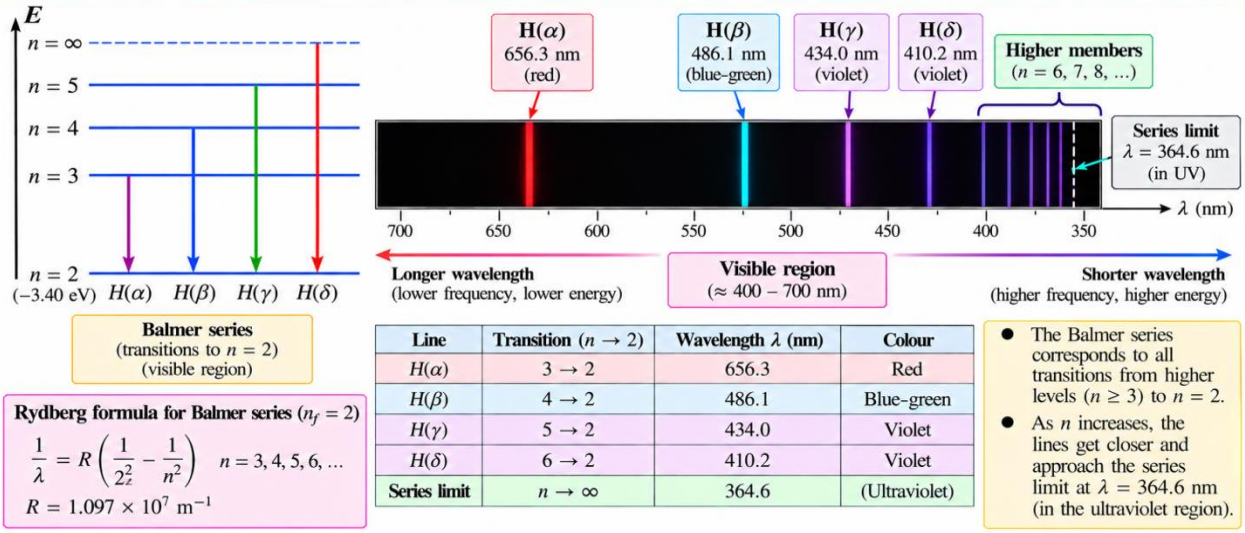
Hδ: $n = 6 \rightarrow n = 2$

3.11 Balmer Series and Visible Spectrum

The first few Balmer lines are visible because their wavelengths lie approximately within the visible range. The series eventually approaches a limiting wavelength as: $n_2 \rightarrow \infty$

This is called the: **Balmer series limit**

Figure 12.11 : Balmer series in the visible region



Balmer series (transitions to $n = 2$) (visible region)

Rydberg formula for Balmer series ($n_f = 2$)

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad n = 3, 4, 5, 6, \ldots$$

$$R = 1.097 \times 10^7 \text{ m}^{-1}$$

Line	Transition ($n \to 2$)	Wavelength λ (nm)	Colour
$H(\alpha)$	$3 \to 2$	656.3	Red
$H(\beta)$	$4 \to 2$	486.1	Blue-green
$H(\gamma)$	$5 \to 2$	434.0	Violet
$H(\delta)$	$6 \to 2$	410.2	Violet
Series limit	$n \to \infty$	364.6	(Ultraviolet)

- The Balmer series corresponds to all transitions from higher levels ($n \geq 3$) to $n = 2$.
- As n increases, the lines get closer and approach the series limit at $\lambda = 364.6$ nm (in the ultraviolet region).

3.12 Paschen Series

For Paschen series: $n_1 = 3$ and: $n_2 = 4, 5, 6, \ldots$

Therefore: $\mathbf{1/\lambda = R\,[1/9 - 1/n_2^2]}$

The Paschen series lies in the: **Infrared region**

3.13 Brackett Series

For Brackett series: $n_1 = 4$ and: $n_2 = 5, 6, 7\ldots$

Therefore: $\mathbf{1/\lambda = R\,[1/16 - 1/n_2^2]}$

The Brackett series lies in the: **Infrared region**

3.14 Pfund Series

For Pfund series: $n_1 = 5$ and: $n_2 = 6, 7, 8\ldots$

Therefore: $\mathbf{1/\lambda = R\,[1/25 - 1/n_2^2]}$

The Pfund series lies in the: **Infrared region**

3.15 Hydrogen Spectral Series – Master Table

Series	Final level n_1	Initial level n_2	Main region
Lyman	1	2, 3, 4, ...	Ultraviolet
Balmer	2	3, 4, 5, ...	Visible
Paschen	3	4, 5, 6, ...	Infrared
Brackett	4	5, 6, 7, ...	Infrared
Pfund	5	6, 7, 8, ...	Infrared

Memory Trick

Remember the final level: **Lyman → 1; Balmer → 2; Paschen → 3; Brackett → 4; Pfund → 5**

3.16 Frequency of Spectral Radiation

From: $E = hf$ and: $E = 13.6\ [1/n_1^2 - 1/n_2^2]$ eV

We obtain: $\mathbf{f = (13.6\ eV/h)\ [1/n_1^2 - 1/n_2^2]}$

Thus: Frequency increases when the energy difference between the two levels increases.

3.17 Wavelength and Frequency Relationship

For electromagnetic radiation: $c = f\lambda$

Therefore: $\boldsymbol{\lambda = c/f}$

Hence: **Higher frequency → shorter wavelength** and: **Lower frequency → longer wavelength**

3.18 Wave number

The reciprocal of wavelength is called: **Wavenumber**

It is represented by: $1/\lambda$

For hydrogen: $1/\lambda = R\ [1/n_1^2 - 1/n_2^2]$

SI Unit: m^{-1}

Wavenumber is particularly useful in spectroscopy.

3.19 Series Limit

For a particular spectral series, the shortest wavelength occurs when: $n_2 \to \infty$

For example, for the Lyman series: $n_1 = 1$ and: $n_2 \to \infty$

Therefore: $1/\lambda min = R$

Hence: $\boldsymbol{\lambda min = 1/R}$ approximately: $\lambda min \approx 91.2$ nm

---

3.20 Series Limit of Balmer Series

For Balmer series: $n_1 = 2$

At the series limit: $n_2 \to \infty$

Therefore: $1/\lambda min = R/4$

Hence: $\lambda min = 4/R$

Approximately: $\boldsymbol{\lambda min \approx 364.6\ nm}$

This is close to the ultraviolet boundary, which explains why the higher Balmer lines converge toward the UV side of the spectrum.

3.21 Longest Wavelength in a Series

The longest wavelength corresponds to the **smallest energy difference** in that series.

Therefore, it occurs for: $n_2 = n_1 + 1$

Example: Balmer Series

The first Balmer line is: $n = 3 \to n = 2$

This corresponds to the longest wavelength in the Balmer series.

3.22 Shortest Wavelength in a Series

The shortest wavelength corresponds to the largest energy difference within the series.

Therefore: $n_2 \to \infty$

This is the **series limit**.

Important Rule

Longest wavelength → first transition
Shortest wavelength → series limit

3.23 Number of Spectral Lines from a Given Excited State

Suppose an atom is excited to the nth energy level.
The maximum possible number of different transitions among the first n levels is:

$$N = n(n-1)/2$$

Example

If: n = 4 then: N = 4(3)/2 = 6
Thus, a maximum of **6 different spectral lines** can be produced if all possible transitions occur.

3.34 Example: Transitions from n = 4

Possible transitions are:
4 → 3
4 → 2
4 → 1
3 → 2
3 → 1
2 → 1
Total:
6 transitions

3.25 Number of Lines in a Particular Series

If an electron is initially excited to level n_2 and we are interested in a particular series ending at n_1, then possible transitions depend on the available higher levels.
For example, if the highest occupied level is: n = 5; the Balmer series can contain transitions:
5 → 2
4 → 2
3 → 2
Therefore: **3 Balmer lines** can be produced from these levels.

3.26 Solved Numerical

Example 1 – Balmer Transition

Question

Calculate the wavelength of radiation emitted when an electron in hydrogen transitions from n = 3 to n = 2.

Solution

Using: $1/\lambda = R_H [1/n_1^2 - 1/n_2^2]$
Here: $n_1 = 2$, $n_2 = 3$
Therefore: $1/\lambda = 1.097 \times 10^7 [1/4 - 1/9]$
Therefore: $1/\lambda \approx 1.097 \times 10^7 \times 5/36$
Hence: $\lambda \approx 6.56 \times 10^{-7}$ m

Therefore: $\lambda \approx 656$ nm

Answer: $\lambda \approx 656$ nm

This is the well-known **Hα line** of the Balmer series.

Solved Example 2 – Lyman Series

Question

Find the wavelength of radiation emitted when an electron falls from n = 2 to n = 1 in hydrogen.

Solution

Here: $n_1 = 1$; $n_2 = 2$

Using: $1/\lambda = R\,[1 - 1/4] = R \times 3/4$

Therefore: $\lambda \approx 121.6$ nm

Answer: $\lambda \approx 121.6$ nm

This radiation belongs to the **Lyman series**.

Solved Example 3 – Energy of Balmer Photon

For transition: $n = 4 \rightarrow n = 2$. Find the energy of the emitted photon.

Solution

Energy of nth level: $E_n = -13.6/n^2$ eV

For n = 4: $E_4 = -13.6/16 = -0.85$ eV

For n = 2: $E_2 = -3.4$ eV

Therefore: E photon = $E_4 - E_2 = -0.85 - (-3.4) = 2.55$ eV

Answer: Photon energy = 2.55 eV

Solved Example 4 – Number of Spectral Lines

Question

An electron is excited to the fourth energy level. What is the maximum number of different spectral lines that can be emitted?

Solution

Using: $N = n(n - 1)/2$

For: $n = 4$; $N = 4 \times 3/2 = 6$

Answer: Maximum number of spectral lines = 6

Solved Example 5 – Hydrogen-Like Ion

Question

Find the ground-state energy of He^+.

Solution

For hydrogen-like species: $E_n = -13.6Z^2/n^2$ eV

For He^+: Z = 2; n = 1

Therefore: $E_1 = -13.6 \times 2^2 = -54.4$ eV

Answer: Ground-state energy of $He^+ = -54.4$ eV

3.27 Quick Revision Table

Quantity	Formula
Photon energy	E = hf
Photon energy	E = hc/λ
Hydrogen energy	$E_n(eV) = -13.6/n^2$
Hydrogen-like energy	$E_n(\quad) = -13.6Z^2/n^2$
Transition energy	$\Delta E = E_{high} - E_{low}$
Rydberg formula	$1/\lambda = R_H [1/n_1^2 - 1/n_2^2]$
Frequency	f = c/λ
Wavenumber	1/λ
Maximum lines	n(n−1)/2

Part 3 Memory Line

"Atomic spectral lines are fingerprints of quantized energy levels."

PART 4 – BOHR MODEL: APPLICATIONS, HYDROGEN-LIKE ATOMS & NUMERICAL APPLICATIONS

4.1 Hydrogen-Like Atoms

A **hydrogen-like atom or ion** is a species containing only one electron.
Examples: H, He^{+}; Li^{2+} ; Be^{3+}
The nuclear charge may be different, but only one electron is present.
For a hydrogen-like species: nuclear charge = +Ze

4.2 Energy of Hydrogen-Like Species

The Bohr energy formula is: $E_n = -13.6\ Z^2/n^2$ eV
Where: Z = atomic number; n = principal quantum number

Important

For hydrogen: Z = 1; $E_n = -13.6/n^2$ eV
For He^{+}: Z = 2; $E_n = -54.4/n^2$ eV
For Li^{2+}: Z = 3; $E_n = -122.4/n^2$ eV

4.3 Ground-State Energy

For the ground state: n = 1
Therefore: $E_1 = -13.6Z^2$ eV

Examples

For H: $E_1 = -13.6$ eV
For He^{+}: $E_1 = -54.4$ eV
For Li^{2+}: $E_1 = -122.4$ eV

4.4 Ionisation Energy of Hydrogen-Like Species

Ionisation means taking the electron from its bound state to: $n = \infty$
At infinity: $E_\infty = 0$
Therefore, ionisation energy from nth orbit is: $E_{ion} = 0 - E_n$
Since: $E_n = -13.6Z^2/n^2$
We obtain: **$E_{ion} = 13.6Z^2/n^2$ eV**

Ground-State Ionisation Energy

For n = 1: **$E_{ion} = 13.6Z^2$ eV**

4.5 Ionisation Energy Examples

Hydrogen: Z = 1; Eion = 13.6 eV
He^{+}: Z = 2; Eion = 13.6 × 4 = 54.4 eV
Li^{2+}: Z = 3; Eion = 13.6 × 9 = 122.4 eV
Important Relationship: $E_{ion} \propto Z^2$
Therefore, higher nuclear charge produces stronger binding.

4.6 Excitation Energy

Excitation means transferring an electron from one allowed energy level to another higher allowed level.

For transition: $n_1 \rightarrow n_2$ where: $n_2 > n_1$

Energy required: $\Delta E = En_2 - En_1$

For a hydrogen-like species: $\mathbf{\Delta E = 13.6Z^2 \left[1/n_1^2 - 1/n_2^2\right] eV}$

4.7 Excitation vs Ionisation

Excitation	Ionisation
Electron moves to another allowed bound state	Electron is removed completely
Final n is finite	Final $n = \infty$
Electron remains bound	Electron becomes free
Excitation requires less energy than ionisation from the same initial state. The limiting case corresponds to ionisation.	Requires ionisation energy
Example: $n = 1 \rightarrow n = 2$	Example: $n = 1 \rightarrow n = \infty$

Important

Ionisation is the limiting case of excitation where: $n_2 \rightarrow \infty$

4.8 Energy Required for First Excitation

For hydrogen:

Ground state: $E_1 = -13.6$ eV

First excited state: $E_2 = -3.4$ eV

Therefore: $\Delta E = E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV

Thus: **First excitation energy of hydrogen = 10.2 eV**

4.9 Excitation Energy for Hydrogen-Like Species

For transition: $1 \rightarrow 2$

Energy required: $\Delta E = 13.6Z^2 [1 - 1/4]$

Therefore: $\mathbf{\Delta E = 10.2Z^2\ eV}$

Examples

Hydrogen: 10.2 eV

He^+: 40.8 eV

Li^{2+}: 91.8 eV

4.10 Photon Absorption

When an atom absorbs a photon, the photon energy must exactly match the difference between two allowed energy levels.

Therefore: $\mathbf{hf = E_2 - E_1}$

For absorption: $E_2 > E_1$

Hence: $hf > 0$

Important Concept
An atom cannot absorb an arbitrary photon energy and still remain in a stationary state.
The photon energy must correspond to an allowed transition.

4.11 Photon Emission

When an electron moves from higher energy level E_{high} to lower energy level E_{low}, a photon is emitted. Its energy is:

$$E_{photon} = E_{high} - E_{low}$$

Or,

$$hf = E_{high} - E_{low}$$

Or,

$$\lambda = \frac{hc}{E_{high} - E_{low}}$$

4.12 Transition Energy and Wavelength

For hydrogen: $E_n = -13.6/n^2$ eV
Therefore, for transition: $n_2 \rightarrow n_1$: $E_{photon} = 13.6\,[1/n_1^2 - 1/n_2^2]$ eV
Since: $E = hc/\lambda$
We obtain: $$\lambda = hc / \Delta E$$
This is often the fastest method for numerical problems.

4.13 Shortcut for Photon Wavelength

When transition energy is given in eV: **λ(nm) = 1240/E(eV)**

Example
If: $\Delta E = 2.48$ eV
Then: $\lambda = 1240/2.48 = 500$ nm

4.14 Hydrogen-Like Spectral Formula

For a hydrogen-like species: $$1/\lambda = R_H Z^2 [1/n_1^2 - 1/n_2^2]$$
Where: $n_2 > n_1$
This shows that: $1/\lambda \propto Z^2$ for the same transition.
Therefore: Z ↑ → wavelength ↓

4.15 Comparing Hydrogen and He^+

Suppose the same transition occurs: $n_2 \rightarrow n_1$ in hydrogen and He^+.
For hydrogen: Z = 1
For He^+: Z = 2
Therefore: $\lambda \propto 1/Z^2$
Hence: $\lambda(He^+)/\lambda(H) = 1/4$
Thus:
He^+ emits radiation of one-fourth the wavelength of hydrogen for the same transition.
Equivalently: $f(He^+) = 4f(H)$

4.16 Radius of Hydrogen-Like Orbit

Formula:

$$r_n = a_0 n^2 / Z$$

Therefore:

$$r_n \propto n^2$$

and:

$$r_n \propto 1/Z$$

For Same n: If Z increases: r decreases.
For Same Z: If n increases: r increases as n^2.

4.17 Important Comparison Table

For the same transition:

Quantity	Dependence on Z
Energy difference	$\propto Z^2$
Frequency	$\propto Z^2$
Wavenumber	$\propto Z^2$
Wavelength	$\propto 1/Z^2$
Ground-state energy magnitude	$\propto Z^2$
Ground-state radius	$\propto 1/Z$

These relations apply to the same transition and within the ideal Bohr model, neglecting reduced-mass and other small corrections.

4.18 Example – Radius Comparison

Compare the ground-state radii of H and He^+.
For H: Z = 1, $r(H) = a_0$
For He^+: Z = 2; $r(He^+) = a_0/2$
Therefore: **$r(H) : r(He^+) = 2 : 1$**

4.19 Velocity of Electron

For hydrogen-like species:
$v_n = Z\alpha c/n$
Therefore:
$v \propto Z/n$
For Same orbit: Higher Z → higher velocity.
For Same atom: Higher n → lower velocity.

4.20 Example – Velocity Comparison

Compare the electron velocities in the first orbit of H and He^+.
For H:

$$v(H) = \alpha c$$

For He^+:

$$v(He^+) = 2\alpha c$$

Therefore:

$$v(He^+)/v(H) = 2$$

So the electron in the first orbit of He^+ moves twice as fast as in hydrogen according to the Bohr model.

4.21 Angular Momentum

Bohr's quantisation condition:

$$\mathbf{L = mvr = nh/2\pi}$$

Therefore: $Ln = nh/2\pi$
For: n = 1: $L_1 = h/2\pi$
For: n = 2: $L_2 = h/\pi$
Therefore: $Ln \propto n$

Important

Angular momentum does **not** depend directly on Z in this expression.

4.22 Frequency of Revolution

The orbital frequency of the electron is: f(orbit) = v/(2πr)
Using:

$$v_n = Z\alpha c/n$$

and:

$$r_n = a_0 n^2/Z$$

We obtain:

$$f(orbit) \propto Z^2/n^3$$

This is mainly a **JEE-level extension**.

4.23 Spectral Series Comparison

For the same series transition in hydrogen-like species: $1/\lambda \propto Z^2$
Therefore:

$$\lambda \propto 1/Z^2$$

Example

If hydrogen emits: λ(H) = 656 nm for a particular transition, then the same transition in He^+ would have: $\lambda(He^+) = 656/4 \approx 164$ nm
This falls in the ultraviolet region.

4.24 Series Limit and Longest Wavelength

For a given series:

Longest wavelength; smallest energy difference
Therefore: $n_2 = n_1 + 1$

Shortest wavelength: Largest energy difference
Therefore: $n_2 \rightarrow \infty$
This distinction is frequently tested.

4.25 Solved Numerical

Example-1. Longest Wavelength in Balmer Series

Balmer series has: $n_1 = 2$
The first transition is: $n_2 = 3 \rightarrow n_1 = 2$
Therefore, this gives the longest wavelength of the Balmer series.
It corresponds to approximately: $\lambda \approx 656$ nm

Example-2. Shortest Wavelength in Balmer Series

For the series limit: $n_2 \rightarrow \infty$
Therefore: $1/\lambda min = R(H)/4$
Thus: $\lambda min \approx 364.6$ nm

4.26 Spectral Lines and Energy-Level Transitions

Every spectral line corresponds to a particular energy difference.
Therefore:
Different transition → different ΔE
Different ΔE → different frequency
Different frequency → different wavelength
Thus:
Different wavelength → different spectral lines

4.27 Number of Possible Spectral Lines

If an electron is excited to the nth level and can make all possible downward transitions:

$N = n(n-1)/2$

Example

$n = 5$
$N = 5 \times 4/2 = 10$
Therefore, maximum:**10 spectral lines** can be produced.

4.28 Number of Lines in a Particular Series

Suppose the highest occupied level is n = 5.

Lyman series: Final level = 1
Possible:
$2 \rightarrow 1$
$3 \rightarrow 1$
$4 \rightarrow 1$
$5 \rightarrow 1$
Therefore: 4 lines

Balmer series: Final level = 2
Possible:
$3 \rightarrow 2$
$4 \rightarrow 2$
$5 \rightarrow 2$
Therefore: 3 lines

Paschen series: Final level = 3
Possible:
$4 \rightarrow 3$
$5 \rightarrow 3$
Therefore: 2 lines

4.29 Important Numerical Strategy

For any spectral numerical, follow these steps:

Step 1: Identify initial and final levels.
Step 2: Write: $E_n = -13.6Z^2/n^2$ eV
Step 3: Find: $\Delta E = E(\text{high}) - E(\text{low})$
Step 4: If wavelength is required: $\lambda(\text{nm}) = 1240/\Delta E(\text{eV})$
Step 5: If frequency is required: $f = \Delta E/h$
This method is often faster than directly using the Rydberg formula.

4.30 Solved Numerical

Example 1 – He^+ Transition

Question

Calculate the wavelength emitted when an electron in He^+ moves from n = 3 to n = 2.

Solution: For He^+: Z = 2
Energy difference: $\Delta E = 13.6 \times 2^2 [1/2^2 - 1/3^2] = 54.4 [1/4 - 1/9] = 54.4 \times 5/36 \approx 7.56$ eV
Therefore: $\lambda = 1240/7.56 \approx 164$ nm
Answer: $\lambda \approx 164$ nm

Example 2 – Ionisation from n = 2

Question

Find the ionisation energy of hydrogen when its electron is initially in n = 2.

Solution: Using:

$$E_{ion} = 13.6/n^2$$

For: n = 2

$$E_{ion} = 13.6/4 = 3.4\ eV$$

Answer: Ionisation energy = 3.4 eV

Example 3 – Excitation of He^+

Question

What minimum energy is required to excite He^+ from n = 1 to n = 2?

Solution: For He^+: Z = 2
$\Delta E = 13.6Z^2 [1 - 1/4] = 13.6 \times 4 \times ¾ = 40.8$ eV
Answer: Excitation energy = 40.8 eV

Example 4 – Comparing Wavelengths

Question

The same transition occurs in H and He^+. If the wavelength emitted by H is 600 nm, find the wavelength emitted by He^+.

Solution: For the same transition: $\lambda \propto 1/Z^2$

For H: $Z_H = 1$

For He^+: $Z_{He} = 2$

Therefore: $\lambda_{He} / \lambda_H = ¼ = 600/4 = 150$ nm

Answer: $\lambda(He^+) = 150$ nm

Example 5 – Maximum Number of Lines

Question

An atom is excited to n = 6. What is the maximum number of spectral lines possible when the electron returns to the ground state through all possible transitions?

Solution: $N = n(n-1)/2 = 6 \times 5/2 = 15$

Answer: Maximum number of spectral lines = 15

Part 4 Memory Line

"For a hydrogen-like species, increasing nuclear charge makes the electron more tightly bound, decreases the orbit radius, increases the energy-level separation, and decreases the wavelength of radiation for the same transition."

PART 5 – BOHR MODEL: LIMITATIONS, SPECTRAL ANALYSIS & ADVANCED APPLICATIONS

5.1 Introduction

Bohr's model successfully explained:

- Stability of hydrogen atom
- Quantised energy levels
- Hydrogen line spectrum
- Radius of allowed orbits
- Energy of electron
- Ionisation energy

However, the model has limitations, especially when applied to more complex atoms and detailed spectral observations.

In this part we will study:

1. Successes and limitations of Bohr's model
2. Spectral-line analysis
3. Excitation and ionisation problems
4. Hydrogen-like ions
5. de Broglie interpretation of Bohr's quantisation
6. Important competitive-exam applications

5.2 Successes of Bohr's Atomic Model

Bohr's model was a major advancement because it explained several experimental observations.

Success 1: Atomic Stability

Electrons in stationary orbits do not continuously radiate energy. Therefore, the electron does not spiral into the nucleus.

Success 2: Hydrogen Spectrum

The model gives: $E_n = -13.6/n^2$ eV

From this, the observed spectral wavelengths can be explained.

Success 3: Quantised Energy Levels

Only certain values of energy are permitted.

Thus: E_1, E_2, E_3, ... are discrete.

Success 4: Ionisation Energy

For ground-state hydrogen: Eion = 13.6 eV which agrees with the observed value.

Success 5: Hydrogen-Like Ions

The model also gives useful results for one-electron systems such as: He^+ ; Li^{2+} ; Be^{3+}

5.3 Limitations of Bohr's Model

Despite its successes, Bohr's model cannot completely describe atomic structure.

Limitation 1: Multi-Electron Atoms

Bohr's model works well for hydrogen-like systems having one electron.
It does not adequately explain the spectra of atoms containing many electrons.
For example: Helium, Lithium, Carbon, Oxygen require more sophisticated quantum-mechanical treatment.

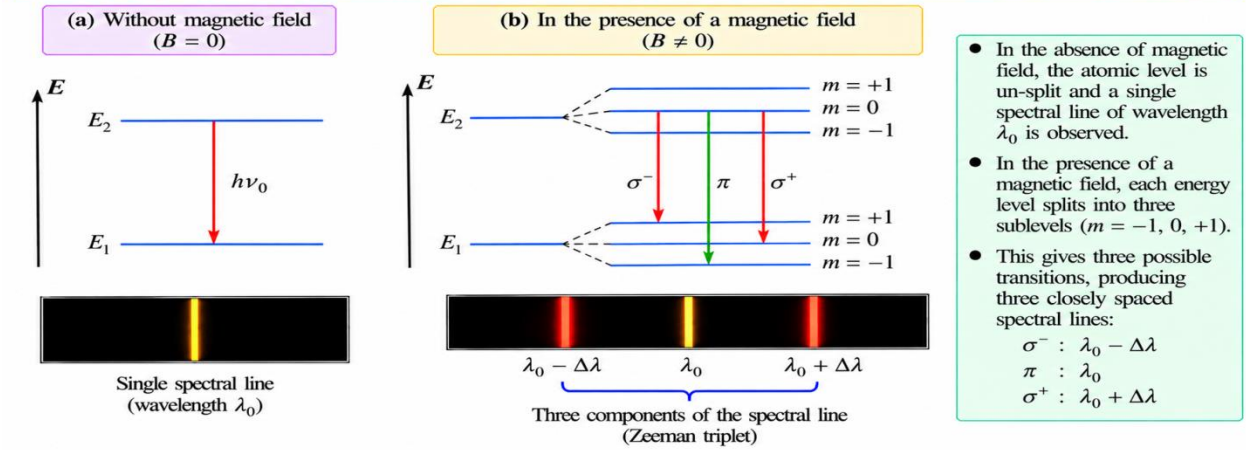
Limitation 2: Fine Structure of Spectral Lines

Some spectral lines that appear as a single line at low resolution are actually composed of closely spaced lines. This is called: **Fine structure**
Bohr's simple model does not fully explain this structure.

Limitation 3: Zeeman Effect

When an atom is placed in a magnetic field, spectral lines may split into several components.
This phenomenon is called: **Zeeman effect**
The simple Bohr model cannot fully explain the observed splitting.

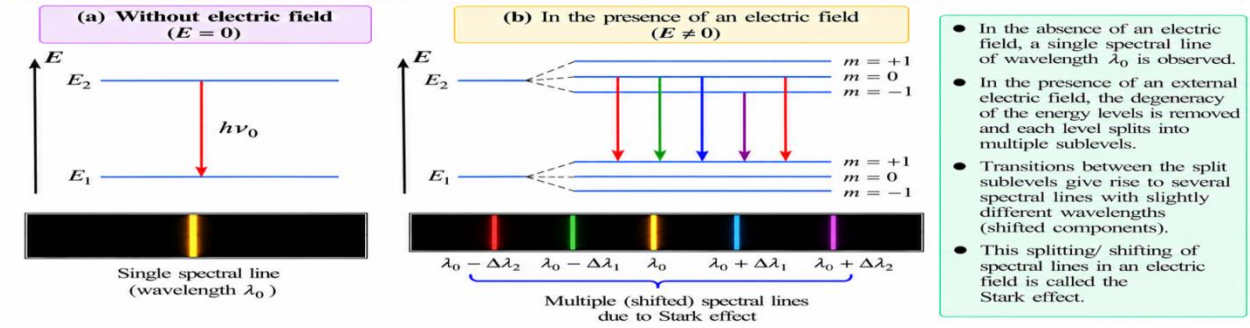
Figure 12.12 : Zeeman splitting of a spectral line



5.4 Stark Effect

When an atom is placed in an external electric field, its spectral lines may split or shift.
This phenomenon is called: **Stark effect**
The simple Bohr model cannot adequately explain the detailed Stark effect.

Figure 12.13 : Stark effect – splitting/ shifting of spectral lines



5.5 Limitation 4: Intensity of Spectral Lines

Bohr's model can predict the allowed energies and hence spectral frequencies, but it does not adequately explain the relative intensities of different spectral lines.

5.6 Limitation 5: Selection Rules

The detailed rules governing which transitions are strongly allowed or forbidden require quantum mechanics.
Bohr's model does not provide a complete explanation of these selection rules.

5.7 Limitation 6: Electron Wave Nature

Bohr described the electron as moving in a definite circular orbit.
Modern quantum mechanics does not describe an electron as following a precise classical orbit around the nucleus.
Instead, the electron is described by a quantum state or orbital.
This is beyond the basic Bohr model.

5.8 Bohr Quantisation and de Broglie Waves

An important connection can be made between Bohr's quantisation condition and de Broglie's matter-wave hypothesis.
Bohr condition:

$$mvr = nh/2\pi$$

Since:

$$p = mv$$

We can write:

$$pr = nh/2\pi$$

Therefore:

$$p(2\pi r) = nh$$

But:

$$\lambda = h/p$$

Therefore:

$$2\pi r = n\lambda$$

This means that the circumference of the allowed orbit contains an integral number of de Broglie wavelengths.

5.9 Standing-Wave Interpretation

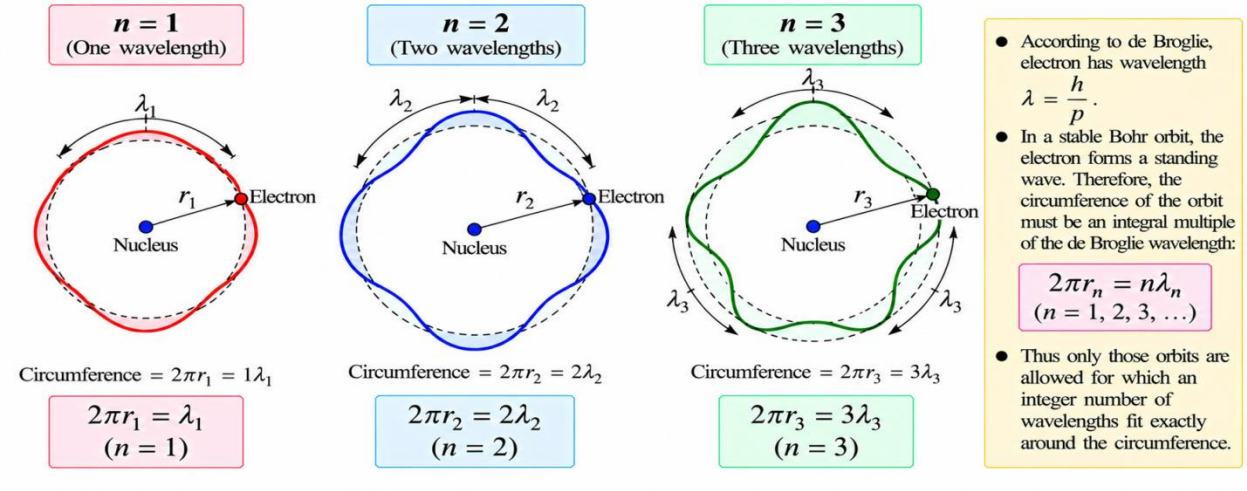
The condition: $2\pi r = n\lambda$ means that the electron wave forms a stable standing wave around the orbit.
For n = 1: $2\pi r = \lambda$
For n = 2: $2\pi r = 2\lambda$
For n = 3: $2\pi r = 3\lambda$ and so on.
Only those orbits are allowed in which the electron wave fits exactly around the circumference.

5.10 Why Arbitrary Orbits Are Not Allowed

Suppose: $2\pi r \neq n\lambda$

Then the matter wave does not join smoothly after completing one revolution.
The wave would not reinforce itself consistently.
Therefore, a stable standing wave is not formed.
Hence, only: $2\pi r = n\lambda$ gives stable allowed states.
This provides a wave-based interpretation of Bohr's quantisation.

Figure 12.14 : de Broglie standing wave interpretation of Bohr orbits



5.11 Derivation of Bohr Quantisation from de Broglie Hypothesis

Starting with:

$$\lambda = h/mv$$

For a stable circular orbit:

$$2\pi r = n\lambda$$

Substitute: $\lambda = h/mv$
Therefore:

$$2\pi r = nh/mv$$

Multiplying by mv: $2\pi mvr = nh$
Hence:

$$\boldsymbol{mvr = nh/2\pi}$$

This is exactly Bohr's angular-momentum quantisation condition.

Important Concept

Bohr's quantisation condition can be interpreted as a standing-wave condition for the electron matter wave.

5.12 Spectral Analysis – A Systematic Method

For any hydrogen or hydrogen-like spectral problem:

Step 1: Identify the atomic number Z.
Step 2: Identify initial and final energy levels.
Step 3: Use: $E_n = -13.6Z^2/n^2$ eV
Step 4: Find energy difference: $\Delta E = E(\text{high}) - E(\text{low})$
Step 5: Use: $E_{photon} = \Delta E$

Step 6: If wavelength is required: $\lambda(nm) = 1240/\Delta E(eV)$
Step 7: If frequency is required: $f = \Delta E/h$

5.13 Solved Numerical

Example 1 – Finding Frequency

Question

An electron in hydrogen falls from n = 3 to n = 2. Find the frequency of emitted radiation.

Solution

Energy difference: $\Delta E = 13.6\ [1/2^2 - 1/3^2] = 13.6 \times 5/36 \approx 1.89$ eV
Convert to joule: $\Delta E \approx 1.89 \times 1.602 \times 10^{-19}$ J $\approx 3.03 \times 10^{-19}$ J
Using: $f = \Delta E/h$ Or $f \approx (3.03 \times 10^{-19}) / (6.626 \times 10^{-34})$
Therefore: $f \approx 4.57 \times 10^{14}$ Hz

Answer: $f \approx 4.57 \times 10^{14}$ Hz

Example 2 – Excitation by Photon

Question

Can a photon of energy 10 eV excite a ground-state hydrogen atom to n = 2?

Solution

Energy required for: $n = 1 \rightarrow n = 2$ is: $\Delta E = 10.2$ eV
Available photon energy: 10 eV
Since: 10 eV < 10.2 eV the photon does not have the required energy.

Answer

No, the transition cannot occur through absorption of this photon.

Important Concept

Atomic absorption is quantised.

Example 3 – Ionisation by Photon

Question

Can a 15 eV photon ionise ground-state hydrogen?

Solution

Ground-state ionisation energy: Eion = 13.6 eV
Incident photon energy: E = 15 eV
Since: 15 > 13.6 ionisation occurs.
The excess energy becomes kinetic energy of the emitted electron: Kmax = 15 − 13.6 = 1.4 eV

Answer:

Ionisation occurs and the electron receives 1.4 eV kinetic energy.

Example 4 – Ionisation from an Excited State

Question

A hydrogen atom is initially in n = 2. A photon of energy 5 eV is incident on it. Can the atom be ionised?

Solution

Ionisation energy from n = 2: Eion = 13.6/4 = 3.4 eV

Incident photon energy: 5 eV
Since: 5 > 3.4 ionisation occurs
Excess energy: K = 5 − 3.4 = 1.6 eV

Answer:

Yes. The emitted electron has 1.6 eV kinetic energy.

Example 5 – Finding the Final Orbit

Question

A hydrogen atom in n = 3 emits a photon of energy 1.89 eV. To which level does the electron transition?

Solution

Energy at n = 3: $E_3 \approx -1.51$ eV
After emission: E(final) = $E_3 - 1.89 \approx -3.40$ eV
For hydrogen: $E_2 = -3.40$ eV
Therefore:

Answer: The electron transitions from n = 3 to n = 2.

5.14 Photon Absorption and Excitation

Suppose a hydrogen atom is initially in: n = 1
It can absorb a photon only if the photon energy equals an allowed energy difference.
For example: $E_2 - E_1 = 10.2$ eV
Therefore a photon of exactly: 10.2 eV can excite the atom: n = 1 → n = 2
Similarly: n = 1 → n = 3 requires: 12.09 eV approximately.

5.15 Ionisation vs Excitation – Numerical Difference

For hydrogen ground state:
First excitation energy: 10.2 eV
Ionisation energy: 13.6 eV
Therefore: Ionisation energy − first excitation energy = 13.6 − 10.2 = 3.4 eV
This 3.4 eV is the energy required to take the electron from: n = 2 → ∞

5.16 Spectral Series and Energy Regions

Series	Final level	Main region
Lyman	1	UV
Balmer	2	Visible
Paschen	3	IR
Brackett	4	IR
Pfund	5	IR

Important

The location of a spectral series is determined by the energy difference associated with transitions.

5.17 Series Limit Interpretation

As: $n_2 \to \infty$ the upper energy level approaches: E = 0

Therefore, the series approaches a limiting frequency.
At this point the electron is approaching the ionisation limit.
Thus: **Series limit is closely connected with ionisation.**

5.18 Frequency at Series Limit

For hydrogen: E(limit) = $13.6/n_1^2$ eV
Therefore: f(limit) = E(limit) / h
For Lyman series: $n_1 = 1$; f(limit) corresponds to the ionisation energy from the ground state.
For Balmer series: $n_1 = 2$; f(limit) corresponds to ionisation from n = 2.

5.19 Wavelength at Series Limit

Since: $\lambda = c/f$ the series-limit wavelength is the shortest wavelength of that series.
Therefore: **Series limit → maximum frequency → minimum wavelength**

5.20 JEE-Level Concept: Same Wavelength

Suppose two hydrogen-like species emit the same wavelength for the same transition.
Since: $1/\lambda \propto Z^2$ the same wavelength for the same n_1 and n_2 requires: $Z_1 = Z_2$
Therefore, two different nuclear charges cannot give the same wavelength for exactly the same transition in the simple hydrogen-like model.

5.21 JEE-Level Concept: Same Energy Difference

For two hydrogen-like species: $\Delta E \propto Z^2$
If the energy difference is the same for the same transition: $Z_1^2 = Z_2^2$
For positive atomic numbers: $Z_1 = Z_2$

5.22 JEE-Level Concept: Same Wavelength with Different Transitions

If two different hydrogen-like systems produce the same wavelength, then:
$Z_1^2 [1/n_1^2 - 1/n_2^2] = Z_2^2 [1/n_3^2 - 1/n_4^2]$
This type of relation can be used in higher-level numerical problems.

5.23 Important Graph: Energy vs Principal Quantum Number

For hydrogen: $E_n = -13.6/n^2$
As n increases: Energy becomes less negative.
The levels become closer together.
As: $n \to \infty$; $E \to 0$
Where:
n = principal quantum number;
E_n = nth orbit total energy

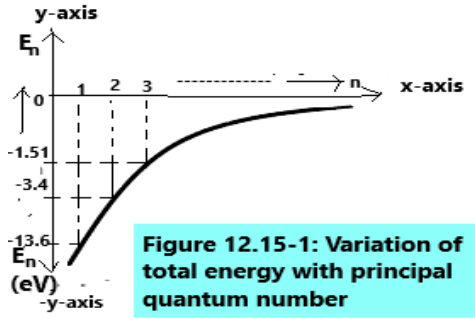


Figure 12.15-1: Variation of total energy with principal quantum number

5.24 Important Graph: Radius vs Principal Quantum Number

For hydrogen: $r_n = a_0 n^2$

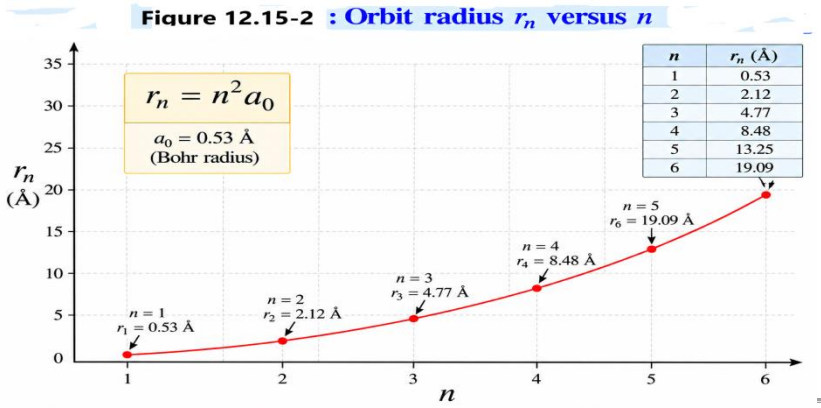
Therefore: $r_n \propto n^2$

As n increases, orbit radius increases rapidly.

Figure to Insert

Figure 12.21: Orbit radius r_n versus n

Show a quadratic increasing curve.



5.25 Important Graph: Energy vs 1/n²

From: $E_n = -13.6/n^2$

We can write: $E_n = -13.6 \times (1/n^2)$

Thus, a graph of En versus $1/n^2$ is a straight line.

Slope: −13.6 eV

This is a useful mathematical interpretation.

n	$1/n^2$	$E_n = -13.6\ eV/n^2$
0	∞	∞
1	1	-13.6
2	0.25	-3.4
3	0.11	-1.51
4	0.06	-0.85
∞	0	0

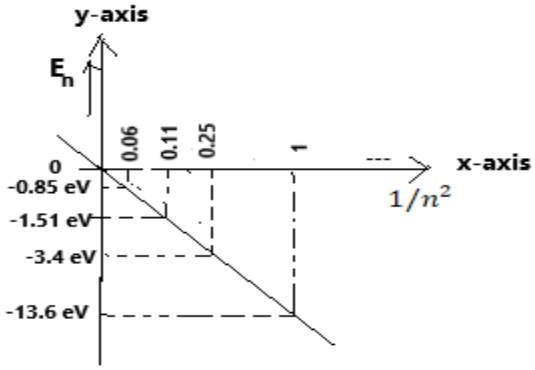


Figure 12.16: Variation of E with $1/n^2$

Part 5 Memory Line

"Bohr's model explains the main energy structure of hydrogen, while de Broglie's wave interpretation explains why only certain circular states can be stable."